The Elements

The human mind is wholly linguistic by function, therefore, the *elements* of human *psychology* are *logic* and *analogic*—the two fundamental branches of reasoning. The start of psychological training starts with learning one logic and one analogic paired in a formal presentation.

The Elements, as a presentation, is a tool for early childhood development. It provides *psychological* tools which will be used for life. Common grammar encompasses the linguistic requirement for a proficiency in *logic*, while geometry encompasses a linguistic requirement for a proficiency in *analogic*.

The pattern, as noted in the material below, is *perception determines conception*, *conception determines will*, *will is determined by our definition* as an environmental acquisition system—whose purpose is the production of those behaviors which maintain and promote the life of the body—which is our defined job and a job common to every member of the species called *man*. *Virtue* is, then, *doing our own work* through the artifice of language.

As every environmental acquisition system of a living organism has an ideal diet, *The Elements* is one of the staples for the environmental acquisition system called *mind*.

Perception

> "Everything which has a function exists for its function." *On the Heavens*, by Aristotle, W.D. Ross.

It is common to divide the human body in terms of *senses*. It is a time honored tradition to do so in many genera of writing—from poetry to science. This method of division is not based upon the first principle of why they are. Also, this method is presented by *enumeration* which leaves open just exactly how many senses we have. Therefore I will divide the human body based upon the first principle of purpose, and the definition itself will determine class inclusion and class exclusion.

Environmental Acquisition Systems

Every living organism survives by crafting things it needs for survival from its environment. Even the act of feeding is, itself, a crafting act. Judgment is also a crafting act.

Definition: An environmental acquisition system of a living organism is that system of an organism which must acquire from the environment an element from some thing and process that element which it has acquired for a product that maintains and promotes the life of that organism.

Those Systems that Acquire Material.

1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

Those Systems that Acquire Form.

4) The Ocular-System.
5) The Vestibular-System.
6) The Procreative-System.
7) The Judgmental-System.

I have presorted the members of the class environmental acquisition systems for a particular reason.

1) We acquire food to survive.
2) We acquire materials to create the necessities of survival.
3) We acquire oxygen for survival.
4) We acquire ocular intelligence of our environment for survival.
5) We acquire gravitational intelligence for our survival.
6) We acquire a mate for our survival.
7) The human mind acquires memory and from that memory abstracts forms of behavior it then applies to the body which is called our behavior. Memory is a recursive function which implies a notion of time. A basic recursive function may be called a temporal map—the planning of future actions. The human mind then acquires behavior for survival. Survival is then determined by behavioral practices, by an individual or a society of individuals.

If one believe that the human mind is wholly linguistic by function, it then becomes clear that the fittest are those behaviors derived from the principles of language itself. Language is derived from perception of the environment, survival is then determined by standards in human behavior called Law. This Law is not the product of neither man nor gods, but as a product of perception.

Standards in behavior called language, derived from perception, is called Law.

Conception

The recursive function of the mind, the notion of time, and a basic temporal map are all fundamental to a functional psychology.

Behavior is the product of a mind, and therefore it has to learn that behavior from past experience. Experiences, as pointed out in the last chapter it gathers not only from the environment, but also with its own body.

In regard to pleasure and pain, it suffers both in order to learn commensurate behaviors in regard to both. Basic behavior is acquisition and avoidance. It must learn when to embrace *each* and when to avoid *each* as a function of the *results* of each *over time*. One of the greatest reason for a great deal of the Judeo-Christian Scripture being involved with prophecy is the importance of the ability to render behavior over time. It is a fundamental requirement of awareness.

One can then realize that a functional psychology has the ability to construct, and reconstruct, temporal maps and it functions best with at least a basic map. This basic map is often afforded one through enculturation. A Universal basic map should be the form of life itself—the basic form of human behavior. A perfect map clearly has the beginning, the middle and the end envisioned. One may call this Alpha and Omega, or simply God—a being he is working to be like. A work he will do his entire life.

An essential to psychology is a well founded trust in the tools by which the mind accomplishes its task, his only tool is language. With a simple one-to-one correspondence with perceptions, one can build standards in language. This one-to-one correspondence can be called the image, or truth. This is how one can say that God is Truth and that Man is created in the image of God. The belief in the efficacy of language to perform our job, to produce human will, which is the only thing a mind can do—its only power, is a requisite psychological advantage. So God, in a metaphor, is Truth and the Almighty. This notion is derivable itself from the *Two-Element Metaphysics*. A dysfunctional psychology, and one that must become parasitic, behaves in such a manner as to formulate this lack of confidence in the tools of human craft. One is actually attempting to impose their own neurotic image upon others. This is a psychological inversion.

The individuals ability to comprehend this temporal map will depend upon how developed that mind is. In the elementary stages of psychology, one sees this map anthropomorphically. It is unavoidable.

It is also clear that a social structure that does not have a map, much less a well defined map, is counterproductive to the population. Only a fool would promote the idea of the separation of religion and state and only a fool would embrace a division of power in church and state.

An ideal social structure, then, has a state supported language center which maintains and promotes both branches of reasoning, logic and analogic, a state supported and recognized religion, and a clear conception that its function is to produce a virtuous population—standards in human behavior. It must also be aware that it cannot afford to be liberal in ignoring these fundamentals nor illiberal in their support. The purpose of the state is to produce, within the individual, a functional psychology as every state is the sum of its individuals.

Fundamental to a good social order is that it is based on human psychology itself. Since reason is the source of power, a good social order is not the respecter of persons, all judgment is derived from the principles of language—both branches, or in a metaphor, the Two Stone Tablets, or again, as the Two Witnesses of God. Thus judgment is not the whim of any so-called half baked judge, but judgment is of the Lord—the principles of language itself—the principles of judgment. It is also founded on the notion that its purpose is to develop judgment in the population, not to rest it from them. The idea is to fully express the functionality of mankind as derived from his own definition. Therefore, responsibility for ones own actions are to be encouraged instead of trying to find the deepest pocket to loot—nor in the trafficking in human life and slavery as is common even in the United States.

What one is aiming for, in the beginning is the transition from an anthropomorphic conceptualization of judgment, be it man or gods, to one that is independent of mankind,—one that is true of reality itself.

Will

The human mind is responsible for the production of human behavior in order to maintain and promote life. In order to do this, it must standardize behavioral sets in order to construct language. This is the master image that resides over man, behavior begets behavior. Identity is what is called *a closed system*. Intelligence is the active awareness that one is responsible for behavior and thus learning standards of behavior in order to do its own work. As the Judeo-Christian Scripture intimates, the image is God. A equals A. I am that I am. The Law of Reciprocity.

Therefore, it is our own work to either learn language or if mankind's understanding is insufficient, to teach the principles of language. By our own biological construction and the definition of any thing this means at least one Logic and one Analogic by which to construct our desired behavior, so that we, as a functional environmental acquisition system of a living organism can do our own work of maintaining and promoting life.

The Elements, in reference to this particular work, targets the Logic known as, common grammar, common arithmetic and algebra. And the Analogic, simple of Euclidean Geometry and a standard set of that for Analog Mathematics.

These two appellations, *Logic* and *Analogic*, are the *Form* of *Law*. *Law* was enumerated when it was first given, as enumeration is how children learn. When the human race matures sufficiently, it will embrace the *Law* again, this time in its form, or *The Spirit of Truth*.

We learn by example, however we learn by example means that learning is derived from the image, that is perception. As we learn by example, we can be tested to see if we have learned by example. This means that when we become functional we can demonstrate that we have learned by example. A test can be constructed, through the artifice of language by which we can demonstrate our own functionality, a functionality which distinguishes us from other animals, be that animal of human stock or not. Such a test has been left for mankind. One of the things that this test demonstrates is the fact that our tests, the test of human will, always reside right in our environment, be that environment close at hand, or over unimaginable distances. Let us examine that test and see what is expected of us as a functional mind.

Truth: An expression of a One-to-One Correspondence.

The Number Of His Name

> "The names of the Bible have been a favorite field for gematry. Most famous is the Number of the Beast, given in the Revelation of St. John (13:18) "Here is wisdom. Let him that has understanding count the number of the beast; for it is the number of a man and his number is six hundred three score and six." In spite of the innumerable researches on this question through the centuries it seems impossible to arrive at any definite solution. Clearly many names will have the same number. In the violent theological feuds of the Reformation it was a

vicious stroke to write the opponent's name in such a way that his number became the fatal 666 of the beast." ***Number Theory and Its History***, O. Ore © 1948

Reasoning is based on standards of behavior called language. Standards of language are based upon a one-to-one correspondence between perception and names, therefore I will start by disregarding gematry, which is not based upon standards of behavior and hold fast to the standard use of letters for the Hebrew numbering system.

Letter Name	Sound	Number value
א Aleph	A,	1.
ב Beth	B,	2.
ג Gimel	G,	3.
ד Daleth	D,	4.
ה He	H,	5.
ו Vav	V,	6.
ז Zayin	Z,	7.
ח Heth	H,	8.
ט Teth	T,	9.
י Yod	Y,	10.
כ Kaph	K,	20.
ל Lamed	L,	30.
מ Mem	M,	40.
נ Nun	N,	50.
ס Samekh	S,	60.
ע Ayin	',	70.
פ Pe	P,	80.
צ Tsade	Ts,	90.
ק Qoph	Q,	100.
ר Resh	R,	200.
ש Shin	Sh,	300.
ת Tav	T,	400.

Numerology chart found in ***From One to Zero***. by George Ifrah.

This produces the number 666.

ת Tav	T,	400.
ר Resh	R,	200.
ס Samekh	S,	60.
ו Vav	V,	6.

Under the standard numbering system, it is the only possible result.

400, 200, 60, 6. TRSV.

There is no word listed in the Hebrew Dictionary that I have. In fact, T is an ending of a word. The construction itself, that "the first shall be last and the last shall be first" is telling me the order of the letters. Every name is constructed by a set of letters, upon which order is imposed to make a particular name. This is part of the phonetic place value notational system of common grammar. This is one time that one is told what order the letters go in. Let us look at another.

> Revelation 13:2 And the beast which I saw was like unto a leopard, and his feet were as the feet of a bear, and his mouth as the mouth of a lion: and the dragon gave him his power, and his seat, and great authority.

Who described themselves in the Book using all three images?

> Hosea 13:7-8. Therefore I will be unto them as a lion; as a leopard by the way will I observe them: I will meet them as a bear that is bereaved of her whelps, and will rend the caul of their heart, and there will I devour them like a lion; the wild beast shall tear them.

The beast is described as God, but the text reveals that it is the name of a man.

> Revelation 12:18 for it is the number of a man;

What is the relationship of God to man given in the scripture?

> Genesis 1:27 So God created man in his own ***image***, in the ***image*** of God created he him; male and female created he them.

Memory of past experience is a fundamental part of language. For a second time one is being told to turn the letters around. Another statement is given here:—

> Revelation 13:14 … "that they should make an ***image*** to the beast…"

Language is based upon the image. This is the third time one is told to turn the letters around.

> Revelation 13:18 "Let him that has understanding ***count*** the number"

Standards in a language system produce a standard result. In other words, put the numbers in counting sequence—a simple arithmetic ***convention*** of counting. This is the fourth time one is told to turn the letters around. Language is based upon *stammering*, i.e., recursion. All of the principles of language are derived from a one-to-one correspondence with perception, that is called an image. In other words, the image, effects truth as a one-to-one correspondence between reality and perception—so too does language. One is being told that the solution to the puzzle of judgment resides in the image, i.e., Truth.

I will simply reverse the order of the letters, 6, 60, 200, 400, or VSRT. Now what Hebrew word does "VSRT" make? "To shutter" (VSR) with the conversive ending (T, turning the past into the future and the future into the past.) Found in the dictionary written by R. Alcalay. We have been introduced to the *shutter* before in the text, here again, we are being referred to past experience;

> And the key of the house of David will I lay upon his shoulder; so he shall open, and none shall shut; and he shall shut, and none shall open.

The key is the image of God, the image is Truth. What does it mean "to shutter" something? In Scripture, they are a heavily used metaphor. Shutters, gates, doors, windows,—those things which regulate the coming and going of things—i.e. regulate behavior. To shutter something means to regulate its behavior. Language depends upon definition and the sum of definitions.

Taking the word back to the constituent definition and adding the conversive metaphor to it, what do we have?—I will show the results of this double locked metaphor.

"To regulate one's behavior so as to turn
the past into the future and to bring the future to pass."

This is a simple biological fact concerning the function of the human mind as one among a group of environmental acquisition systems of the human body. It was locked to man's understanding solely through the use of standards in language. The mind's ability to function alone determines if man is cursed or if man is blessed, it alone determines if man is functional or just another beast. The mind regulates behavior through standards of behavior which are learned by experience in order to maintain and promote our life. Language is the simple establishment of a one-to-one correspondence of perception with symbols for the same purpose of any environmental acquisition system of a living organism. The accomplishment of this product determines if that environmental acquisition is functional and is doing its job. If it is not functional, then it cannot contribute to the body whole the product of life. That organism, by biological definition, is doomed to die. The scriptural metaphors in regard to God, life and death are all derived from biological fact. From this alone one can determine if the source of what appears to be mythology has been in fact generated by a living mind. It proves that whatever men call God, is actually the product of a living organism—not a delusional and dysfunctional mind. It also demonstrates that whatever it is, it has survived a great deal longer than the human species.

Language is not defective, it is the only power a mind has. The mind can either use that power or it cannot. The mind is either functional or it is not.

Thus we study language in order to effect specific behaviors aimed at maintaining and promoting our life. Thus the principles of language are *Law* by which our behavior, as a life form in the Universe, is expressed. This Law, in the division of language called *Logic* and in the division of language called *Analogic* are the Two Stone Tables of God, the Two Pillars before the Promised land of Life. True religion is simply the use of metaphor by which dysfunctional minds may one day achieve functionality. True religion complies with the definition of the mind of man in regard to its definition.

The Two-Element Metaphysics, is the foundation of Law, from which any law claiming to be valid must be derived from. This is the rightful foundational constitution over which a people express their life. Judgment is of the Lord, it is not the dictates of any group of individuals.

Judgment day, a day in man's future, is the day that man learns judgment, in body, mind and soul. It is simply a biological fact.

A Two-Element Metaphysics.

Let us review the domain of behaviors that we, as mind, are responsible for learning and effecting.

Definition 1: An environmental acquisition system of a living organism is that system of an organism which must acquire from the environment an element from some thing and process that element which it has acquired for a product that maintains and promotes the life of that organism.

Those Systems that Acquire Material.

1) The Digestive-System.
2) The Manipulative-System.
3) The Respiratory-System.

Those Systems that Acquire Form.

4) The Ocular-System.
5) The Vestibular-System.
6) The Procreative-System.
7) The Judgmental-System.

This denotes that all of reasoning is derived from the definition of a thing. Another way of saying this same thing is that all language is derived from perception—just like every other environmental acquisition system of a living organism. Perception provides every environmental acquisition system with its material. The mind is not different in this respect. We are a beast of seven eyes and seven horns. We have these seven wives, seven mothers of life, or seven whores. Our behavior is done unto them as we are ourselves. In other words, the whore of Babylon, is our own mind. A metaphor meant for children.

Definition 2: *A thing* is any *difference* what-so-ever within any *form*, *shape*, *boundary* or *limit*.

Therefore, by definition, the *elements* of a thing are *form* and *material*. This is how we can say that language is based upon a *Two Element Metaphysics*, or in the simple as *The Elements*.

Some of our environmental acquisition systems abstract form and others abstract the material in a form.

A common notion then is, just like every environmental acquisition system, the distinction between material and form, or in other words, a container and the contained, or in another metaphor, as male and female.

As a thing is any material in any form, in order to have a functional understanding of language as a craft, it is requisite to realize that neither the material of a thing, nor the form of a thing is a thing. The material of a thing is not the thing of which it is the material. Nor is the form of a thing the thing of which it is the form.

We craft from the elements of things in order to make things. The elements of a thing are not the thing of which they are the elements. One has to instill the notion that we are a craftsman, and a craftsman knows the elements of his craft as well as the forms of behavior to effect his craft.

By the definition of a thing, if we start with a form, we must supply some material to that form in order to make some thing. By the definition of a thing, if we start with material we must add form to that material in order to make some thing. The first form of reasoning then, is called Logic. The second is called Analogic. Their principles are the two Tablets of Law. And as both are derived from one and the same thing, they can only say the same thing. This, then, is the two witnesses of God—Logic and Analogic.

The only difference between Logic and Analogic is which element is a given, and which element must be applied or supplied in order to make some thing.

Logic.

Legend: ATEM, *A Two-Element Metaphysics*. EE, *Euclid's Elements*.

Starting with the definition of a thing;

Definition 2[ATEM]: *A thing* is any *difference* what-so-ever within any *form*, *shape*, *boundary* or *limit*.

Logic is that branch of reasoning which has the *element form* as a *given* and the *material* for those forms is *supplied*.

What does this mean?

The most common Logic, is common grammar. Any of the given forms of this language may be set into a one-to-one correspondence with a semi-standard form set derived from an alphabet. As form is not a difference, any of the forms used for common grammar, be it vocal, written, signed, smoke signals, or radio waves are commensurate as forms for the language. The chosen standard form set for common grammar is the written set. This form set is the most capable of standardization.

The written form set for common grammar is composed of a symbol set called an alphabet composed of letters. This set is augmented by a form set called punctuation. The arrangement of letters may be seen as a primitive type of place value notation. This place value notation, however, is geared toward phonetic sounds of the voice. It may then be called a phonetic value notation. It is not perfect, but it is passable. This system has not been standardized and is handed down much as stories and histories have been handed down in primitive cultures. In this regard, Logic is still in a primitive state of development.

Memory is the material which is associated with these forms. Together, memory which is the material difference and the given forms, called words, or names, make words. Thus one can take it that what is supplied in Logic is actually implied in the language. All of the differences associated with the given forms, other than syntax, in Logic, is implied. It is also clear that until there are recognized standards of experience to associate with those forms, failure in communication is a constant byproduct of attempts to reason.

Syntax is standards in the progression of name after name. Syntax, on its surface is linear. Dimensional differences in syntax is implied within this linear string through modifications of the names themselves. This is also true, for example, in arithmetic and algebra.

Names:

What can we name? The ability to name, one may call *assertion* and *denial*. It is derived from the concept of *same* and *difference*, or again, *form* and *material*. This concept is derived from perception. A naming convention proceeds once we realize that we may *assert names* and *deny names*. One can call this the *Law of the Excluded Middle*.

A thing may be called either *A* or it may not be called *A*.

The Law of the Excluded Middle may be seen as a light bulb turning on. It is the recognition that one can standardize a simple behavior—the construction of naming conventions. So, what can we name once we finally realize that we can name?

In this regard we go back to our definition:

Definition 2[ATEM]: *A thing* is any *difference* what-so-ever within any *form*, *shape*, *boundary* or *limit*.

We can name things and we can name each of the two elements of a thing.

Common Notion 1[EE]: *Things equal to the same thing are equal to each other.*

In other words, as applied to logic:

Common Notion 1: *The names of things equated to the same thing are equal to each other.*

Common Notion 1 is a straightforward application of the awareness of *form* and *material*, *same* and *difference*, i.e., perception. A thing does not change because we have created methods of naming it. That we have discovered the simple mental motion of assertion and denial. Naming a thing is a form of behavior, and form is not a difference.

After the standard behavior of an agreed upon symbolic set, be it vocal, written, or whatever, Logic proceeds with the convention of establishing a one-to-one correspondence between a symbolic naming convention to the memory of abstractions. Failure to maintain this convention means that language is not possible. Language is only functional to the degree that this convention is maintained. It follows that human

behavior is only functional to the degree this convention is maintained. It then becomes clear that what men call legal systems, systems aimed at the destruction of conventions of language are corrupted. The naming convention is a standard of agreed upon behavior.

Claims that language is wholly conventional and thus not valid, is only the declaration that one is refusing to comply with standards of behavior that make language possible. This refusal may be deliberate, or it may stem from ignorance, or both, the desire to remain ignorant.

By definition, we can then see that logic consists in two distinct naming conventions. Naming a thing directly and naming a thing as the sum of the names of that things elements.

Let us call the convention which names a thing as a whole, the *Subject Naming Convention—Subjects* for short.

Let us call the convention which names a thing by concatenating the names of that things elements, the *Predicate Naming Convention Predicates* for short.

Maintaining the conventions is simply be denoting their equality.

Definition: Definition is the preservation of the social convention by which we record, recognize, and employ the equality between the name of a thing, *Subjects*, and the names of that things materials and the names of that things forms which contain those materials, *Predicates*.

Assertion an *denial* is a recursive function. Which means we can simply use it again. We have two distinct naming conventions and we can assert and deny their equality the same as when we asserted and denied the original names to what they originally denoted. Thus there are three basic assertive sentence units.

We can assert that a Subject is equal to a Subject.

Mary is laughing.

We can assert that a Subject is equal to a Predicate.

Mary is a girl.

We can assert that a Predicate is equal to a Predicate.

That girl is my friend.

There are also three basic denial sentence units. One must realize that denial can be a simple *not* or it can name the actual difference. The most respected logicians admire the wood-pecker with there recursive not, not, not, etc, but I am not an ornithologist.

Subject is not equal to Subject.

Mary is up there!

Subject is not equal to Predicate.

Mary has a lamb.

Predicate is not equal to Predicate.

A cat is up that tree.

One of the most elementary methods of adding units of assertion and denial is by enclosing a subject between predicate pairs.

Mary has a little lamb.

And another involves the predication of time by constructing name sets to indicate that single assertion. In this case the simple assertion, time is past.

Mary had a little lamb.

Parsing is the deconstruction of a complex sentence into all of its units of assertion and denials. Grammar is learned and taught by adding and subtracting these units of assertion and denials, just like in common arithmetic. Resolving what was given into units, and constructing compounds with given sets of units.

Let this outline suffice for a temporal map for the construction of formal common grammar. It can be seen by the definition of a thing, and by my demonstration, that the simple parts of grammar are three. I am not part of the enlightenment so I fail to appreciate the Baroque style of grammar expression.

BOOK I.

OF

EUCLID'S ELEMENTS

TRANSLATED FROM THE TEXT OF HEIBERG

BY

SIR THOMAS L. HEATH,

K. C. B., K. C. V. O., F. R. S.,

SC. D. CAMB., HON. D. SC. OXFORD

HONORARY FELLOW (SOMETIME FELLOW) OF TRINITY COLLEGE CAMBRIDGE

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REFORMATTED AND ABRIDGED
FOR STUDY OF
***THE ELEMENTS*.**
BY JOHN CLARK.

BOOK I.

DEFINITIONS.

1. A **POINT** IS THAT WHICH HAS NO PART.
2. A **LINE** IS BREADTHLESS LENGTH.
3. THE EXTREMITIES OF A LINE ARE **POINTS**.
4. A **STRAIGHT LINE** IS A LINE WHICH LIES EVENLY WITH THE POINTS ON ITSELF.
5. A **SURFACE** IS THAT WHICH HAS LENGTH AND BREADTH ONLY.
6. THE EXTREMITIES OF A SURFACE ARE **LINES**.
7. A **PLANE SURFACE** IS A SURFACE WHICH LIES EVENLY WITH THE STRAIGHT LINES ON ITSELF.
8. A **PLANE ANGLE** IS THE INCLINATION TO ONE ANOTHER OF TWO LINES IN A PLANE WHICH MEET ONE ANOTHER AND DO NOT LIE IN A STRAIGHT LINE.
9. AND WHEN THE LINES CONTAINING THE ANGLE ARE STRAIGHT, THE ANGLE IS CALLED **RECTILINEAL**.
10. WHEN A STRAIGHT LINE SET UP ON A STRAIGHT LINE MAKES THE ADJACENT ANGLES EQUAL, TO ONE ANOTHER, EACH, OF THE EQUAL ANGLES IS **RIGHT**, AND THE STRAIGHT LINE STANDING ON THE OTHER IS CALLED A **PERPENDICULAR** TO THAT ON WHICH IT STANDS.
11. AN **OBTUSE ANGLE** IS AN ANGLE GREATER THAN A RIGHT ANGLE.
12. AN **ACUTE ANGLE** IS AN ANGLE LESS THAN A RIGHT ANGLE.
13. A **BOUNDARY** IS THAT WHICH IS AN EXTREMITY OF ANYTHING.
14. A **FIGURE** IS THAT WHICH IS CONTAINED BY ANY BOUNDARY OR BOUNDARIES.
15. A **CIRCLE** IS A PLANE FIGURE CONTAINED BY ONE LINE SUCH THAT ALL THE STRAIGHT LINES FALLING UPON IT FROM ONE POINT AMONG THOSE LYING WITHIN THE FIGURE ARE EQUAL, TO ONE ANOTHER;
16. AND THE POINT IS CALLED THE **CENTRE** OF THE CIRCLE.
17. A **DIAMETER** OF THE CIRCLE IS ANY STRAIGHT LINE DRAWN THROUGH THE CENTRE AND TERMINATED IN BOTH DIRECTIONS BY THE CIRCUMFERENCE OF THE CIRCLE, AND SUCH A STRAIGHT LINE, ALSO, BISECTS THE CIRCLE.
18. A **SEMICIRCLE** IS THE FIGURE CONTAINED BY THE DIAMETER AND THE CIRCUMFERENCE CUT OFF BY IT. AND THE CENTRE OF THE SEMICIRCLE IS THE SAME AS THAT OF THE CIRCLE.
19. **RECTILINEAL FIGURES** ARE THOSE WHICH ARE CONTAINED BY STRAIGHT LINES, **TRILATERAL** FIGURES BEING THOSE CONTAINED BY THREE, **QUADRILATERAL** THOSE CONTAINED BY FOUR, AND **MULTILATERAL** THOSE CONTAINED BY MORE THAN FOUR STRAIGHT LINES.

20. Of trilateral figures, an **equilateral triangle** is that which has its three sides equal, an **isosceles triangle** that which has two of its sides alone equal, and a **scalene triangle** that which has its three sides unequal.

21. Further, of trilateral figures, a **right-angled triangle** is that which has a right angle, an **obtuse-angled triangle** that which has an obtuse angle, and an **acute-angled triangle** that which has its three angles acute.

22. Of quadrilateral figures, a **square** is that which is both equilateral and right-angled; an **oblong** that which is right-angled but not equilateral; a **rhombus** that which is equilateral but not right-angled; and a **rhomboid** that which has its opposite sides and angles equal, to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called **trapezia**.

23. **Parallel** straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction.

Postulates.

Let the following be postulated:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any centre and distance.
4. That all right angles are equal, to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Common Notions.

1. Things which are equal, to the same thing are, also, equal, to one another.

2. If equals be added to equals, the wholes are equal.

3. If equals be subtracted from equals, the remainders are equal.

[7] 4. Things which coincide with one another are equal, to one another.

[8] 5. The whole is greater than the part.

Notes.

Definition 1.

A point *is that which has no part*.

Notes.

Definition 2.

A LINE *IS BREADTHLESS LENGTH*.

Notes.

Definition 3.

THE EXTREMITIES OF A LINE ARE POINTS.

Notes.

Definition 4.

A STRAIGHT LINE *is a line which lies evenly with the points on itself*.

Notes.

Definition 5.

A SURFACE *is that which has length and breadth only*.

Notes.

Definition 6.

The extremities of a surface are lines.

Notes.

Definition 7.

A plane *surface is a surface which lies evenly with the straight lines on itself.*

Notes.

Definition 8.

A plane angle *is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line*.

Notes.

Definition 9.

And when the lines containing the angle are straight, *the angle is called* rectilineal.

Notes.

Definition 10.

When a straight line set up on a straight line makes the adjacent angles equal, to one another, each, of the equal angles is RIGHT, *and the straight line standing on the other is called a* PERPENDICULAR *to that on which it stands*.

Notes.

Definition 11.

An OBTUSE ANGLE *is an angle greater than a right angle*.

Notes.

Definition 12.

12. *An* ACUTE ANGLE *is an angle less than a right angle*.

Notes.

Definition 13.

A BOUNDARY *is that which is an extremity of anything*.

Notes.

Definition 14.

A figure *is that which is contained by any boundary or boundaries*.

Notes.

Definition 15.

15. a circle *is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal, to one another*;

Notes.

Definition 16.

16. *And the point is called the* centre *of the circle.*

Notes.

Definition 17.

A diameter *of the circle is any straight line drawn through the centre and terminated in both directions by the circumference of the circle, and such a straight line, also, bisects the circle.*

Notes.

Definition 18.

A semicircle *is the figure contained by the diameter and the circumference cut off by it. And the centre of the semicircle is the same as that of the circle.*

Notes.

Definition 19.

19. Rectilineal figures *are those which are contained by straight lines*, trilateral figures *being those contained by three*, quadrilateral *those contained by four*, and multilateral *those contained by more than four straight lines*.

Notes.

Definition 20.

20. *Of trilateral figures, an* equilateral triangle *is that which has its three sides equal, an* isosceles triangle *that which has two of its sides alone equal, and a* scalene triangle *that which has its three sides unequal.*

Notes.

Definition 21.

21. *Further, of trilateral figures, a* right-angled triangle *is that which has a right angle, an* obtuse-angled triangle *that which has an obtuse angle, and an* acute-angled triangle *that which has its three angles acute.*

NOTES.

DEFINITION 22.

Of quadrilateral figures, a SQUARE *is that which is both equilateral and right-angled*; *an* OBLONG *that which is right-angled but not equilateral*; *a* RHOMBUS *that which is equilateral but not right-angled*; *and a* RHOMBOID *that which has its opposite sides and angles equal, to one another but is neither equilateral nor right-angled. And let quadrilaterals other than these be called* TRAPEZIA.

NOTES.

DEFINITION 23.

PARALLEL *straight lines are straight lines which*, *being in the same plane and being produced indefinitely in both directions*, *do not meet one another in either direction*.

Notes.

Postulate 1.

Let the following be postulated: to draw a straight line from any point to any point.

Notes.

Postulate 2.

To produce a finite straight line continuously in a straight line.

Notes.

Postulate 3.

To describe a circle with any centre and distance.

Notes.

Postulate 4.

That all right angles are equal, to one another.

Notes.

Postulate 5.

That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

Notes.

Common Notion 1.

Things which are equal, to the same thing are, also, equal, to one another.

Notes.

Common Notions 2.

2. *If equals be added to equals, the wholes are equal.*

Notes.

Common Notions 3.

3. *If equals be subtracted from equals, the remainders are equal.*

Notes.

Common Notion 4.

Things which coincide with one another are equal, to one another.

Notes.

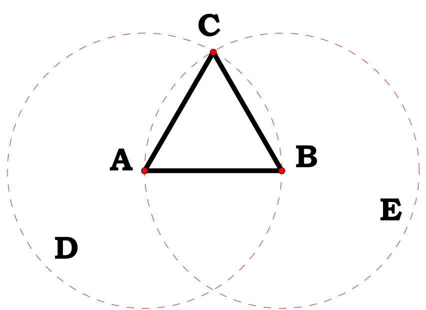
Common Notion 5.

The whole is greater than the part.

BOOK I.

PROPOSITIONS.

PROPOSITION 1.



ON A GIVEN FINITE STRAIGHT LINE TO CONSTRUCT AN EQUILATERAL TRIANGLE.

LET,
AB BE THE GIVEN FINITE STRAIGHT LINE.

THUS IT IS REQUIRED,
TO CONSTRUCT AN EQUILATERAL TRIANGLE ON
THE STRAIGHT LINE, *AB*.

[POST. 3]

WITH,
CENTRE, *A*, AND DISTANCE, *AB*,

LET,
THE CIRCLE, *BCD*, BE DESCRIBED;

[POST. 3]

AGAIN WITH,
CENTRE, *B*, AND DISTANCE, *BA*,

LET,
THE CIRCLE, *ACE*, BE DESCRIBED;

[POST. 1]

AND FROM,
THE POINT, *C*, IN WHICH THE CIRCLES CUT ONE ANOTHER,
TO THE POINTS, *A*, *B*,

LET,
THE STRAIGHT LINES, *CA*, *CB*, BE JOINED.

[DEF. 15]

NOW, SINCE,
THE POINT, *A*, IS THE CENTRE OF THE CIRCLE, *CDB*,
AC IS EQUAL, TO *AB*.

[DEF. 15]

AGAIN SINCE ,
THE POINT, *B*, IS THE CENTRE OF THE CIRCLE, *CAE*,
BC IS EQUAL, TO *BA*.

BUT,
AC WAS, ALSO, PROVED EQUAL, TO *AB*;

[*C. N.* 1]

THEREFORE,
EACH, OF THE STRAIGHT LINES, *AC*, *BC*, IS EQUAL, TO *AB*.

AND,
THINGS WHICH ARE EQUAL, TO THE SAME THING, ARE, ALSO,
EQUAL, TO ONE ANOTHER;

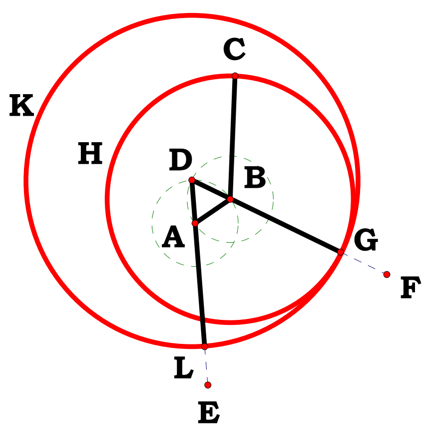
THEREFORE,
CA IS, ALSO, EQUAL, TO *CB*.

THEREFORE,
THE THREE STRAIGHT LINES,
CA, *AB*, *BC*, ARE EQUAL, TO ONE ANOTHER.

THEREFORE,
THE TRIANGLE, *ABC*, IS EQUILATERAL; AND
IT HAS BEEN CONSTRUCTED ON
THE GIVEN FINITE STRAIGHT LINE, *AB*.

(BEING) WHAT IT WAS REQUIRED *TO* DO.

PROPOSITION 2.



TO PLACE, AT A GIVEN POINT (AS AN EXTREMITY), A STRAIGHT LINE EQUAL, TO A GIVEN STRAIGHT LINE.

LET,
A BE THE GIVEN POINT,

AND,
BC, THE GIVEN STRAIGHT LINE.

THUS IT IS REQUIRED,
TO PLACE, AT THE POINT, *A*,
(AS AN EXTREMITY),
A STRAIGHT LINE EQUAL, TO THE GIVEN STRAIGHT LINE, *BC*.

[POST. 1]

FROM,
THE POINT, *A*, TO THE POINT, *B*,

LET,
THE STRAIGHT LINE, *AB*, BE JOINED; AND

[I. 1]

ON IT LET,
THE EQUILATERAL TRIANGLE, *DAB*, BE CONSTRUCTED.

[POST. 2]

LET,
THE STRAIGHT LINES, *AE*, *BF*,
BE PRODUCED IN A STRAIGHT LINE WITH *DA*, *DB*;

[POST. 3]

WITH,
CENTRE, *B*, AND DISTANCE, *BC*,

LET,
THE CIRCLE, *CGH*, BE DESCRIBED;

[POST. 3]

AND AGAIN, WITH,
CENTRE, *D*, AND DISTANCE, *DG*,

LET,
THE CIRCLE, *GKL*, BE DESCRIBED.

THEN, SINCE,
THE POINT, *B*, IS THE CENTRE OF THE CIRCLE, *CGH*,
BC IS EQUAL, TO *BG*.

AGAIN, SINCE,
THE POINT, *D*, IS THE CENTRE OF THE CIRCLE, *GKL*,
DL IS EQUAL, TO *DG*,

AND, IN THESE,
DA IS EQUAL, TO *DB*;

[*C. N.* 3]

THEREFORE,
THE REMAINDER, *AL* IS EQUAL, TO
THE REMAINDER, *BG*,

BUT, ALSO,
BC WAS PROVED EQUAL, TO *BG*;

THEREFORE, EACH, OF,
THE STRAIGHT LINES, *AL*, *BC*, IS EQUAL, TO *BG*.

[*C. N.* 1]

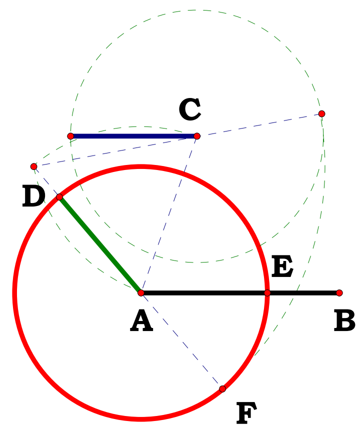
AND,
THINGS WHICH ARE EQUAL,
TO THE SAME THING ARE, ALSO, EQUAL, TO ONE ANOTHER;

THEREFORE,
AL IS, ALSO, EQUAL, TO *BC*.

THEREFORE,
AT THE GIVEN POINT, *A*,
THE STRAIGHT LINE, *AL*, IS PLACED EQUAL, TO
THE GIVEN STRAIGHT LINE, *BC*.

(BEING) WHAT IT WAS REQUIRED TO DO.

Proposition 3.



Given two unequal straight lines, to cut off from the greater, a straight line equal, to the less.

Let,
AB, *C*, be
the two given unequal straight lines,

and let,
AB be the greater of them.

Thus it is required,
to cut off from *AB*, the greater,
a straight line equal, to *C*, the less.

[I. 2]

[Post. 3]

Let,
at the point, *A*,
AD be placed equal, to the straight line, *C*; and
with centre, *A*, and distance, *AD*,

let,
the circle, *DEF*, be described.

[Def. 15]

Now, since,
the point, *A*, is the centre of the circle, *DEF*,
AE is equal, to *AD*.

But,
C is, also, equal, to *AD*.

[*C. N.* 1]

Therefore,
each, of the straight lines, *AE*, *C*, is equal, to *AD*;

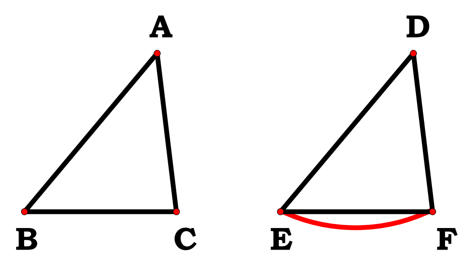
so that,
AE is, also, equal, to *C*.

Therefore,
given the two straight lines *AB*, *C*,

from,
AB, the greater,
AE has been cut off equal, to *C*, the less.

(Being) what it was required to do.

PROPOSITION 4.



IF TWO TRIANGLES HAVE THE TWO SIDES EQUAL, TO TWO SIDES RESPECTIVELY, AND HAVE THE ANGLES CONTAINED BY THE EQUAL STRAIGHT LINES EQUAL, THEY WILL, ALSO, HAVE THE BASE EQUAL, TO THE BASE, THE TRIANGLE WILL BE EQUAL, TO THE TRIANGLE, AND THE REMAINING ANGLES WILL BE EQUAL, TO THE REMAINING ANGLES RESPECTIVELY, NAMELY THOSE WHICH THE EQUAL SIDES SUBTEND.

LET,
ABC, *DEF*, BE TWO TRIANGLES HAVING
THE TWO SIDES, *AB*, *AC*, EQUAL, TO
THE TWO SIDES, *DE*, *DF*, RESPECTIVELY,

NAMELY,
AB TO *DE* AND
AC TO *DF*, AND
THE ANGLE, *BAC*, EQUAL, TO THE ANGLE, *EDF*.

I SAY THAT;
THE BASE, *BC*, IS, ALSO, EQUAL, TO THE BASE, *EF*,
THE TRIANGLE, *ABC*, WILL BE EQUAL
TO THE TRIANGLE, *DEF*, AND
THE REMAINING ANGLES WILL BE EQUAL, TO
THE REMAINING ANGLES RESPECTIVELY,

NAMELY,
THOSE WHICH THE EQUAL SIDES SUBTEND,

THAT IS,
THE ANGLE, *ABC*, TO THE ANGLE, *DEF*, AND
THE ANGLE, *ACB*, TO THE ANGLE, *DFE*.

FOR, IF,
THE TRIANGLE, *ABC*, BE APPLIED TO THE TRIANGLE, *DEF*,

AND IF,
THE POINT, *A*, BE PLACED ON THE POINT, *D*, AND
THE STRAIGHT LINE, *AB*, ON *DE*,

THEN,
THE POINT, *B*, WILL, ALSO, COINCIDE WITH *E*,

BECAUSE,
AB IS EQUAL, TO *DE*.

AGAIN,
AB COINCIDING WITH *DE*,
THE STRAIGHT LINE, *AC*, WILL, ALSO, COINCIDE WITH *DF*,

BECAUSE,
THE ANGLE, *BAC*, IS EQUAL, TO THE ANGLE, *EDF*;

HENCE, ALSO,
THE POINT, *C*, WILL COINCIDE WITH THE POINT, *F*,

BECAUSE,
AC IS AGAIN EQUAL, TO *DF*.

BUT, ALSO,
B COINCIDED WITH *E*;

HENCE,
THE BASE, *BC*, WILL COINCIDE WITH THE BASE, *EF*.

[FOR IF,
WHEN *B* COINCIDES WITH *E* AND
C WITH *F*,
THE BASE, *BC*, DOES NOT COINCIDE WITH THE BASE, *EF*.

THEN,
TWO STRAIGHT LINES WILL ENCLOSE A SPACE:

WHICH,
IS IMPOSSIBLE.

[*C. N.* 4]

THEREFORE,
THE BASE, *BC*, WILL COINCIDE WITH, *EF*] AND
WILL BE EQUAL, TO IT.

THUS,
THE WHOLE TRIANGLE, *ABC*, WILL COINCIDE WITH
THE WHOLE TRIANGLE, *DEF*, AND
WILL BE EQUAL, TO IT.

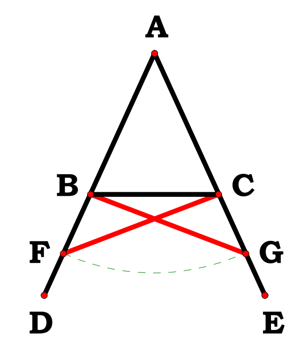
AND,
THE REMAINING ANGLES WILL, ALSO, COINCIDE WITH

the remaining angles, and will be equal, to them,
the angle, ABC, to the angle, DEF, and
the angle, ACB, to the angle, DFE.

Therefore etc.

(Being) what it was required to prove.

Proposition 5.



In isosceles triangles the angles at the base are equal, to one another, and, if the equal straight lines be produced further, the angles under the base will be equal, to one another.

[Post. 2]

Let,
ABC be an isosceles triangle having
the side, AB, equal, to the side, AC;

and let,
the straight lines, BD, CE, be produced further
in a straight line with AB, AC.

I say that;
the angle, ABC, is equal, to the angle, ACB, and
the angle, CBD, to the angle, BCE.

Let,
a point, F, be taken at random, on BD;

[I. 3]

let from,
AE, the greater,
AG be cut off equal, to AF, the less;

[Post. 1]

and let,
the straight lines, FC, GB, be joined.

Then, since,
AF is equal, to AG, and
AB to AC, the two sides,
FA, AC, are equal, to the two sides, GA, AB,
respectively; and
they contain a common angle, the angle, FAG.

Therefore,
the base, FC, is equal, to the base, GB, and
the triangle, AFC, is equal, to the triangle, AGB, and

THE REMAINING ANGLES
WILL BE EQUAL, TO THE REMAINING ANGLES, RESPECTIVELY,

NAMELY,
THOSE WHICH THE EQUAL SIDES SUBTEND,

[I. 4]

THAT IS,
THE ANGLE, *ACF*, TO THE ANGLE, *ABG*, AND
THE ANGLE, *AFC*, TO THE ANGLE, *AGB*.

AND, SINCE,
THE WHOLE, *AF*, IS EQUAL, TO THE WHOLE, *AG*, AND
IN THESE, *AB* IS EQUAL, TO *AC*,
THE REMAINDER, *BF*, IS EQUAL, TO THE REMAINDER, *CG*.

BUT,
FC WAS, ALSO, PROVED EQUAL, TO *GB*;

THEREFORE,
THE TWO SIDES, *BF*, *FC*, ARE EQUAL, TO
THE TWO SIDES, *CG*, *GB*, RESPECTIVELY; AND
THE ANGLE, *BFC*, IS EQUAL, TO THE ANGLE, *CGB*, WHILE
THE BASE, *BC*, IS COMMON TO THEM;

THEREFORE,
THE TRIANGLE, *BFC*, IS, ALSO, EQUAL, TO
THE TRIANGLE, *CGB*, AND
THE REMAINING ANGLES WILL BE EQUAL, TO
THE REMAINING ANGLES, RESPECTIVELY,

NAMELY,
THOSE WHICH THE EQUAL SIDES SUBTEND;

THEREFORE,
THE ANGLE, *FBC*, IS EQUAL, TO THE ANGLE, *GCB*, AND
THE ANGLE, *BCF*, TO THE ANGLE, *CBG*.

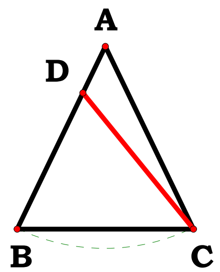
ACCORDINGLY, SINCE,
THE WHOLE ANGLE, *ABG*, WAS PROVED EQUAL, TO
THE ANGLE, *ACF*, AND
IN THESE THE ANGLE, *CBG*, IS EQUAL, TO THE ANGLE, *BCF*,
THE REMAINING ANGLE,
ABC, IS EQUAL, TO THE REMAINING ANGLE, *ACB*; AND
THEY ARE AT THE BASE OF THE TRIANGLE, *ABC*.

BUT,
THE ANGLE,
FBC, WAS, ALSO, PROVED EQUAL, TO THE ANGLE, *GCB*; AND
THEY ARE UNDER THE BASE.

THEREFORE ETC.

Q. E. D.

PROPOSITION 6.



IF IN A TRIANGLE TWO ANGLES BE EQUAL, TO ONE ANOTHER, THE SIDES WHICH SUBTEND THE EQUAL ANGLES WILL, ALSO, *BE EQUAL, TO ONE ANOTHER.*

LET,
 ABC, BE A TRIANGLE HAVING
 THE ANGLE, *ABC*, EQUAL, TO THE ANGLE, *ACB*;

I SAY THAT;
 THE SIDE, *AB*, IS, ALSO, EQUAL, TO THE SIDE, *AC*.

FOR, IF,
 AB IS UNEQUAL, TO *AC*,

THEN,
 ONE OF THEM IS GREATER.

LET,
 AB BE GREATER;

AND LET FROM,
 AB, THE GREATER,
 DB BE CUT OFF EQUAL, TO *AC*, THE LESS;

LET,
 DC BE JOINED.

THEN, SINCE,
 DB IS EQUAL, TO *AC*, AND
 BC IS COMMON,
 THE TWO SIDES, *DB*, *BC*, ARE EQUAL, TO
 THE TWO SIDES, *AC*, *CB*, RESPECTIVELY; AND
 THE ANGLE, *DBC*, IS EQUAL, TO THE ANGLE, *ACB*;

THEREFORE,
 THE BASE, *DC*, IS EQUAL, TO THE BASE, *AB*, AND
 THE TRIANGLE, *DBC*, WILL BE EQUAL, TO THE TRIANGLE, *ACB*,
 THE LESS TO THE GREATER:

WHICH,
 IS ABSURD.

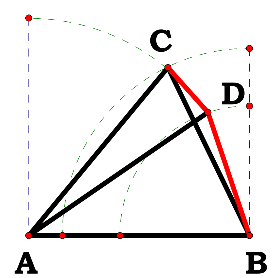
THEREFORE,
 AB IS NOT UNEQUAL, TO *AC*;

THEREFORE,
 IT IS EQUAL, TO IT.

THEREFORE ETC.

Q. E. D.

PROPOSITION 7.



GIVEN TWO STRAIGHT LINES CONSTRUCTED ON A STRAIGHT LINE (FROM ITS EXTREMITIES) AND MEETING IN A POINT, THERE CANNOT BE CONSTRUCTED ON THE SAME STRAIGHT LINE (FROM ITS EXTREMITIES), AND ON THE SAME SIDE OF IT, TWO OTHER STRAIGHT LINES MEETING IN ANOTHER POINT AND EQUAL, TO THE FORMER TWO RESPECTIVELY, NAMELY EACH TO THAT WHICH HAS THE SAME EXTREMITY WITH IT.

FOR, IF POSSIBLE, GIVEN,
TWO STRAIGHT LINES, *AC*, *CB*,
CONSTRUCTED ON THE STRAIGHT LINE, *AB*, AND
MEETING AT THE POINT, *C*,

LET,
TWO OTHER STRAIGHT LINES, *AD*, *DB*,
BE CONSTRUCTED, ON THE SAME STRAIGHT LINE, *AB*,
ON THE SAME SIDE OF IT,
MEETING IN ANOTHER POINT, *D*, AND
EQUAL, TO THE FORMER TWO, RESPECTIVELY,

NAMELY,
EACH TO THAT WHICH HAS THE SAME EXTREMITY WITH IT,

SO THAT,
CA IS EQUAL, TO *DA*,

WHICH,
HAS THE SAME EXTREMITY, *A*, WITH IT, AND
CB TO *DB*, WHICH HAS THE SAME EXTREMITY, *B*, WITH IT;

AND LET,
CD BE JOINED.

[I. 5]

THEN, SINCE,
AC IS EQUAL, TO *AD*,
THE ANGLE, *ACD*, IS, ALSO, EQUAL, TO THE ANGLE, *ADC*;

THEREFORE,
THE ANGLE, *ADC*, IS GREATER THAN THE ANGLE, *DCB*;

THEREFORE,
THE ANGLE, *CDB*, IS MUCH GREATER THAN THE ANGLE, *DCB*.

AGAIN, SINCE,
CB IS EQUAL, TO *DB*,
THE ANGLE, *CDB*, IS, ALSO, EQUAL, TO THE ANGLE, *DCB*.

BUT,
IT WAS, ALSO, PROVED MUCH GREATER THAN IT:

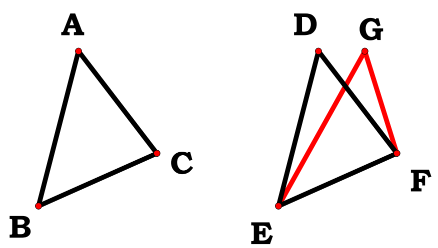
WHICH,
IS IMPOSSIBLE.

THEREFORE ETC.

Q. E. D.

Proposition 8.

If two triangles have the two sides equal, to two sides respectively, and have, also, *the base equal, to the base, they will,* also, *have the angles equal which are contained by the equal straight lines.*



Let,

ABC, *DEF* be two triangles having

the two sides, *AB*, *AC*, equal, to

the two sides, *DE*, *DF*, respectively,

namely,

AB to *DE*, and *AC* to *DF*;

and let,

them have the base, *BC*, equal, to the base, *EF*;

I say that;

the angle, *BAC*, is, also, equal, to the angle, *EDF*.

For, if,

the triangle, *ABC*, be applied to the triangle, *DEF*,

and if,

the point, *B*, be placed on the point, *E*, and

the straight line, *BC*, on *EF*,

the point, *C*, will, also, coincide with *F*,

because,

BC is equal, to *EF*.

Then,

BC coinciding with *EF*,

BA, *AC* will, also, coincide with *ED*, *DF*;

for, if,

the base, *BC*, coincides with the base, *EF*, and

the sides, *BA*, *AC*, do not coincide with, *ED*, *DF*,

but,

fall beside them as *EG*, *GF*,

then,

given two straight lines constructed on

a straight line (from its extremities), and

meeting in a point,

there will have been constructed on

the same straight line (from its extremities), and

on the same side of it,

two other straight lines meeting in another point, and

equal, to the former two, respectively,

namely,

each to that which has the same extremity with it.

[I. 7]

But,

they cannot be so constructed.

Therefore,

it is not possible that,

if,

the base, *BC*, be applied to the base, *EF*,

the sides, *BA*, *AC*, should not coincide with *ED*, *DF*;

therefore,

they will coincide,

so that,

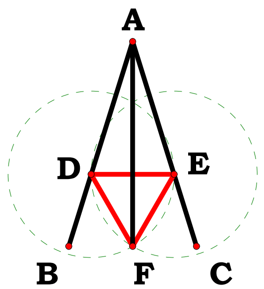
the angle, *BAC*, will, also, coincide with

the angle, *EDF*, and will be equal, to it.

If therefore etc.

Q. E. D.

PROPOSITION 9.



TO BISECT A GIVEN RECTILINEAL ANGLE.

LET,
THE ANGLE, *BAC*, BE
THE GIVEN RECTILINEAL ANGLE.

THUS IT IS REQUIRED,
TO BISECT IT.

LET,
A POINT, *D*, BE TAKEN AT RANDOM, ON *AB*;

[I. 3]

LET,
AE BE CUT OFF FROM *AC* EQUAL, TO *AD*;

LET,
DE BE JOINED, AND ON *DE*,

LET,
THE EQUILATERAL TRIANGLE, *DEF*, BE CONSTRUCTED;

LET,
AF BE JOINED.

I SAY THAT;
THE ANGLE, *BAC*, HAS BEEN BISECTED BY
THE STRAIGHT LINE, *AF*.

FOR, SINCE,
AD IS EQUAL, TO *AE*, AND *AF* IS COMMON,
THE TWO SIDES, *DA*, *AF*, ARE EQUAL, TO
THE TWO SIDES, *EA*, *AF*, RESPECTIVELY.

AND,
THE BASE, *DF*, IS EQUAL, TO THE BASE, *EF*;

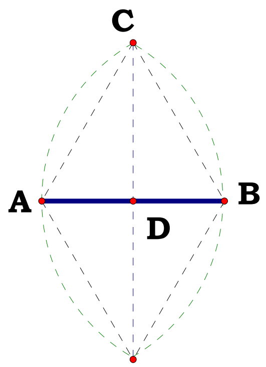
THEREFORE,
THE ANGLE, *DAF*, IS EQUAL, TO THE ANGLE, *EAF*.

THEREFORE,
THE GIVEN RECTILINEAL ANGLE, *BAC*,
HAS BEEN BISECTED BY THE STRAIGHT LINE, *AF*.

Q. E. F.

PROPOSITION 10.

TO BISECT A GIVEN FINITE STRAIGHT LINE.



LET,
AB BE THE GIVEN FINITE STRAIGHT LINE.

THUS IT IS REQUIRED,
TO BISECT THE FINITE STRAIGHT LINE *AB*.

[I. 1]

LET,
THE EQUILATERAL TRIANGLE, *ABC*, BE CONSTRUCTED ON IT,

[I. 9]

AND LET,
THE ANGLE, *ACB*, BE BISECTED BY THE STRAIGHT LINE, *CD*;

I SAY THAT;
THE STRAIGHT LINE, *AB*, HAS BEEN BISECTED AT THE POINT, *D*.

FOR, SINCE,
AC IS EQUAL, TO *CB*, AND *CD* IS COMMON,
THE TWO SIDES, *AC*, *CD*, ARE EQUAL, TO
THE TWO SIDES, *BC*, *CD*, RESPECTIVELY; AND
THE ANGLE, *ACD*, IS EQUAL, TO THE ANGLE, *BCD*;

[I. 4]

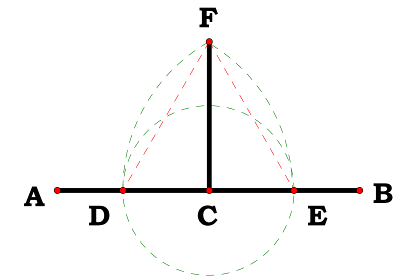
THEREFORE,
THE BASE, *AD*, IS EQUAL, TO THE BASE, *BD*.

THEREFORE,
THE GIVEN FINITE STRAIGHT LINE, *AB*,
HAS BEEN BISECTED AT *D*.

Q. E. F.

PROPOSITION 11.

TO DRAW A STRAIGHT LINE AT RIGHT ANGLES TO A GIVEN STRAIGHT LINE FROM A GIVEN POINT ON IT.



LET,
AB BE THE GIVEN STRAIGHT LINE,

AND,
C THE GIVEN POINT ON IT.

THUS IT IS REQUIRED,
TO DRAW FROM THE POINT, *C*, A STRAIGHT LINE AT
RIGHT ANGLES TO THE STRAIGHT LINE, *AB*.

LET,
A POINT, *D*, BE TAKEN AT RANDOM, ON *AC*;

[I. 3]

LET,
CE BE MADE EQUAL, TO *CD*;

[I. 1]

LET,
ON *DE*, THE EQUILATERAL TRIANGLE, *FDE*, BE CONSTRUCTED,

AND LET,
FC BE JOINED;

I SAY THAT;
THE STRAIGHT LINE, *FC*, HAS BEEN DRAWN AT
RIGHT ANGLES TO THE GIVEN STRAIGHT LINE, *AB*, FROM
C, THE GIVEN POINT ON IT.

FOR, SINCE,
DC IS EQUAL, TO *CE*,

AND,
CF IS COMMON,
THE TWO SIDES, *DC*, *CF*, ARE EQUAL, TO
THE TWO SIDES, *EC*, *CF*, RESPECTIVELY; AND
THE BASE, *DF*, IS EQUAL, TO THE BASE, *FE*;

[I. 8]

THEREFORE,
THE ANGLE, *DCF*, IS EQUAL, TO THE ANGLE, *ECF*; AND
THEY ARE ADJACENT ANGLES.

[DEF. 10]

BUT,
WHEN A STRAIGHT LINE SET UP ON A STRAIGHT LINE MAKES
THE ADJACENT ANGLES EQUAL, TO ONE ANOTHER,
EACH, OF THE EQUAL ANGLES IS RIGHT;

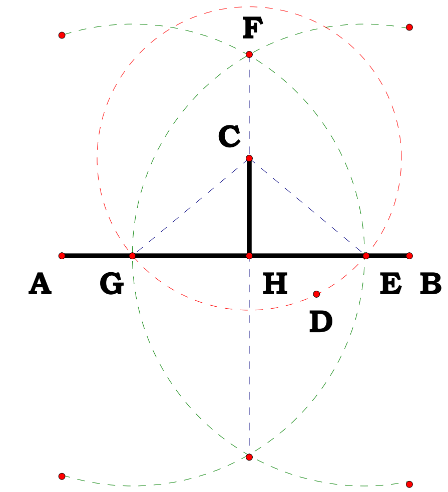
THEREFORE,
EACH, OF THE ANGLES, *DCF*, *FCE*, IS RIGHT.

THEREFORE,
THE STRAIGHT LINE, *CF*, HAS BEEN DRAWN AT
RIGHT ANGLES TO THE GIVEN STRAIGHT LINE, *AB*,
FROM THE GIVEN POINT, *C*, ON IT.

Q. E. F.

PROPOSITION 12.

TO A GIVEN INFINITE STRAIGHT LINE, FROM A GIVEN POINT WHICH IS NOT ON IT, TO DRAW A PERPENDICULAR STRAIGHT LINE.



LET,
AB BE THE GIVEN INFINITE STRAIGHT LINE, AND
C, THE GIVEN POINT WHICH IS NOT ON IT;

THUS IT IS REQUIRED,
TO DRAW TO THE GIVEN INFINITE STRAIGHT LINE, *AB*,
FROM THE GIVEN POINT, *C*, WHICH IS NOT ON IT,
A PERPENDICULAR STRAIGHT LINE.

FOR LET, AT RANDOM,
A POINT, *D*, BE TAKEN ON
THE OTHER SIDE OF THE STRAIGHT LINE, *AB*, AND
WITH CENTRE, *C*, AND DISTANCE, *CD*,

[POST. 3]

LET,
THE CIRCLE, *EFG*, BE DESCRIBED;

[I. 10]

LET,
THE STRAIGHT LINE, *EG*, BE BISECTED, AT *H*,

[POST 1]

AND LET,
THE STRAIGHT LINES, *CG*, *CH*, *CE*, BE JOINED.

I SAY THAT;
CH HAS BEEN DRAWN PERPENDICULAR TO
THE GIVEN INFINITE STRAIGHT LINE, *AB*, FROM
THE GIVEN POINT, *C*, WHICH IS NOT ON IT.

FOR, SINCE,
GH IS EQUAL, TO *HE*, AND
HC IS COMMON,
THE TWO SIDES, *GH*, *HC*, ARE EQUAL, TO

THE TWO SIDES, *EH*, *HC*, RESPECTIVELY; AND
THE BASE, *CG*, IS EQUAL, TO THE BASE, *CE*;

[I. 8]

THEREFORE,
THE ANGLE, *CHG*, IS EQUAL, TO THE ANGLE, *EHC*.
AND THEY ARE ADJACENT ANGLES.

[DEF. 10]

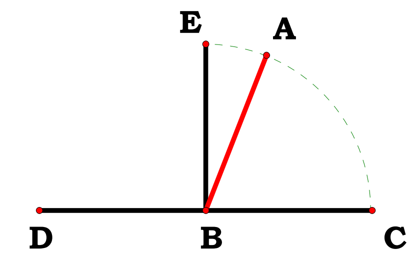
BUT,
WHEN A STRAIGHT LINE SET UP ON A STRAIGHT LINE MAKES
THE ADJACENT ANGLES EQUAL, TO ONE ANOTHER,
EACH, OF THE EQUAL ANGLES IS RIGHT, AND
THE STRAIGHT LINE STANDING ON
THE OTHER IS CALLED A PERPENDICULAR TO THAT
ON WHICH IT STANDS.

THEREFORE,
CH HAS BEEN DRAWN PERPENDICULAR TO
THE GIVEN INFINITE STRAIGHT LINE, *AB*, FROM
THE GIVEN POINT, *C*, WHICH IS NOT ON IT.

Q. E. F.

PROPOSITION 13.

IF A STRAIGHT LINE SET UP ON A STRAIGHT LINE MAKE ANGLES, IT WILL MAKE EITHER TWO RIGHT ANGLES OR ANGLES EQUAL, TO TWO RIGHT ANGLES.



FOR LET,
ANY STRAIGHT LINE, *AB*, SET UP ON
THE STRAIGHT LINE, *CD*, MAKE
THE ANGLES, *CBA*, *ABD*;

I SAY THAT;
THE ANGLES, *CBA*, *ABD*, ARE EITHER TWO RIGHT ANGLES, OR
EQUAL, TO TWO RIGHT ANGLES.

NOW, IF,
THE ANGLE, *CBA*, IS EQUAL, TO THE ANGLE, *ABD*,

[DEF. 10]

THEN,
THEY ARE TWO RIGHT ANGLES.

[I. 11]

BUT, IF NOT, LET,
BE, BE DRAWN FROM THE POINT, *B*, AT RIGHT ANGLES, TO *CD*;

THEREFORE,
THE ANGLES, *CBE*, *EBD*, ARE TWO RIGHT ANGLES.

THEN, SINCE,
THE ANGLE, *CBE*, IS EQUAL, TO THE TWO ANGLES, *CBA*, *ABE*,

LET,
THE ANGLE, *EBD*, BE ADDED TO EACH;

[*C. N.* 2]

THEREFORE,
THE ANGLES, *CBE*, *EBD*, ARE EQUAL, TO
THE THREE ANGLES, *CBA*, *ABE*, *EBD*.

AGAIN, SINCE,
THE ANGLE, *DBA*, IS EQUAL, TO THE TWO ANGLES, *DBE*, *EBA*,

LET,
THE ANGLE, *ABC*, BE ADDED TO EACH;

[*C. N.* 2]

therefore,

the angles, *DBA*, *ABC*, are equal, to
the three angles, *DBE*, *EBA*, *ABC*.

[*C. N.* 1]

But,

the angles, *CBE*, *EBD*, were, also, proved equal, to
the same three angles; and
things which are equal, to the same thing are also
equal, to one another;

therefore,

the angles, *CBE*, *EBD*, are, also, equal, to
the angles, *DBA*, *ABC*.

But,

the angles, *CBE*, *EBD*, are two right angles;

therefore,

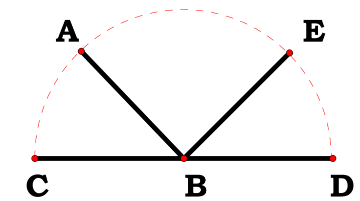
the angles, *DBA*, *ABC*, are, also, equal, to
two right angles.

Therefore etc.

Q. E. D.

Proposition 14.

If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal, to two right angles, the two straight lines will be in a straight line with one another.



For, let

with any straight line, *AB*, and at the point, *B*, on it,
the two straight lines, *BC*, *BD*,
not lying on the same side, make
the adjacent angles, *ABC*, *ABD*, equal, to
two right angles;

I say that;

BD is in a straight line with *CB*.

For, if,

BD is not in a straight line with *BC*,

let,

BE, be in a straight line with *CB*.

[I. 13]

Then, since,

the straight line, *AB*, stands on
the straight line, *CBE*,
the angles, *ABC*, *ABE*, are equal, to two right angles.

But,

the angles, *ABC*, *ABD*, are, also, equal, to
two right angles;

[Post. 4 and *C. N.* 1]

therefore,

the angles,
CBA, *ABE*, are equal, to the angles, *CBA*, *ABD*.

Let,

the angle, *CBA*, be subtracted from each;

[*C. N.* 3]

THEREFORE,
THE REMAINING ANGLE, *ABE*, IS EQUAL, TO
THE REMAINING ANGLE, *ABD*,
THE LESS TO THE GREATER:

WHICH,
IS IMPOSSIBLE.

THEREFORE,
BE IS NOT IN A STRAIGHT LINE WITH *CB*.

SIMILARLY, WE CAN PROVE THAT,
NEITHER IS ANY OTHER STRAIGHT LINE EXCEPT, *BD*.

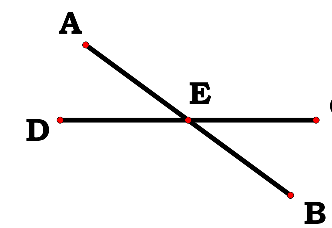
THEREFORE,
CB IS IN A STRAIGHT LINE WITH *BD*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 15.

IF TWO STRAIGHT LINES CUT ONE ANOTHER, THEY MAKE THE VERTICAL ANGLES EQUAL, TO ONE ANOTHER.



FOR LET,
THE STRAIGHT LINES, *AB*, *CD*, CUT ONE ANOTHER AT
THE POINT, *E*;

I SAY THAT;
THE ANGLE, *AEC*, IS EQUAL, TO THE ANGLE, *DEB*, AND
THE ANGLE, *CEB*, TO THE ANGLE, *AED*.

FOR, SINCE,
THE STRAIGHT LINE, *AE*, STANDS ON
THE STRAIGHT LINE, *CD*, MAKING
THE ANGLES, *CEA*, *AED*,

[I. 13]
THE ANGLES, *CEA*, *AED*, ARE EQUAL, TO TWO RIGHT ANGLES.

AGAIN, SINCE,
THE STRAIGHT LINE, *DE*, STANDS ON THE STRAIGHT LINE, *AB*,
MAKING THE ANGLES, *AED*, *DEB*,

[I. 13]
THE ANGLES, *AED*, *DEB*, ARE EQUAL, TO TWO RIGHT ANGLES.

BUT,
THE ANGLES, *CEA*, *AED*, WERE, ALSO, PROVED EQUAL, TO
TWO RIGHT ANGLES;

[POST. 4 AND *C. N.* 1]

THEREFORE,
THE ANGLES, *CEA*, *AED*, ARE EQUAL, TO
THE ANGLES, *AED*, *DEB*.

LET,
THE ANGLE, *AED*, BE SUBTRACTED FROM EACH;

[*C. N.* 3]

THEREFORE,
THE REMAINING ANGLE, *CEA*, IS EQUAL, TO

THE REMAINING ANGLE, *BED*.

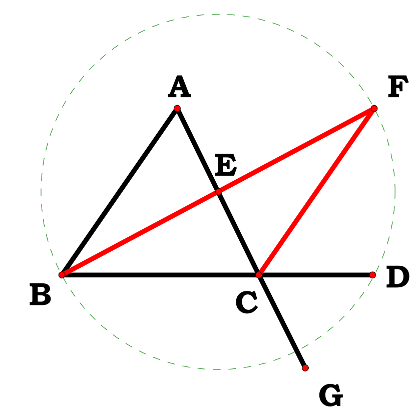
SIMILARLY, IT CAN BE PROVED THAT,
THE ANGLES, *CEB*, *DEA*, ARE, ALSO, EQUAL.

THEREFORE ETC.

Q. E. D.

[PORISM.
FROM THIS IT IS MANIFEST THAT, IF TWO STRAIGHT LINES CUT ONE ANOTHER, THEY WILL MAKE THE ANGLES AT THE POINT OF SECTION EQUAL, TO FOUR RIGHT ANGLES.]

PROPOSITION 16.



IN ANY TRIANGLE, IF ONE OF THE SIDES BE PRODUCED, THE EXTERIOR ANGLE IS GREATER THAN EITHER OF THE INTERIOR AND OPPOSITE ANGLES.

LET,
ABC BE A TRIANGLE,

AND LET,
ONE SIDE OF IT, *BC*, BE PRODUCED TO *D*;

I SAY THAT;
THE EXTERIOR ANGLE, *ACD*, IS GREATER THAN EITHER OF THE INTERIOR AND OPPOSITE ANGLES, *CBA*, *BAC*.

[I. 10]

LET,
AC BE BISECTED AT *E*.

AND LET,
BE, BE JOINED, AND
PRODUCED IN A STRAIGHT LINE TO *F*;

[I. 3]

LET,
EF BE MADE EQUAL, TO *BE*.

[POST. 1]

LET,
FC BE JOINED,

[POST. 2]

AND LET,
AC BE DRAWN THROUGH TO *G*.

THEN, SINCE,
AE IS EQUAL, TO *EC*, AND
BE TO *EF*,
THE TWO SIDES, *AE*, *EB*, ARE EQUAL, TO
THE TWO SIDES, *CE*, *EF*, RESPECTIVELY; AND
THE ANGLE, *AEB*, IS EQUAL, TO THE ANGLE, *FEC*,

[I. 15]

FOR,

THEY ARE VERTICAL ANGLES.

[I. 4]

THEREFORE,

THE BASE, *AB*, IS EQUAL, TO THE BASE, *FC*, AND
THE TRIANGLE, *ABE*, IS EQUAL, TO THE TRIANGLE, *CFE*, AND
THE REMAINING ANGLES ARE EQUAL, TO
THE REMAINING ANGLES, RESPECTIVELY,

NAMELY,

THOSE WHICH THE EQUAL SIDES SUBTEND;

THEREFORE,

THE ANGLE, *BAE*, IS EQUAL, TO THE ANGLE, *ECF*.

[*C. N.* 5]

BUT,

THE ANGLE, *ECD*, IS GREATER THAN THE ANGLE, *ECF*;

THEREFORE,

THE ANGLE, *ACD*, IS GREATER THAN THE ANGLE, *BAE*.

[I. 15]

SIMILARLY ALSO, IF,

BC BE BISECTED, THE ANGLE *BCG*,

THAT IS,

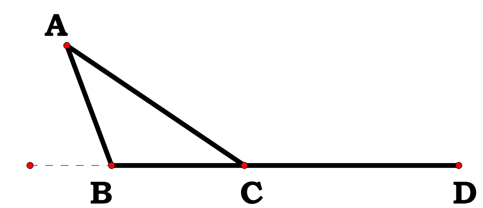
THE ANGLE, *ACD*, CAN BE PROVED GREATER THAN
THE ANGLE, *ABC*, AS WELL.

THEREFORE ETC.

Q. E. D.

PROPOSITION 17.

IN ANY TRIANGLE, TWO ANGLES TAKEN TOGETHER IN ANY MANNER ARE LESS THAN TWO RIGHT ANGLES.



LET,

ABC BE A TRIANGLE;

I SAY THAT;

TWO ANGLES OF THE TRIANGLE, *ABC*, TAKEN TOGETHER,
IN ANY MANNER,
ARE LESS THAN TWO RIGHT ANGLES.

[POST. 2]

FOR LET,

BC BE PRODUCED TO *D*.

THEN, SINCE,

THE ANGLE,
ACD, IS AN EXTERIOR ANGLE OF THE TRIANGLE, *ABC*,
IT IS GREATER THAN THE INTERIOR AND OPPOSITE ANGLE, *ABC*.

LET,

THE ANGLE, *ACB*, BE ADDED TO EACH;

THEREFORE,

THE ANGLES, *ACD*, *ACB*, ARE GREATER THAN
THE ANGLES, *ABC*, *BCA*.

[I.13]

BUT

THE ANGLES, *ACD*, *ACB*, ARE EQUAL, TO TWO RIGHT ANGLES.

THEREFORE,

THE ANGLES, *ABC*, *BCA*, ARE LESS THAN TWO RIGHT ANGLES.

SIMILARLY WE CAN PROVE, ALSO, THAT,

THE ANGLES, *BAC*, *ACB*, ARE LESS THAN TWO RIGHT ANGLES,

AND,

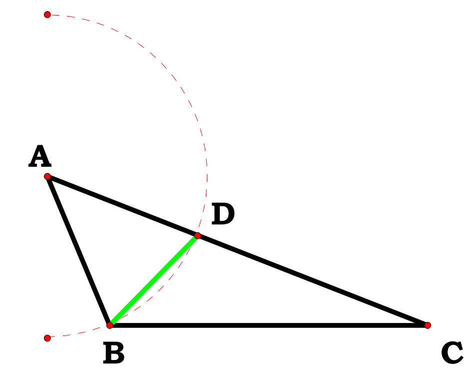
SO ARE THE ANGLES, *CAB*, *ABC*, AS WELL.

THEREFORE ETC.

Q. E. D.

Proposition 18.

In any triangle the greater side subtends the greater angle.



For let,

ABC be a triangle
having the side, *AC*,
greater than *AB*;

I say that;

the angle, *ABC*, is, also, greater than the angle, *BCA*.

For, since,

AC is greater than *AB*,

[I. 3]

let,

AD be made equal, to *AB*.

and let,

BD be joined.

[I. 16]

Then, since,

the angle, *ADB*, is an exterior angle of
the triangle, *BCD*, it is greater than
the interior and opposite angle, *DCB*.

But,

the angle, *ADB*, is equal, to the angle, *ABD*,

since,

the side, *AB*, is equal, to *AD*;

therefore,

the angle, *ABD*, is, also, greater than the angle, *ACB*;

therefore,

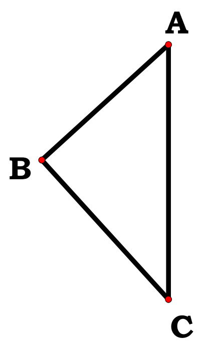
the angle, *ABC*, is much greater than the angle, *ACB*.

Therefore etc.

Q. E. D.

Proposition 19.

In any triangle the greater angle is subtended by the greater side.



Let,

ABC be a triangle
having the angle, ABC, greater than
the angle, BCA;

I say that;

the side, AC, is, also, greater than the side, AB.

For, if not,

AC is either equal, to AB, or less.

Now,

AC is not equal, to AB;

[I. 5]

for then,

the angle, ABC, would, also, have been equal, to
the angle, ACB;

but,

it is not;

therefore,

AC is not equal, to AB.

Neither,

is AC less than AB,

[I. 18]

for then,

the angle, ABC, would, also, have been less than
the angle, ACB,

but,

it is not;

therefore,

AC is not less than AB.

And,

it was proved that it is not equal either.

Therefore,

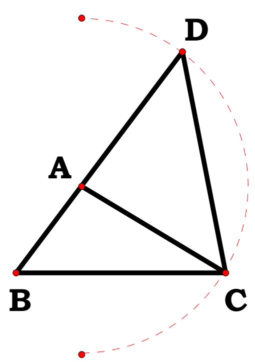
AC is greater than AB.

Therefore etc.

Q. E. D.

Proposition 20.

In any triangle two sides taken together in any manner are greater than the remaining one.



For let,

ABC be a triangle;

I say that;

in the triangle, *ABC*, two sides taken together,
in any manner, are greater than the remaining one,

namely,

BA, *AC* greater than *BC*,
AB, *BC* greater than *AC*,
BC, *CA* greater than *AB*.

For let,

BA be drawn through to the point, *D*,

let,

DA be made equal, to *CA*,

and let,

DC be joined.

[I. 5]

Then, since,

DA is equal, to *AC*,
the angle, *ADC*, is, also, equal, to the angle, *ACD*;

[*C. N.* 5]

therefore,

the angle, *BCD*, is greater than the angle, *ADC*.

[I. 19]

And, since,

DCB is a triangle having
the angle, *BCD*, greater than the angle, *BDC*, and
the greater angle is subtended by the greater side,

therefore,

DB is greater than *BC*.

But,

DA is equal, to *AC*;

therefore,

BA, *AC* are greater than *BC*.

Similarly, also, we can prove that,

AB, *BC* are greater than *CA*,

and,

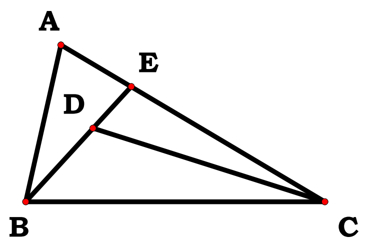
BC, *CA* than *AB*.

Therefore etc.

Q. E. D.

PROPOSITION 21.

IF ON ONE OF THE SIDES OF A TRIANGLE, FROM ITS EXTREMITIES, THERE BE CONSTRUCTED TWO STRAIGHT LINES MEETING WITHIN THE TRIANGLE, THE STRAIGHT LINES SO CONSTRUCTED WILL BE LESS THAN THE REMAINING TWO SIDES OF THE TRIANGLE, BUT WILL CONTAIN A GREATER ANGLE.



LET,
ON *BC*, ONE OF THE SIDES OF THE TRIANGLE, *ABC*,

FROM,
ITS EXTREMITIES, *B*, *C*,
THE TWO STRAIGHT LINES, *BD*, *DC*, BE CONSTRUCTED
MEETING WITHIN THE TRIANGLE;

I SAY THAT;
BD, *DC* ARE LESS THAN THE REMAINING TWO SIDES OF
THE TRIANGLE, *BA*, *AC*,

BUT,
CONTAIN AN ANGLE, *BDC*, GREATER THAN THE ANGLE, *BAC*.

FOR LET,
BD BE DRAWN THROUGH TO *E*.

[I. 20]

THEN, SINCE,
IN ANY TRIANGLE,
TWO SIDES ARE GREATER THAN THE REMAINING ONE,

THEREFORE,
IN THE TRIANGLE, *ABE*,
THE TWO SIDES, *AB*, *AE*, ARE GREATER THAN *BE*.

LET,
EC BE ADDED TO EACH;

THEREFORE,
BA, *AC* ARE GREATER THAN *BE*, *EC*.

AGAIN, SINCE,
IN THE TRIANGLE, *CED*,
THE TWO SIDES, *CE*, *ED*, ARE GREATER THAN *CD*,

LET,
DB BE ADDED TO EACH;

THEREFORE,
CE, *EB* ARE GREATER THAN *CD*, *DB*.

BUT,
BA, *AC* WERE PROVED GREATER THAN *BE*, *EC*;

THEREFORE,
BA, *AC* ARE MUCH GREATER THAN *BD*, *DC*.

[I. 16]

AGAIN, SINCE,
IN ANY TRIANGLE,
THE EXTERIOR ANGLE IS GREATER THAN
THE INTERIOR AND OPPOSITE ANGLE,

THEREFORE,
IN THE TRIANGLE, *CDE*, THE EXTERIOR ANGLE, *BDC*,
IS GREATER THAN THE ANGLE, *CED*.

FOR THE SAME REASON, MOREOVER,
IN THE TRIANGLE, *ABE*, ALSO,
THE EXTERIOR ANGLE, *CEB*, IS GREATER THAN
THE ANGLE, *BAC*.

BUT,
THE ANGLE, *BDC*, WAS PROVED GREATER THAN
THE ANGLE, *CEB*;

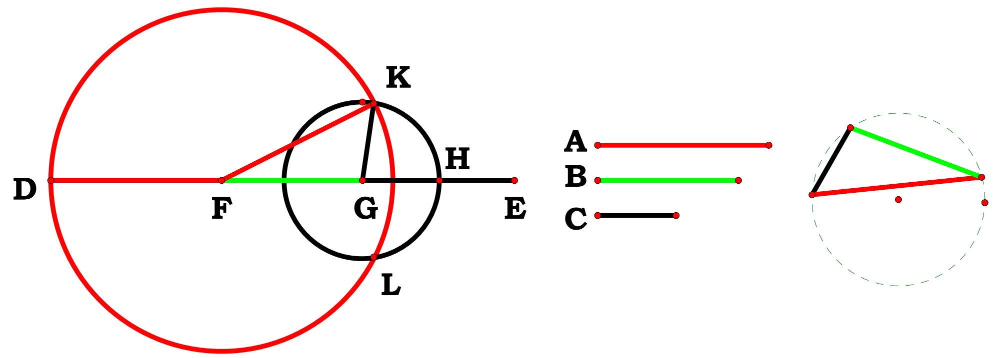
THEREFORE,
THE ANGLE *BDC* IS MUCH GREATER THAN THE ANGLE *BAC*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 22.

OUT OF THREE STRAIGHT LINES, WHICH ARE EQUAL, TO THREE GIVEN STRAIGHT LINES, TO CONSTRUCT A TRIANGLE: THUS IT IS NECESSARY THAT TWO OF THE STRAIGHT LINES TAKEN TOGETHER IN ANY MANNER SHOULD BE GREATER THAN THE REMAINING ONE. [I. 20]



LET,
THE THREE GIVEN STRAIGHT LINES BE *A*, *B*, *C*,

AND LET,
OF THESE, TWO TAKEN TOGETHER, IN ANY MANNER,
BE GREATER THAN THE REMAINING ONE,

NAMELY,
A, *B* GREATER THAN *C*,
A, *C* GREATER THAN *B*, AND,
B, *C* GREATER THAN *A*;

THUS IT IS REQUIRED,
TO CONSTRUCT A TRIANGLE,
OUT OF STRAIGHT LINES, EQUAL, TO *A*, *B*, *C*.

LET,
THERE BE SET OUT A STRAIGHT LINE, *DE*, TERMINATED AT *D*,

BUT,
OF INFINITE LENGTH IN THE DIRECTION OF *E*,

[I. 3]

AND LET,
DF BE MADE EQUAL, TO *A*,
FG EQUAL, TO *B*, AND,
GH EQUAL, TO *C*.

LET,
WITH CENTRE, *F*, AND DISTANCE, *FD*,
THE CIRCLE, *DKL*, BE DESCRIBED;

AGAIN, LET,
WITH CENTRE, *G*, AND DISTANCE, *GH*,
THE CIRCLE, *KLH*, BE DESCRIBED;

AND LET,
KF, *KG*, BE JOINED;

I SAY THAT;
THE TRIANGLE, *KFG*, HAS BEEN CONSTRUCTED,
OUT OF THREE STRAIGHT LINES, EQUAL, TO *A*, *B*, *C*.

FOR, SINCE,
THE POINT, *F*, IS THE CENTRE OF THE CIRCLE, *DKL*,
FD IS EQUAL, TO *FK*.

BUT,
FD IS EQUAL, TO *A*;

THEREFORE,
KF IS, ALSO, EQUAL, TO *A*.

AGAIN, SINCE,
THE POINT, *G*, IS THE CENTRE OF THE CIRCLE, *LKH*,
GH IS EQUAL, TO *GK*.

BUT,
GH IS EQUAL, TO *C*;

THEREFORE,
KG IS, ALSO, EQUAL, TO *C*.

AND,
FG IS, ALSO, EQUAL, TO *B*;

THEREFORE,
THE THREE STRAIGHT LINES, *KF*, *FG*, *GK*, ARE EQUAL, TO
THE THREE STRAIGHT LINES, *A*, *B*, *C*.

THEREFORE,
OUT OF THE THREE STRAIGHT LINES, *KF*, *FG*, *GK*,

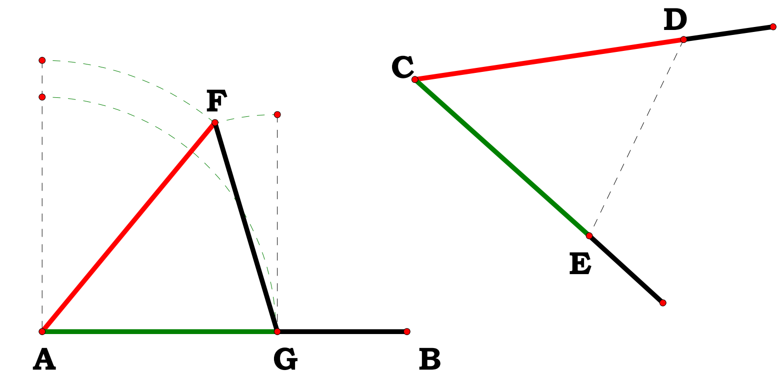
WHICH,
ARE EQUAL, TO,

THE THREE GIVEN STRAIGHT LINES, *A*, *B*, *C*,
THE TRIANGLE, *KFG*, HAS BEEN CONSTRUCTED.

Q. E. F.

Proposition 23.

On a given straight line and at a point on it to construct a rectilineal angle equal, to a given rectilineal angle.



Let,
AB BE THE GIVEN STRAIGHT LINE,
A THE POINT ON IT,

AND,
THE ANGLE, *DCE*, THE GIVEN RECTILINEAL ANGLE;

THUS IT IS REQUIRED,
TO CONSTRUCT ON THE GIVEN STRAIGHT LINE, *AB*,

AND,
AT THE POINT, *A*, ON IT,
A RECTILINEAL ANGLE EQUAL, TO
THE GIVEN RECTILINEAL ANGLE, *DCE*.

AT RANDOM, LET,
ON THE STRAIGHT LINES, *CD*, *CE*,
THE POINTS, *D*, *E*, BE TAKEN; RESPECTIVELY

LET,
DE BE JOINED,

[I. 22]

AND,
OUT OF THREE STRAIGHT LINES, WHICH ARE EQUAL, TO
THE THREE STRAIGHT LINES, *CD*, *DE*, *CE*,

LET,
THE TRIANGLE, *AFG*, BE CONSTRUCTED,

in such a way, that,
CD is equal, to *AF*,
CE to *AG*,

and further,
DE to *FG*

[I. 8]

Then, since,
the two sides,
DC, *CE*, are equal, to the two sides, *FA*, *AG*,
respectively,

and,
the base, *DE*, is equal, to the base, *FG*,
the angle, *DCE*, is equal, to the angle, *FAG*,

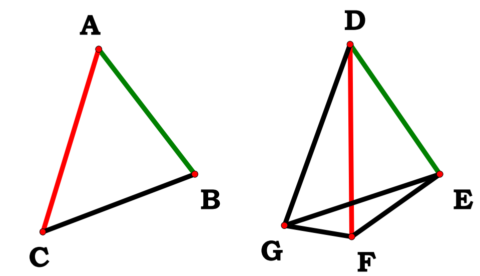
Therefore,
on the given straight line, *AB*,

and
at the point *A* on it,
the rectilineal angle, *FAG*, has been constructed,
equal, to the given rectilineal angle, *DCE*.

Q E F

Proposition 24.

If two triangles have the two sides equal, to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will, also, have the base greater than the base.



Let,
ABC, *DEF* be two triangles having
the two sides, *AB*, *AC*, equal, to
the two sides, *DE*, *DF*, respectively,

namely,
AB to *DE*, and
AC to *DF*,

and let,
the angle, at *A*, be greater than the angle, at *D*;

I say that;
the base, *BC*, is, also, greater than the base, *EF*.

For, since,
the angle, *BAC*, is greater than the angle, *EDF*,

let,
there be constructed, on the straight line, *DE*,

[I. 23]

and,
at the point, *D*, on it,
the angle, *EDG*, equal, to the angle, *BAC*;

let,
DG be made equal, to either of
the two straight lines, *AC*, *DF*,

and let,
EG, *FG* be joined.

[I. 4]

Then, since ,
AB is equal, to *DE*, and,
AC to *DG*,

THE TWO SIDES, *BA*, *AC*, ARE EQUAL, TO
THE TWO SIDES, *ED*, *DG*, RESPECTIVELY;

AND

THE ANGLE, *BAC*, IS EQUAL, TO THE ANGLE, *EDG*;

THEREFORE,

THE BASE, *BC*, IS EQUAL, TO THE BASE, *EG*.

[I. 5]

AGAIN, SINCE,

DF IS EQUAL, TO *DG*,
THE ANGLE, *DGF*, IS, ALSO, EQUAL, TO THE ANGLE, *DFG*;

THEREFORE,

THE ANGLE, *DFG*, IS GREATER THAN THE ANGLE, *EGF*.

THEREFORE,

THE ANGLE, *EFG*, IS MUCH GREATER THAN THE ANGLE, *EGF*.

AND, SINCE,

EFG IS A TRIANGLE
HAVING THE ANGLE, *EFG*, GREATER THAN THE ANGLE, *EGF*,

[I. 19]

AND,

THE GREATER ANGLE IS SUBTENDED BY THE GREATER SIDE,
THE SIDE, *EG*, IS, ALSO, GREATER THAN *EF*.

BUT,

EG IS EQUAL, TO *BC*.

THEREFORE,

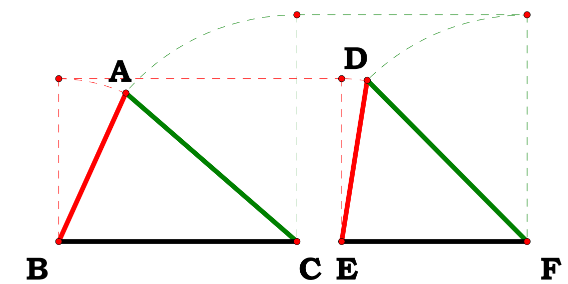
BC IS, ALSO, GREATER THAN *EF*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 25.

IF TWO TRIANGLES HAVE THE TWO SIDES EQUAL, TO TWO SIDES RESPECTIVELY, BUT HAVE THE BASE GREATER THAN THE BASE, THEY WILL, ALSO, *HAVE THE ONE OF THE ANGLES CONTAINED BY THE EQUAL STRAIGHT LINES GREATER THAN THE OTHER.*



LET,

ABC, *DEF*, BE TWO TRIANGLES HAVING
THE TWO SIDES, *AB*, *AC*, EQUAL, TO
THE TWO SIDES, *DE*, *DF*, RESPECTIVELY,

NAMELY,

AB TO *DE*, AND
AC TO *DF*;

AND LET,

THE BASE, *BC*, BE GREATER THAN THE BASE, *EF*;

I SAY THAT;

THE ANGLE, *BAC*, IS, ALSO, GREATER THAN THE ANGLE, *EDF*.

FOR,

IF NOT,

THEN,

IT IS EITHER EQUAL, TO IT OR LESS.

NOW,

THE ANGLE, *BAC*, IS NOT EQUAL, TO THE ANGLE, *EDF*;

[I. 4]

FOR THEN,

THE BASE, *BC*, WOULD, ALSO, HAVE BEEN EQUAL, TO
THE BASE, *EF*,

BUT,

IT IS NOT;

THEREFORE,

THE ANGLE, *BAC*, IS NOT EQUAL, TO THE ANGLE, *EDF*.

AGAIN,

NEITHER IS THE ANGLE, *BAC*, LESS THAN THE ANGLE, *EDF*;

[I. 24]

FOR THEN,

THE BASE, *BC*, WOULD, ALSO, HAVE BEEN LESS THAN
THE BASE, *EF*,

BUT,

IT IS NOT;

THEREFORE,

THE ANGLE, *BAC*, IS NOT LESS THAN THE ANGLE, *EDF*.

BUT,

IT WAS PROVED THAT IT IS NOT EQUAL EITHER;

THEREFORE,

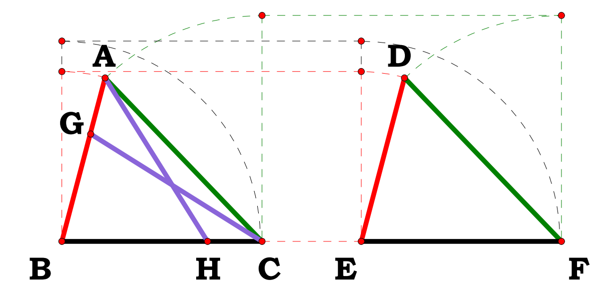
THE ANGLE, *BAC*, IS GREATER THAN THE ANGLE, *EDF*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 26.

IF TWO TRIANGLES HAVE THE TWO ANGLES EQUAL, TO TWO ANGLES RESPECTIVELY, AND ONE SIDE EQUAL, TO ONE SIDE, NAMELY, EITHER THE SIDE ADJOINING THE EQUAL ANGLES, OR THAT SUBTENDING ONE OF THE EQUAL ANGLES, THEY WILL, ALSO, HAVE THE REMAINING SIDES EQUAL, TO THE REMAINING SIDES AND THE REMAINING ANGLE TO THE REMAINING ANGLE.



LET,

ABC, *DEF*, BE TWO TRIANGLES HAVING
THE TWO ANGLES, *ABC*, *BCA*, EQUAL, TO
THE TWO ANGLES, *DEF*, *EFD*, RESPECTIVELY,

NAMELY,

THE ANGLE, *ABC*, TO THE ANGLE, *DEF*, AND
THE ANGLE, *BCA*, TO THE ANGLE *EFD*;

AND LET,

THEM, ALSO, HAVE ONE SIDE EQUAL, TO ONE SIDE,
FIRST THAT ADJOINING THE EQUAL ANGLES,

NAMELY,

BC TO *EF*;

I SAY THAT;

THEY WILL, ALSO, HAVE THE REMAINING SIDES EQUAL, TO
THE REMAINING SIDES, RESPECTIVELY,

NAMELY,

AB TO *DE*, AND
AC TO *DF*, AND
THE REMAINING ANGLE TO THE REMAINING ANGLE,

NAMELY,

THE ANGLE, *BAC*, TO THE ANGLE, *EDF*.

FOR, IF,

AB IS UNEQUAL, TO *DE*,
ONE OF THEM IS GREATER.

LET

AB BE GREATER,

and let,
BG be made equal, to *DE*;

and let,
GC be joined.

Then, since,
BG is equal, to *DE*, and
BC to *EF*,
the two sides, *GB*, *BC*, are equal, to
the two sides, *DE*, *EF*, respectively; and
the angle, *GBC*, is equal, to the angle, *DEF*;

[I. 4]

therefore,
the base, *GC*, is equal, to the base, *DF*, and
the triangle, *GBC*, is equal, to the triangle, *DEF*, and
the remaining angles will be equal
to the remaining angles,

namely,
those which the equal sides subtend;

therefore,
the angle, *GCB*, is equal, to the angle, *DFE*.

But by hypothesis,
the angle, *DFE*, is equal, to the angle, *BCA*;

therefore,
the angle, *BCG*, is equal, to the angle, *BCA*,
the less to the greater:

which,
is impossible.

Therefore,
AB is not unequal, to *DE*,

and therefore,
is equal, to it.

But,
BC is, also, equal, to *EF*;

therefore,
the two sides, *AB*, *BC*, are equal, to
the two sides, *DE*, *EF*, respectively, and
the angle, *ABC*, is equal, to the angle, *DEF*;

[I. 4]

therefore,
the base, *AC*, is equal, to the base, *DF*, and
the remaining angle, *BAC*, is equal, to
the remaining angle, *EDF*.

Again, let,
the sides subtending equal angles be equal,
as *AB* to *DE*;

I say again that,
the remaining sides will be equal, to
the remaining sides,

namely,
AC to *DF*, and
BC to *EF*,

and further,
the remaining angle, *BAC*, is equal, to
the remaining angle, *EDF*.

For, if,
BC is unequal, to *EF*,

then,
one of them is greater.

Let, if possible,
BC be greater,

and let,
BH be made equal, to *EF*;

let,
AH be joined.

Then, since,
BH is equal, to *EF*, and
AB to *DE*,

THE TWO SIDES, *AB*, *BH*, ARE EQUAL, TO
THE TWO SIDES, *DE*, *EF*, RESPECTIVELY,

AND,
THEY CONTAIN EQUAL ANGLES;

[I. 4]

THEREFORE,
THE BASE, *AH*, IS EQUAL, TO THE BASE, *DF*, AND
THE TRIANGLE, *ABH*, IS EQUAL, TO THE TRIANGLE, *DEF*, AND
THE REMAINING ANGLES WILL BE EQUAL, TO
THE REMAINING ANGLES,

NAMELY,
THOSE WHICH THE EQUAL SIDES SUBTEND;

THEREFORE,
THE ANGLE, *BHA*, IS EQUAL, TO THE ANGLE, *EFD*.

BUT,
THE ANGLE, *EFD*, IS EQUAL, TO THE ANGLE, *BCA*;

[I. 16]

THEREFORE,
IN THE TRIANGLE, *AHC*,
THE EXTERIOR ANGLE, *BHA*, IS EQUAL, TO THE INTERIOR, AND
OPPOSITE ANGLE, *BCA*:

WHICH,
IS IMPOSSIBLE.

THEREFORE,
BC IS NOT UNEQUAL, TO *EF*,

AND THEREFORE,
IS EQUAL, TO IT.

BUT,
AB IS, ALSO, EQUAL, TO *DE*;

THEREFORE,
THE TWO SIDES, *AB*, *BC*, ARE EQUAL, TO
THE TWO SIDES, *DE*, *EF*, RESPECTIVELY, AND
THEY CONTAIN EQUAL ANGLES;

[I. 4]

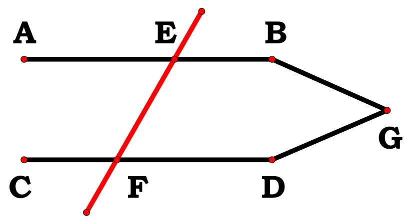
THEREFORE,
THE BASE, *AC*, IS EQUAL, TO THE BASE, *DF*,
THE TRIANGLE, *ABC*, EQUAL, TO THE TRIANGLE, *DEF*, AND
THE REMAINING ANGLE, *BAC*, EQUAL, TO
THE REMAINING ANGLE, *EDF*.

THEREFORE ETC.

Q. E. D.

Proposition 27.

If a straight line falling on two straight lines make the alternate angles equal, to one another, the straight lines will be parallel to one another.



For let,
the straight line, *EF*, falling on
the two straight lines, *AB*, *CD*,
make the alternate angles, *AEF*, *EFD*, equal, to
one another;

I say that;
AB is parallel to *CD*.

For,
if not,

then,
AB, *CD*, when produced, will meet either in
the direction of *B*, *D*, or
towards *A*, *C*.

Let,
them be produced and meet,
in the direction of *B*, *D*, at *G*.

[I. 16]

Then,
in the triangle, *GEF*,
the exterior angle, *AEF*, is equal, to
the interior and opposite angle, *EFG*:

which,
is impossible.

Therefore,
AB, *CD* when produced will not meet in
the direction of *B*, *D*.

Similarly it can be proved that,
neither will they meet towards *A*, *C*.

[Def. 23]

But
straight lines,
which do not meet in either direction, are parallel;

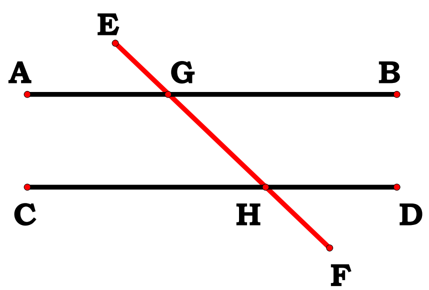
therefore,
AB is parallel to *CD*.

Therefore etc.

Q. E. D.

Proposition 28.

If a straight line falling on two straight lines make the exterior angle equal, to the interior and opposite angle on the same side, or the interior angles on the same side equal, to two right angles, the straight lines will be parallel to one another.



For let,

the straight line, *EF*,
falling on the two straight lines, *AB*, *CD*,
make the exterior angle, *EGB*, equal, to
the interior and opposite angle, *GHD*,

or,

the interior angles on the same side,

namely,

BGH, *GHD*, equal, to two right angles;

I say that;

AB is parallel to *CD*.

For, since,

the angle, *EGB*, is equal, to the angle, *GHD*,

[I. 15]

while,

the angle, *EGB*, is equal, to the angle, *AGH*,
the angle, *AGH*, is, also, equal, to the angle, *GHD*;

and,

they are alternate;

[I. 27]

therefore,

AB is parallel to *CD*.

Again, since ,

the angles, *BGH*, *GHD*, are equal, to two right angles,

[I. 13]

and,

the angles, *AGH*, *BGH*, are, also, equal, to
two right angles,
the angles, *AGH*, *BGH*, are equal, to
the angles, *BGH*, *GHD*.

Let,

the angle, *BGH*, be subtracted from each;

therefore,

the remaining angle, *AGH*, is equal, to
the remaining angle, *GHD*;

and,

they are alternate;

[I. 27]

therefore,

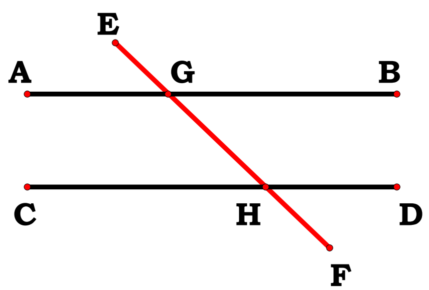
AB is parallel to *CD*.

Therefore etc.

Q. E. D.

Proposition 29.

A straight line falling on parallel straight lines makes the alternate angles equal, to one another, the exterior angle equal, to the interior and opposite angle, and the interior angles on the same side equal, to two right angles.



For let,
the straight line, *EF*, fall on
the parallel straight lines, *AB*, *CD*;

I say that;
it makes the alternate angles, *AGH*, *GHD*, equal,
the exterior angle, *EGB*, equal, to the interior and
opposite angle, *GHD*, and
the interior angles on the same side,

namely,
BGH, *GHD*, equal, to two right angles.

For, if,
the angle, *AGH*, is unequal, to the angle, *GHD*,
one of them is greater.

Let,
the angle, *AGH*, be greater.

Let,
the angle *BGH* be added to each;

therefore,
the angles, *AGH*, *BGH*, are greater than
the angles, *BGH*, *GHD*.

[I. 13]

But
the angles, *AGH*, *BGH*, are equal, to two right angles;

therefore,
the angles, *BGH*, *GHD*, are less than two right angles.

[Post 5]

But,
straight lines, produced indefinitely,

from,
angles less than two right angles, meet;

therefore,
AB, *CD*, if produced indefinitely, will meet;

but,
they do not meet,

because,
by hypothesis, they are parallel.

Therefore,
the angle, *AGH*, is not unequal, to the angle, *GHD*,

and therefore,
is equal, to it.

[I. 15]

Again,
the angle, *AGH*, is equal, to the angle, *EGB*;

[*C. N.* 1]

therefore,
the angle, *EGB*, is, also, equal, to the angle, *GHD*.

Let,
the angle, *BGH*, be added to each;

[*C. N.* 2]

therefore,
the angles, *EGB*, *BGH*, are equal, to
the angles, *BGH*, *GHD*.

[I. 13]

But,
the angles, *EGB*, *BGH*, are equal, to two right angles;

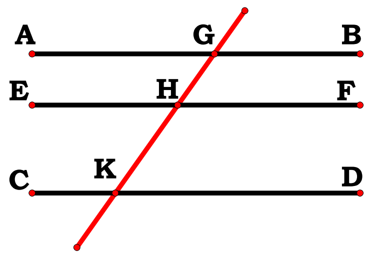
therefore,
the angles, *BGH*, *GHD*, are, also, equal, to
two right angles.

Therefore etc.

Q. E. D.

PROPOSITION 30.

STRAIGHT LINES PARALLEL TO THE SAME STRAIGHT LINE ARE, ALSO, *PARALLEL TO ONE ANOTHER*.



LET,
EACH, OF THE STRAIGHT LINES,
AB, *CD*, BE PARALLEL TO *EF*;

I SAY THAT;
AB IS, ALSO, PARALLEL TO *CD*.

FOR LET,
THE STRAIGHT LINE, *GK*, FALL UPON THEM.

[I. 29]

THEN, SINCE,
THE STRAIGHT LINE, *GK*, HAS FALLEN ON
THE PARALLEL STRAIGHT LINES, *AB*, *EF*,
THE ANGLE, *AGK*, IS EQUAL, TO THE ANGLE, *GHF*.

[I. 29]

AGAIN, SINCE,
THE STRAIGHT LINE, *GK*, HAS FALLEN ON
THE PARALLEL STRAIGHT LINES, *EF*, *CD*,
THE ANGLE, *GHF*, IS EQUAL, TO THE ANGLE, *GKD*.

BUT
THE ANGLE,
AGK, WAS, ALSO, PROVED EQUAL, TO THE ANGLE, *GHF*;

[*C. N.* 1]

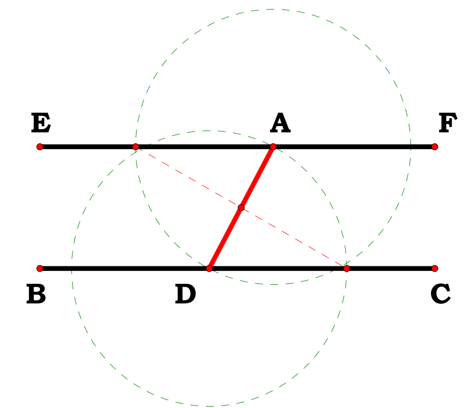
THEREFORE,
THE ANGLE, *AGK*, IS, ALSO, EQUAL, TO THE ANGLE, *GKD*; AND
THEY ARE ALTERNATE.

THEREFORE,
AB IS PARALLEL TO *CD*.

Q. E. D.

PROPOSITION 31.

THROUGH A GIVEN *POINT TO DRAW A STRAIGHT LINE PARALLEL TO* A GIVEN *STRAIGHT LINE*.



LET,
A, BE THE GIVEN POINT, AND,
BC, THE GIVEN STRAIGHT LINE;

THUS IT IS REQUIRED,
TO DRAW THROUGH THE POINT, *A*,
A STRAIGHT LINE PARALLEL TO THE STRAIGHT LINE, *BC*.

LET, AT RANDOM,
ON *BC*, A POINT *D* BE TAKEN,

AND LET,
AD BE JOINED;

[I. 23]

LET,
ON THE STRAIGHT LINE, *DA*, AND
AT THE POINT, *A*, ON IT,
THE ANGLE, *DAE*, BE CONSTRUCTED EQUAL, TO
THE ANGLE, *ADC*;

AND LET,
THE STRAIGHT LINE, *AF*, BE PRODUCED,
IN A STRAIGHT LINE, WITH *EA*.

THEN, SINCE,
THE STRAIGHT LINE, *AD*, FALLING ON
THE TWO STRAIGHT LINES, *BC*, *EF*, HAS MADE
THE ALTERNATE ANGLES, *EAD*, *ADC*, EQUAL, TO ONE ANOTHER.

[I. 27]

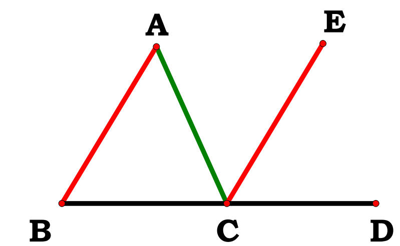
THEREFORE,
EAF IS PARALLEL TO *BC*,

THEREFORE,
THROUGH THE GIVEN POINT, *A*, AND
PARALLEL TO THE GIVEN STRAIGHT LINE, *BC*;
THE STRAIGHT LINE, *EAF*, HAS BEEN DRAWN.

Q. E. F.

PROPOSITION 32.

IN ANY TRIANGLE, IF ONE OF THE SIDES BE PRODUCED, THE EXTERIOR ANGLE IS EQUAL, TO THE TWO INTERIOR AND OPPOSITE ANGLES, AND THE THREE INTERIOR ANGLES OF THE TRIANGLE ARE EQUAL, TO TWO RIGHT ANGLES.



LET,
ABC BE A TRIANGLE,

AND LET,
ONE SIDE OF IT, *BC*, BE PRODUCED TO *D*;

I SAY THAT;
THE EXTERIOR ANGLE, *ACD*, IS EQUAL, TO
THE TWO INTERIOR AND OPPOSITE ANGLES, *CAB*, *ABC*, AND
THE THREE INTERIOR ANGLES OF THE TRIANGLE,
ABC, *BCA*, *CAB*, ARE EQUAL, TO TWO RIGHT ANGLES.

[I. 31]

FOR LET,
CE BE DRAWN THROUGH
THE POINT, *C*, PARALLEL TO THE STRAIGHT LINE, *AB*,

[I. 29]

THEN, SINCE,
AB IS PARALLEL TO *CE*, AND
AC HAS FALLEN UPON THEM,
THE ALTERNATE ANGLES,
BAC, *ACE*, ARE EQUAL, TO ONE ANOTHER.

[I. 29]

AGAIN, SINCE,
AB IS PARALLEL TO *CE*,

AND,
THE STRAIGHT LINE, *BD*, HAS FALLEN UPON THEM,
THE EXTERIOR ANGLE, *ECD*, IS EQUAL, TO
THE INTERIOR AND OPPOSITE ANGLE, *ABC*.

BUT,
THE ANGLE,
ACE, WAS, ALSO, PROVED EQUAL, TO THE ANGLE, *BAC*;

THEREFORE,
THE WHOLE ANGLE, *ACD*, IS EQUAL, TO
THE TWO INTERIOR AND OPPOSITE ANGLES, *BAC*, *ABC*.

LET,
THE ANGLE, *ACB*, BE ADDED TO EACH;

THEREFORE,
THE ANGLES, *ACD*, *ACB*, ARE EQUAL, TO
THE THREE ANGLES, *ABC*, *BCA*, *CAB*.

[I. 13]

BUT,
THE ANGLES, *ACD*, *ACB*, ARE EQUAL, TO TWO RIGHT ANGLES;

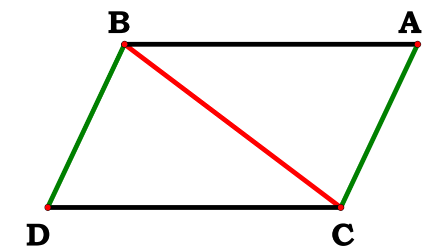
THEREFORE,
THE ANGLES, *ABC*, *BCA*, *CAB*, ARE, ALSO, EQUAL, TO
TWO RIGHT ANGLES.

THEREFORE ETC.

Q. E. D.

PROPOSITION 33.

THE STRAIGHT LINES JOINING EQUAL AND PARALLEL STRAIGHT LINES (*AT THE EXTREMITIES WHICH ARE*) *IN THE SAME DIRECTIONS* (*RESPECTIVELY*) *ARE THEMSELVES*, ALSO, *EQUAL AND PARALLEL*.



LET,
AB, *CD* BE EQUAL AND PARALLEL,

AND LET,
THE STRAIGHT LINES, *AC*, *BD*, JOIN THEM
(AT THE EXTREMITIES WHICH ARE)
IN THE SAME DIRECTIONS (RESPECTIVELY);

I SAY THAT;
AC, *BD* ARE, ALSO, EQUAL AND PARALLEL.

LET,
BC BE JOINED.

[I. 29]

THEN, SINCE,
AB IS PARALLEL TO *CD*, AND
BC HAS FALLEN UPON THEM,
THE ALTERNATE ANGLES, *ABC*, *BCD*, ARE EQUAL, TO
ONE ANOTHER.

AND, SINCE,
AB IS EQUAL, TO *CD*, AND
BC IS COMMON,
THE TWO SIDES, *AB*, *BC*, ARE EQUAL, TO
THE TWO SIDES, *DC*, *CB*; AND
THE ANGLE, *ABC*, IS EQUAL, TO THE ANGLE, *BCD*;

[I. 4]

THEREFORE,
THE BASE, *AC*, IS EQUAL, TO THE BASE, *BD*, AND
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DCB*, AND
THE REMAINING ANGLES WILL BE EQUAL, TO
THE REMAINING ANGLES, RESPECTIVELY,

NAMELY,

THOSE WHICH THE EQUAL SIDES SUBTEND;

THEREFORE,

THE ANGLE, *ACB*, IS EQUAL, TO THE ANGLE, *CBD*.

[I. 27]

AND, SINCE,

THE STRAIGHT LINE, *BC*, FALLING ON
THE TWO STRAIGHT LINES, *AC*, *BD*, HAS MADE
THE ALTERNATE ANGLES EQUAL, TO ONE ANOTHER,
AC IS PARALLEL TO *BD*.

AND,

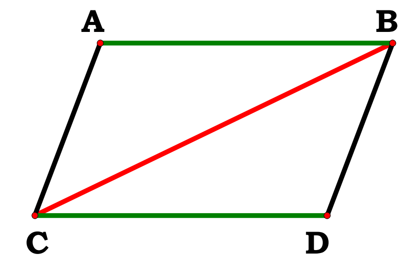
IT WAS, ALSO, PROVED EQUAL, TO IT.

THEREFORE ETC.

Q. E. D.

PROPOSITION 34.

IN PARALLELOGRAMMIC AREAS THE OPPOSITE SIDES AND ANGLES ARE EQUAL, TO ONE ANOTHER, AND THE DIAMETER BISECTS THE AREAS.



LET,

ACDB BE A PARALLELOGRAMMIC AREA, AND
BC ITS DIAMETER;

I SAY THAT;

THE OPPOSITE SIDES AND ANGLES OF THE PARALLELOGRAM,
ACDB, ARE EQUAL, TO ONE ANOTHER, AND
THE DIAMETER, *BC*, BISECTS IT.

[I. 29]

FOR, SINCE,

AB IS PARALLEL TO *CD*, AND
THE STRAIGHT LINE, *BC*, HAS FALLEN UPON THEM,
THE ALTERNATE ANGLES, *ABC*, *BCD*, ARE EQUAL, TO
ONE ANOTHER.

[I. 29]

AGAIN, SINCE,

AC IS PARALLEL TO *BD*, AND
BC HAS FALLEN UPON THEM,
THE ALTERNATE ANGLES, *ACB*, *CBD*, ARE EQUAL, TO
ONE ANOTHER.

[I. 26]

THEREFORE,

ABC, *DCB* ARE TWO TRIANGLES HAVING
THE TWO ANGLES, *ABC*, *BCA*, EQUAL, TO
THE TWO ANGLES, *DCB*, *CBD*, RESPECTIVELY, AND
ONE SIDE EQUAL, TO ONE SIDE,

NAMELY,

THAT ADJOINING THE EQUAL ANGLES, AND
COMMON TO BOTH OF THEM, *BC*;

THEREFORE,

THEY WILL, ALSO, HAVE THE REMAINING SIDES EQUAL, TO

THE REMAINING SIDES, RESPECTIVELY, AND,
THE REMAINING ANGLE TO THE REMAINING ANGLE;

THEREFORE,
THE SIDE, *AB*, IS EQUAL, TO *CD*, AND
AC TO *BD*, AND FURTHER,
THE ANGLE, *BAC*, IS EQUAL, TO THE ANGLE, *CDB*.

[*C. N.* 2]

AND, SINCE,
THE ANGLE, *ABC*, IS EQUAL, TO THE ANGLE, *BCD*, AND
THE ANGLE, *CBD*, TO THE ANGLE, *ACB*,
THE WHOLE ANGLE, *ABD*, IS EQUAL, TO
THE WHOLE ANGLE, *ACD*.

AND,
THE ANGLE, *BAC*, WAS, ALSO, PROVED EQUAL, TO
THE ANGLE, *CDB*.

THEREFORE,
IN PARALLELOGRAMMIC AREAS, THE OPPOSITE SIDES AND
ANGLES ARE EQUAL, TO ONE ANOTHER.

I SAY, NEXT, THAT;
THE DIAMETER, ALSO, BISECTS THE AREAS.

FOR, SINCE,
AB IS EQUAL, TO *CD*, AND
BC IS COMMON,

THE TWO SIDES,
AB, *BC*, ARE EQUAL, TO THE TWO SIDES, *DC*, *CB*,
RESPECTIVELY; AND
THE ANGLE, *ABC*, IS EQUAL, TO THE ANGLE, *BCD*;

[I. 4]

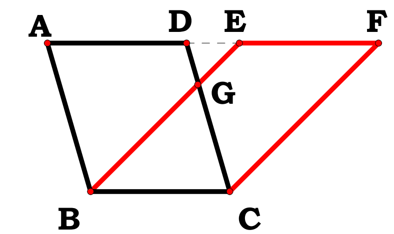
THEREFORE,
THE BASE, *AC*, IS, ALSO, EQUAL, TO *DB*, AND
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DCB*.

THEREFORE,
THE DIAMETER, *BC*, BISECTS THE PARALLELOGRAM, *ACDB*.

Q. E. D.

PROPOSITION 35.

PARALLELOGRAMS WHICH ARE ON THE SAME BASE AND IN THE SAME PARALLELS ARE EQUAL, TO ONE ANOTHER.



LET,
ABCD, *EBCF*, BE PARALLELOGRAMS ON
THE SAME BASE, *BC*, AND
IN THE SAME PARALLELS, *AF*, *BC*;

I SAY THAT;
ABCD IS EQUAL, TO THE PARALLELOGRAM, *EBCF*.

[I. 34]

FOR, SINCE,
ABCD IS A PARALLELOGRAM,
AD IS EQUAL, TO *BC*.

[*C. N.* 1]

FOR THE SAME REASON ALSO,
EF IS EQUAL, TO *BC*,

SO THAT,
AD IS, ALSO, EQUAL, TO *EF*; AND
DE IS COMMON;

[*C. N.* 2]

THEREFORE,
THE WHOLE, *AE*, IS EQUAL, TO THE WHOLE, *DF*.

[I. 34]

BUT,
AB IS, ALSO, EQUAL, TO *DC*;

[I. 29]

THEREFORE,
THE TWO SIDES, *EA*, *AB*, ARE EQUAL, TO
THE TWO SIDES, *FD*, *DC*, RESPECTIVELY, AND
THE ANGLE, *FDC*, IS EQUAL, TO THE ANGLE, *EAB*,
THE EXTERIOR TO THE INTERIOR;

[I. 4]

THEREFORE,

THE BASE, *EB*, IS EQUAL, TO THE BASE, *FC*, AND
THE TRIANGLE, *EAB*, WILL BE EQUAL, TO THE TRIANGLE, *FDC*.

LET,

DGE BE SUBTRACTED FROM EACH;

[*C. N.* 3]

THEREFORE,

THE TRAPEZIUM, *ABGD*, WHICH REMAINS, IS EQUAL, TO
THE TRAPEZIUM, *EGCF*, WHICH REMAINS.

LET,

THE TRIANGLE, *GBC*, BE ADDED TO EACH;

[*C. N.* 2]

THEREFORE,

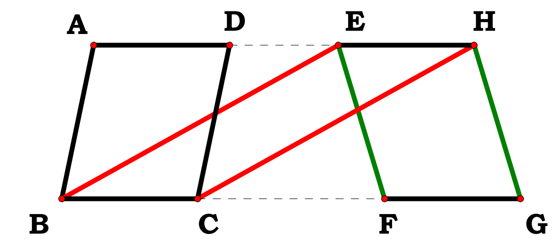
THE WHOLE PARALLELOGRAM, *ABCD*, IS EQUAL, TO
THE WHOLE PARALLELOGRAM, *EBCF*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 36.

PARALLELOGRAMS WHICH ARE ON EQUAL BASES AND IN THE SAME PARALLELS ARE EQUAL, TO ONE ANOTHER.



LET,

ABCD, *EFGH* BE PARALLELOGRAMS
WHICH ARE ON EQUAL BASES, *BC*, *FG*, AND
IN THE SAME PARALLELS, *AH*, *BG*;

I SAY THAT;

THE PARALLELOGRAM, *ABCD*, IS EQUAL, TO *EFGH*.

FOR LET,

BE, *CH* BE JOINED.

[*C. N.* 1]

THEN, SINCE,

BC IS EQUAL, TO *FG*, WHILE ,
FG IS EQUAL, TO *EH*,
BC IS, ALSO, EQUAL, TO *EH*.

BUT,

THEY ARE, ALSO, PARALLEL.

AND,

EB, *HC*, JOIN THEM;

[I. 33]

BUT,

STRAIGHT LINES JOINING EQUAL AND PARALLEL STRAIGHT LINES
(AT THE EXTREMITIES WHICH ARE) IN THE SAME DIRECTIONS
(RESPECTIVELY) ARE EQUAL AND PARALLEL.

[I. 34]

THEREFORE,

EBCH IS A PARALLELOGRAM.

[I. 35]

AND,

IT IS EQUAL, TO *ABCD*;

FOR,

IT HAS THE SAME BASE, *BC*, WITH IT, AND
IS IN THE SAME PARALLELS, *BC*, *AH* WITH IT.

[I. 35]

FOR THE SAME REASON ALSO,

EFGH IS EQUAL, TO THE SAME *EBCH*

[*C. N.* 1]

SO THAT

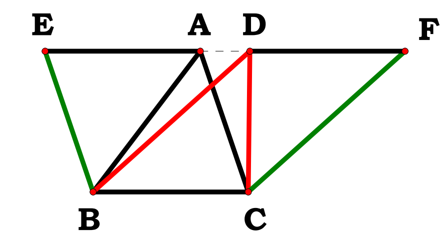
THE PARALLELOGRAM, *ABCD*, IS, ALSO, EQUAL, TO *EFGH*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 37.

TRIANGLES WHICH ARE ON THE SAME BASE AND IN THE SAME PARALLELS ARE EQUAL, TO ONE ANOTHER.



LET,

ABC, *DBC* BE TRIANGLES ON THE SAME BASE, *BC*, AND
IN THE SAME PARALLELS *AD*, *BC*;

I SAY THAT;

THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DBC*.

LET,

AD BE PRODUCED IN BOTH DIRECTIONS TO *E*, *F*;

[I. 31]

LET,

THROUGH *B*,
BE, BE DRAWN PARALLEL TO *CA*,

[I. 31]

AND LET,

THROUGH *C*,
CF BE DRAWN PARALLEL TO *BD*.

THEN,

EACH, OF THE FIGURES;
EBCA, *DBCF* IS A PARALLELOGRAM; AND
THEY ARE EQUAL,

[I. 35]

FOR,

THEY ARE ON THE SAME BASE, *BC*, AND
IN THE SAME PARALLELS, *BC*, *EF*.

MOREOVER,

THE TRIANGLE, *ABC*, IS HALF OF THE PARALLELOGRAM, *EBCA*;

[I. 34]

FOR,

THE DIAMETER, *AB*, BISECTS IT.

AND,

THE TRIANGLE, *DBC*, IS HALF OF THE PARALLELOGRAM, *DBCF*;

[I. 34]

FOR,

THE DIAMETER, *DC*, BISECTS IT.
[BUT THE HALVES OF EQUAL THINGS ARE EQUAL, TO ONE ANOTHER.]

THEREFORE,

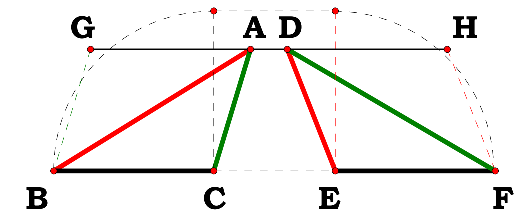
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DBC*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 38.

TRIANGLES WHICH ARE ON EQUAL BASES AND IN THE SAME PARALLELS ARE EQUAL, TO ONE ANOTHER.



LET,

ABC, *DEF* BE TRIANGLES ON EQUAL BASES, *BC*, *EF*, AND
IN THE SAME PARALLELS, *BF*, *AD*;

I SAY THAT;

THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DEF*.

FOR LET,

AD BE PRODUCED IN BOTH DIRECTIONS TO *G*, *H*;

[I. 31]

LET,

THROUGH *B*,
BG BE DRAWN PARALLEL TO *CA*,

AND LET,

THROUGH *F*,
FH BE DRAWN PARALLEL TO *DE*.

THEN,

EACH, OF THE FIGURES,
GBCA, *DEFH*, IS A PARALLELOGRAM; AND
GBCA IS EQUAL, TO *DEFH*;

[I. 36]

FOR,

THEY ARE ON EQUAL BASES, *BC*, *EF*, AND
IN THE SAME PARALLELS, *BF*, *GH*.

[I. 34]

MOREOVER,

THE TRIANGLE, *ABC*, IS HALF OF
THE PARALLELOGRAM, *GBCA*;

FOR,

THE DIAMETER *AB* BISECTS IT.

[I. 34]

AND,

THE TRIANGLE, *FED*, IS HALF OF
THE PARALLELOGRAM, *DEFH*,

FOR,

THE DIAMETER *DF* BISECTS IT.
[BUT THE HALVES OF EQUAL THINGS ARE EQUAL, TO ONE ANOTHER.]

THEREFORE,

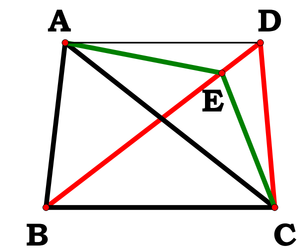
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DEF*.

THEREFORE ETC.

Q. E. D.

PROPOSITION 39.

EQUAL TRIANGLES WHICH ARE ON THE SAME BASE AND ON THE SAME SIDE ARE, ALSO, *IN THE SAME PARALLELS*.



LET,

ABC, *DBC* BE EQUAL TRIANGLES WHICH ARE ON
THE SAME BASE, *BC*, AND
ON THE SAME SIDE OF IT;

[I SAY THAT;

THEY ARE, ALSO, IN THE SAME PARALLELS.]

AND [FOR] LET,

AD BE JOINED;

I SAY THAT;

AD IS PARALLEL TO *BC*.

FOR,

IF NOT,

[I. 31]

LET,

AE BE DRAWN THROUGH THE POINT, *A*, PARALLEL TO
THE STRAIGHT LINE, *BC*,

AND LET,

EC BE JOINED.

THEREFORE,

THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *EBC*;

[I. 37]

FOR,

IT IS ON THE SAME BASE, *BC*, WITH IT, AND,
IN THE SAME PARALLELS.

BUT,

ABC IS EQUAL, TO *DBC*;

[*C. N.* 1]

THEREFORE,

DBC IS, ALSO, EQUAL, TO *EBC*,

THE GREATER TO THE LESS:

WHICH,
IS IMPOSSIBLE.

THEREFORE,
AE IS NOT PARALLEL TO *BC*.

SIMILARLY, WE CAN PROVE THAT;
NEITHER IS ANY OTHER STRAIGHT LINE EXCEPT *AD*;

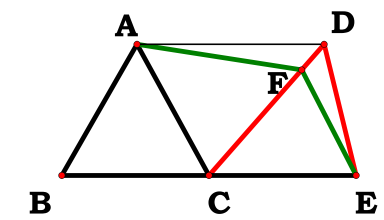
THEREFORE,
AD IS PARALLEL TO *BC*.

THEREFORE ETC.

Q. E. D.

[PROPOSITION 40.

EQUAL TRIANGLES WHICH ARE ON EQUAL BASES AND ON THE SAME SIDE ARE, ALSO, *IN THE SAME PARALLELS*.



LET,
ABC, *CDE* BE EQUAL TRIANGLES ON
EQUAL BASES, *BC*, *CE*, AND
ON THE SAME SIDE.

I SAY THAT;
THEY ARE, ALSO, IN THE SAME PARALLELS.

FOR LET,
AD BE JOINED;

I SAY THAT;
AD IS PARALLEL TO *BE*.

FOR,
IF NOT,

[I. 31]

LET
AF BE DRAWN, THROUGH *A*, PARALLEL TO *BE*,

AND LET
FE BE JOINED.

THEREFORE,
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *FCE*;

[I. 38]

FOR
THEY ARE ON EQUAL BASES, *BC*, *CE*, AND,
IN THE SAME PARALLELS, *BE*, *AF*.

BUT
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *DCE*;

[*C N*. 1]

THEREFORE,
THE TRIANGLE, *DCE*, IS, ALSO, EQUAL, TO THE TRIANGLE, *FCE*,
THE GREATER TO THE LESS:

which
is impossible.

Therefore,
AF is not parallel to *BE*.

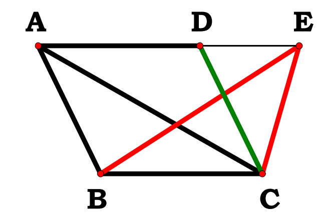
Similarly, we can prove that;
neither is any other straight line except *AD*;

therefore,
AD is parallel to *BE*.

Therefore etc.

Q. E. D.]

Proposition 41.



If a parallelogram have the same base with a triangle and be in the same parallels, the parallelogram is double of the triangle.

For let,
the parallelogram, *ABCD*, have the same base, *BC* with the triangle, *EBC*,

and let,
it be in the same parallels, *BC*, *AE*;

I say that;
the parallelogram,
ABCD, is double of the triangle, *BEC*.

For let,
AC be joined.

Then,
the triangle, *ABC*, is equal, to the triangle, *EBC*;

[I. 37]

for,
it is on the same base, *BC*, with it, and
in the same parallels, *BC*, *AE*.

[I. 34]

But,
the parallelogram,
ABCD, is double of the triangle, *ABC*;

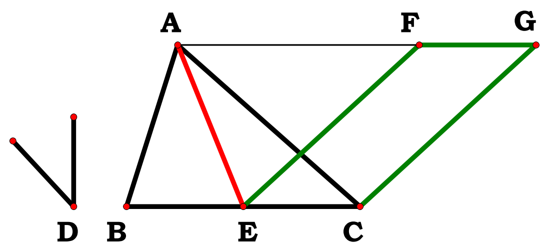
for,
the diameter, *AC*, bisects it;

so,
that the parallelogram,
ABCD, is, also, double of the triangle, *EBC*.

Therefore etc.

Q. E. D.

Proposition 42.



To construct,
in a given rectilineal angle,
a parallelogram equal,
to a given triangle.

Let,
ABC be the given triangle, and
D, the given rectilineal angle;

thus it is required,
to construct in the rectilineal angle, *D*,
a parallelogram equal, to the triangle, *ABC*.

Let,
BC be bisected at *E*,

and let,
AE be joined;

[I. 23]

on,
the straight line, *EC*, and
at the point, *E*, on it,

let,
the angle, *CEF*, be constructed equal, to the angle, *D*,

[I. 31]

let
through *A*,
AG be drawn parallel to *EC*,

and let
through *C*,
CG be drawn parallel to *EF*.

Then
FECG is a parallelogram.

[I. 38]

And, since,
BE is equal, to *EC*,
the triangle, *ABE*, is, also, equal, to the triangle, *AEC*,

for,
they are on equal bases, *BE*, *EC*, and
in the same parallels, *BC*, *AG*;

therefore,
the triangle, *ABC*, is double of the triangle, *AEC*.

[I. 41]

But,
the parallelogram,
FECG, is, also, double of the triangle, *AEC*,

for,
it has the same base with it, and
is in the same parallels with it;

therefore,
the parallelogram,
FECG, is equal, to the triangle, *ABC*.

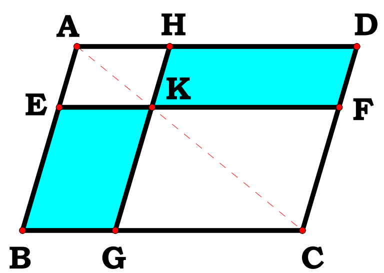
And,
it has the angle, *CEF*, equal, to the given angle, *D*.

Therefore,
the parallelogram, *FECG*,
has been constructed equal, to
the given triangle, *ABC*,
in the angle, *CEF*, which is equal, to *D*.

Q. E. F.

PROPOSITION 43.

IN ANY PARALLELOGRAM THE COMPLEMENTS OF THE PARALLELOGRAMS ABOUT THE DIAMETER ARE EQUAL, TO ONE ANOTHER.



LET,

ABCD BE A PARALLELOGRAM, AND
AC, ITS DIAMETER; AND ABOUT *AC*,

LET,

EH, *FG* BE PARALLELOGRAMS, AND
BK, *KD*, THE SO-CALLED COMPLEMENTS;

I SAY THAT;

THE COMPLEMENT, *BK*, IS EQUAL, TO THE COMPLEMENT, *KD*.

[I. 34]

FOR, SINCE,

ABCD IS A PARALLELOGRAM, AND *AC* ITS DIAMETER,
THE TRIANGLE, *ABC*, IS EQUAL, TO THE TRIANGLE, *ACD*.

AGAIN, SINCE,

EH IS A PARALLELOGRAM, AND *AK* IS ITS DIAMETER,
THE TRIANGLE, *AEK*, IS EQUAL, TO THE TRIANGLE, *AHK*.

FOR THE SAME REASON,

THE TRIANGLE, *KFC*, IS, ALSO, EQUAL, TO *KGC*.

NOW, SINCE,

THE TRIANGLE, *AEK*, IS EQUAL, TO THE TRIANGLE, *AHK*, AND
KFC TO *KGC*,

[*C. N.* 2]

THE TRIANGLE, *AEK*, TOGETHER WITH *KGC* IS EQUAL, TO
THE TRIANGLE, *AHK*, TOGETHER WITH *KFC*.

AND,

THE WHOLE TRIANGLE, *ABC*, IS, ALSO, EQUAL, TO
THE WHOLE, *ADC*;

[*C. N.* 3]

THEREFORE,

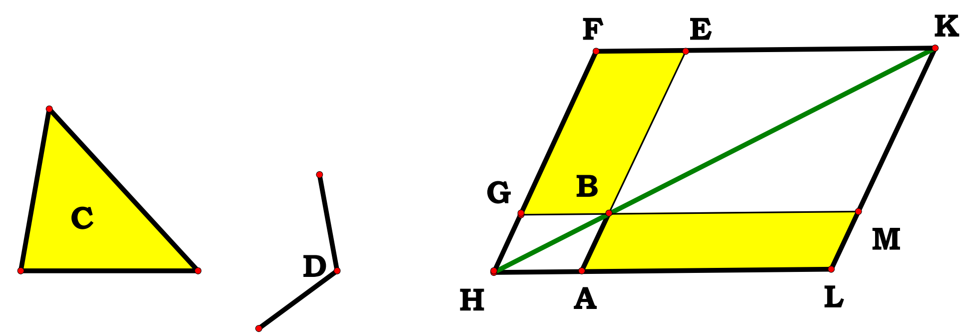
THE COMPLEMENT, *BK*, WHICH REMAINS, IS EQUAL, TO
THE COMPLEMENT, *KD*, WHICH REMAINS.

THEREFORE ETC.

Q. E. D.

Proposition 44.

To a given straight line to apply, in a given rectilineal angle, a parallelogram equal, to a given triangle.



Let,

AB, be the given straight line,

C, the given triangle, and

D, the given rectilineal angle;

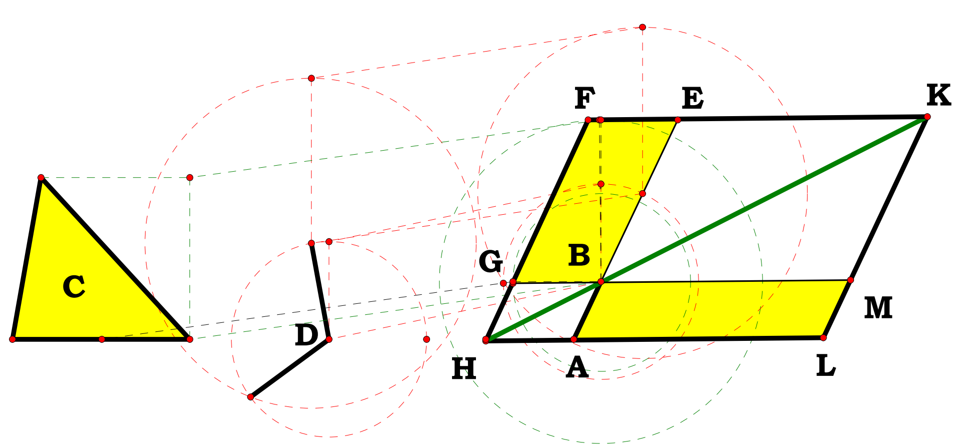
thus it is required,

to apply to the given straight line, *AB*,

in an angle equal, to the angle, *D*,

a parallelogram equal, to the given triangle, *C*.

[I. 42]



Let,

the parallelogram, *BEFG*, be constructed equal, to

the triangle *C*,

in the angle, *EBG*, which is equal, to *D*;

let,

it be placed,

so that;

BE is in a straight line with *AB*;

let,

FG be drawn through, to *H*,

[I. 31]

and let,

AH be drawn, through *A*, parallel to either *BG* or *EF*.

Let,

HB be joined.

[I. 29]

Then, since,

the straight line, *HF*, falls upon the parallels, *AH*, *EF*,

the angles, *AHF*, *HFE*, are equal, to two right angles.

Therefore,

the angles, *BHG*, *GFE*, are less than two right angles;

[Post. 5]

and,

straight lines, produced indefinitely,

from angles less than two right angles, meet;

therefore,

HB, *FE*, when produced, will meet.

Let,

them be produced, and meet at *K*;

[I. 31]

let through,

the point, *K*, *KL*, be drawn parallel to

either, *EA* or *FH*,

and let,

HA, *GB* be produced to the points, *L*, *M*.

Then,

HLKF is a parallelogram,

HK is its diameter, and
AG, *ME* are parallelograms, and
LB, *BF*, the so-called complements, about *HK*;

[I. 43]

therefore,
LB is equal, to *BF*.

But,
BF is equal, to the triangle *C*;

[*C. N.* 1]

therefore,
LB is, also, equal, to *C*.

[I. 15]

And, since,
the angle, *GBE*, is equal, to the angle, *ABM*,

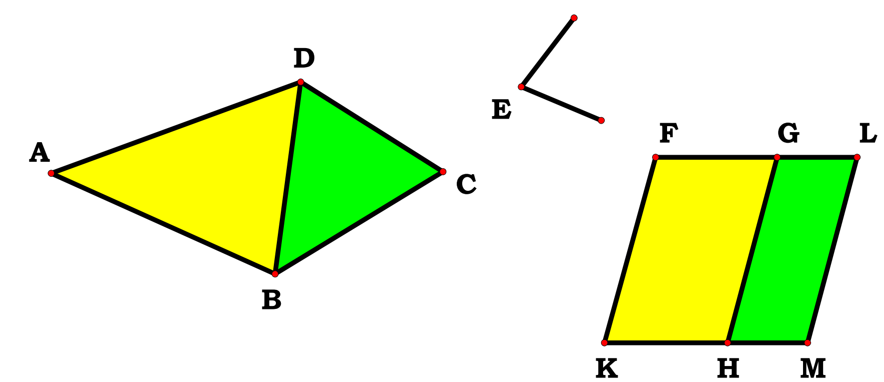
while,
the angle, *GBE*, is equal, to *D*,
the angle, *ABM*, is, also, equal, to the angle, *D*.

Therefore,
the parallelogram, *LB*, equal, to the given triangle, *C*,
has been applied to the given straight line, *AB*,
in the angle, *ABM*, which is equal, to *D*.

Q. E. F.

Proposition 45.

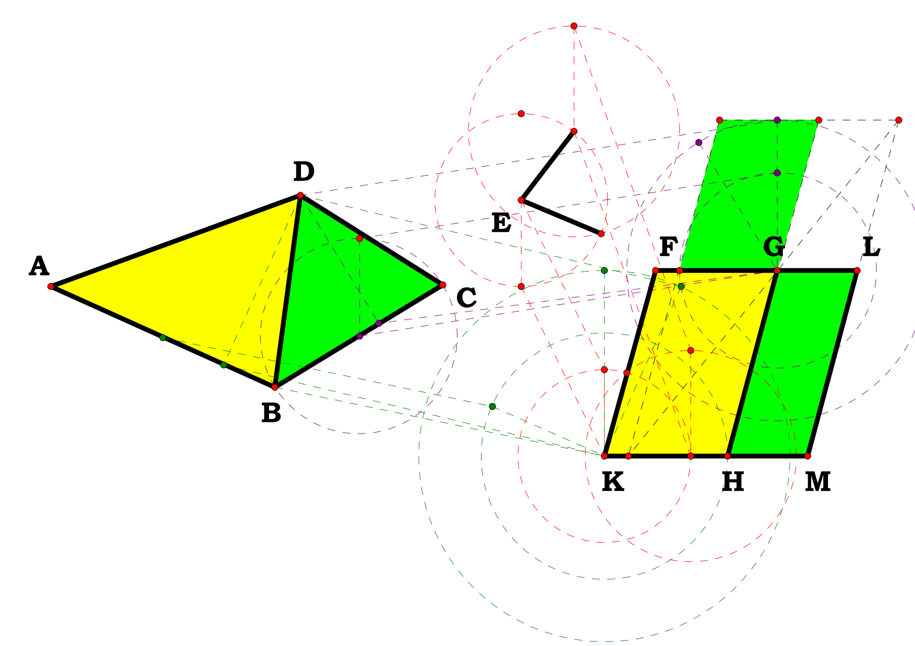
To construct, in a given rectilineal angle, a parallelogram equal, to a given rectilineal figure.



Let,
ABCD be the given rectilineal figure, and
E, the given rectilineal angle;

thus it is required,
to construct, in the given angle, *E*,
a parallelogram equal, to
the rectilineal figure, *ABCD*.

[I. 42]



Let,

DB be joined,

and let,

the parallelogram, *FH*, be constructed equal, to
the triangle, *ABD*, in the angle, *HKF*,

which,

is equal, to *E*;

[I. 44]

let,

the parallelogram, *GM*, equal, to
the triangle, *DBC*, be applied to
the straight line, *GH*, in
the angle, *GHM*, which is equal, to *E*.

[*C. N.* 1]

Then, since,

the angle, *E*, is equal, to each, of
the angles, *HKF*, *GHM*,
the angle, *HKF*, is, also, equal, to the angle, *GHM*.

Let,

the angle, *KHG*, be added to each;

therefore,

the angles, *FKH*, *KHG*, are equal, to
the angles, *KHG*, *GHM*.

[I. 29]

But,

the angles, *FKH*, *KHG*, are equal, to two right angles;

therefore,

the angles, *KHG*, *GHM*, are, also, equal, to
two right angles.

[I. 14]

Thus,

with a straight line, *GH*, and
at the point, *H*, on it,
the two straight lines, *KH*, *HM*,
not lying on the same side,
make the adjacent angles equal, to two right angles;

therefore,

KH is in a straight line with *HM*.

[I. 29]

And, since,

the straight line, *HG*, falls upon
the parallels, *KM*, *FG*,
the alternate angles, *MHG*, *HGF*, are equal, to
one another.

Let,

the angle *HGL* be added to each;

[*C. N.* 2]

therefore,

the angles, *MHG*, *HGL*, are equal, to
the angles, *HGF*, *HGL*.

[I. 29]

But,

the angles, *MHG*, *HGL*, are equal, to two right angles;

[*C. N.* 1]

therefore,

the angles, *HGF*, *HGL*, are, also, equal, to
two right angles.

[I. 14]

Therefore,

FG is in a straight line with *GL*.

[I. 34]

And, since,

FK is equal and parallel to *HG*, and
HG to *ML*,

[*C. N.* 1; I. 30]

also,

KF IS EQUAL AND PARALLEL TO *ML*; AND
THE STRAIGHT LINES, *KM*, *FL*, JOIN THEM,
(AT THEIR EXTREMITIES);

[I. 33]

THEREFORE,
KM, *FL* ARE, ALSO, EQUAL AND PARALLEL.

THEREFORE,
KFLM IS A PARALLELOGRAM.

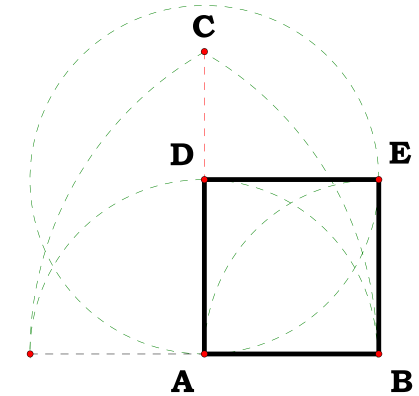
AND, SINCE,
THE TRIANGLE, *ABD*, IS EQUAL, TO
THE PARALLELOGRAM, *FH*, AND
DBC TO *GM*,
THE WHOLE RECTILINEAL FIGURE,
ABCD, IS EQUAL, TO THE WHOLE PARALLELOGRAM, *KFLM*.

THEREFORE,
THE PARALLELOGRAM,
KFLM, HAS BEEN CONSTRUCTED EQUAL, TO
THE GIVEN RECTILINEAL FIGURE, *ABCD*,
IN THE ANGLE, *FKM*, WHICH IS EQUAL, TO THE GIVEN ANGLE, *E*.

Q. E. F.

PROPOSITION 46.

ON A GIVEN STRAIGHT LINE TO DESCRIBE A SQUARE.



LET,
AB BE THE GIVEN STRAIGHT LINE;

THUS IT IS REQUIRED,
TO DESCRIBE A SQUARE, ON
THE STRAIGHT LINE, *AB*.

[I. 11]

LET,
AC BE DRAWN AT RIGHT ANGLES TO
THE STRAIGHT LINE, *AB* , FROM THE POINT *A* ON IT,

AND LET,
AD BE MADE EQUAL, TO *AB*;

LET,
THROUGH THE POINT *D*,
DE BE DRAWN PARALLEL TO *AB*,

[I. 31]

AND LET,
THROUGH THE POINT, *B*,
BE, BE DRAWN PARALLEL TO *AD*.

THEREFORE,
ADEB IS A PARALLELOGRAM;

[I. 34]

THEREFORE,
AB IS EQUAL, TO *DE*, AND,
AD TO *BE*.

BUT,
AB IS EQUAL, TO *AD*;

THEREFORE,
THE FOUR STRAIGHT LINES,
BA, *AD*, *DE*, *EB*, ARE EQUAL, TO ONE ANOTHER;

THEREFORE,

THE PARALLELOGRAM, *ADEB*, IS EQUILATERAL.

I SAY NEXT THAT;
IT IS, ALSO, RIGHT-ANGLED.

[I. 29]

FOR, SINCE,
THE STRAIGHT LINE, *AD*, FALLS UPON
THE PARALLELS, *AB*, *DE*,
THE ANGLES, *BAD*, *ADE*, ARE EQUAL, TO TWO RIGHT ANGLES.

BUT,
THE ANGLE *BAD* IS RIGHT;

THEREFORE,
THE ANGLE *ADE* IS, ALSO, RIGHT.

[I. 34]

AND,
IN PARALLELOGRAMMIC AREAS THE OPPOSITE SIDES, AND,
ANGLES ARE EQUAL, TO ONE ANOTHER;

THEREFORE,
EACH, OF THE OPPOSITE ANGLES, *ABE*, *BED*, IS, ALSO, RIGHT.

THEREFORE,
ADEB IS RIGHT-ANGLED.

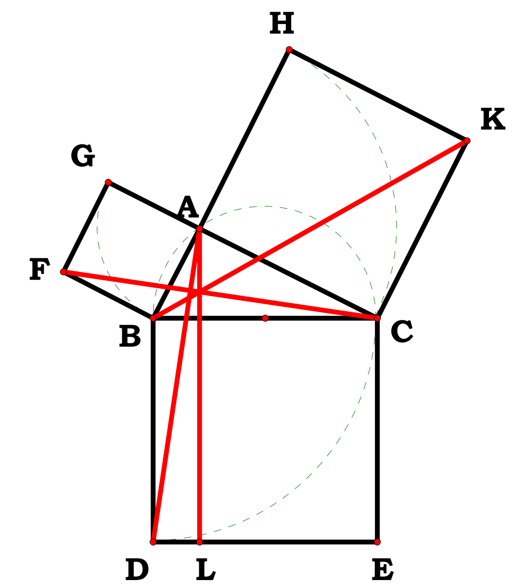
AND,
IT WAS, ALSO, PROVED EQUILATERAL.

THEREFORE,
IT IS A SQUARE; AND
IT IS DESCRIBED ON THE STRAIGHT LINE, *AB*.

Q. E. F.

PROPOSITION 47.

IN RIGHT-ANGLED TRIANGLES THE SQUARE, ON THE SIDE SUBTENDING THE RIGHT ANGLE IS EQUAL, TO THE SQUARES ON THE SIDES CONTAINING THE RIGHT ANGLE.



LET,
ABC BE
A RIGHT-ANGLED TRIANGLE

HAVING,
THE ANGLE, *BAC*, RIGHT;

I SAY THAT;
THE SQUARE, ON *BC*, IS EQUAL, TO
THE SQUARES, ON *BA*, *AC*.

[I. 46]

FOR LET,
THERE BE DESCRIBED, ON *BC*, THE SQUARE, *BDEC*, AND
ON *BA*, *AC*, THE SQUARES, *GB*, *HC*;

LET,
THROUGH *A*,
AL BE DRAWN PARALLEL TO EITHER, *BD* OR *CE*,

AND LET,
AD, *FC* BE JOINED.

THEN, SINCE,
EACH, OF THE ANGLES, *BAC*, *BAG* IS RIGHT,

IT FOLLOWS THAT;
WITH A STRAIGHT LINE *BA*, AND
AT THE POINT *A* ON IT,
THE TWO STRAIGHT LINES, *AC*, *AG*,
NOT LYING ON THE SAME SIDE, MAKE
THE ADJACENT ANGLES EQUAL, TO TWO RIGHT ANGLES;

[I. 14]

THEREFORE,
CA IS IN A STRAIGHT LINE WITH *AG*.

For the same reason,
BA is, also, in a straight line with *AH*.

And, since,
the angle, *DBC*, is equal, to the angle, *FBA*:

for,
each is right:

let,
the angle, *ABC*, be added to each;

[*C. N.* 2]

therefore,
the whole angle, *DBA*, is equal, to
the whole angle, *FBC*,

And, since,
DB is equal, to *BC*, and
FB to *BA*,

[I. 4]

therefore,
the two sides, *AB*, *BD*, are equal, to
the two sides, *FB*, *BC*, respectively, and
the angle, *ABD*, is equal, to the angle, *FBC*;

therefore,
the base, *AD*, is equal, to the base, *FC*, and
the triangle, *ABD*, is equal, to the triangle, *FBC*.

[I. 41]

Now,
the parallelogram, *BL*, is double of
the triangle, *ABD*,

for,
they have the same base, *BD*, and
are in the same parallels, *BD*, *AL*.

[I. 41]

And,
the square, *GB*, is double of the triangle, *FBC*,

for,
they again have the same base, *FB*, and
are in the same parallels, *FB*, *GC*,
[But the doubles of equals are equal, to one another.]

Therefore,
the parallelogram, *BL*, is, also, equal, to
the square, *GB*.

Similarly,
if *AE*, *BK* be joined,
the parallelogram,
CL, can, also, be proved equal, to the square, *HC*;

[*C. N.* 2]

therefore,
the whole square, *BDEC*, is equal, to
the two squares, *GB*, *HC*.

And,
the square, *BDEC*, is described, on *BC*, and
the squares, *GB*, *HC*, on *BA*, *AC*.

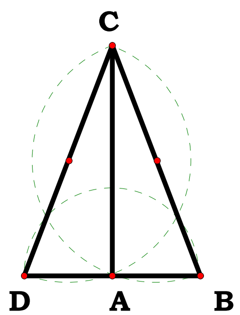
Therefore,
the square, on the side, *BC*, is equal, to
the squares, on the sides, *BA*, *AC*.

Therefore etc.

Q. E. D.

PROPOSITION 48.

IF IN A TRIANGLE THE SQUARE, ON ONE OF THE SIDES BE EQUAL, TO THE SQUARES ON THE REMAINING TWO SIDES OF THE TRIANGLE, THE ANGLE CONTAINED BY THE REMAINING TWO SIDES OF THE TRIANGLE IS RIGHT.



FOR,
IN THE TRIANGLE, *ABC*,

LET,
THE SQUARE, ON ONE SIDE, *BC*, BE EQUAL, TO
THE SQUARES, ON THE SIDES, *BA*, *AC*;

I SAY THAT;
THE ANGLE, *BAC*, IS RIGHT.

FOR LET,
AD, BE DRAWN FROM THE POINT, *A*,
AT RIGHT ANGLES TO THE STRAIGHT LINE, *AC*,

LET,
AD BE MADE EQUAL, TO *BA*,

AND LET,
DC BE JOINED.

SINCE,
DA IS EQUAL, TO *AB*,
THE SQUARE, ON *DA*, IS, ALSO, EQUAL, TO THE SQUARE, ON *AB*.

LET,
THE SQUARE, ON *AC*, BE ADDED TO EACH;

THEREFORE,
THE SQUARES, ON *DA*, *AC*, ARE EQUAL, TO
THE SQUARES, ON *BA*, *AC*.

[I. 47]

BUT,
THE SQUARE, ON *DC*, IS EQUAL, TO THE SQUARES, ON *DA*, *AC*,

FOR,
THE ANGLE, *DAC*, IS RIGHT; AND
THE SQUARE, ON *BC*, IS EQUAL, TO THE SQUARES, ON *BA*, *AC*,

FOR,
THIS IS THE HYPOTHESIS;

THEREFORE,
THE SQUARE, ON *DC*, IS EQUAL, TO THE SQUARE, ON *BC*,

SO THAT,
THE SIDE, *DC*, IS, ALSO, EQUAL, TO *BC*.

[I. 8]

AND, SINCE,
DA IS EQUAL, TO *AB*, AND
AC IS COMMON,
THE TWO SIDES, *DA*, *AC*, ARE EQUAL, TO
THE TWO SIDES, *BA*, *AC*; AND
THE BASE, *DC*, IS EQUAL, TO THE BASE, *BC*;

THEREFORE,
THE ANGLE, *DAC*, IS EQUAL, TO THE ANGLE, *BAC*,

BUT,
THE ANGLE *DAC* IS RIGHT;

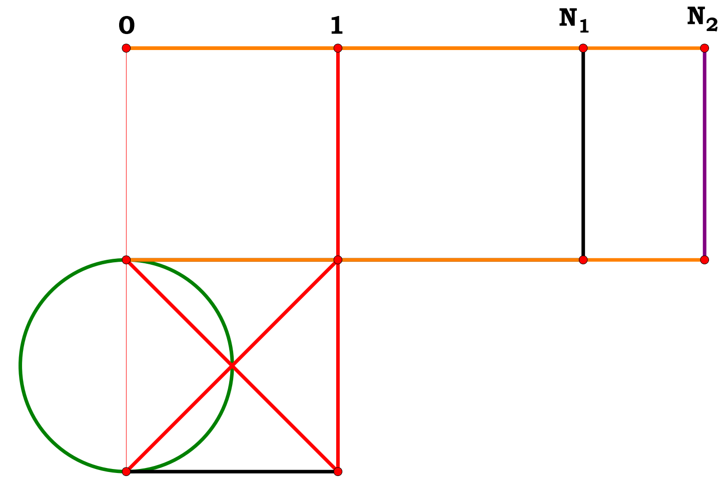
THEREFORE,
THE ANGLE, *BAC*, IS, ALSO, RIGHT.

THEREFORE ETC.

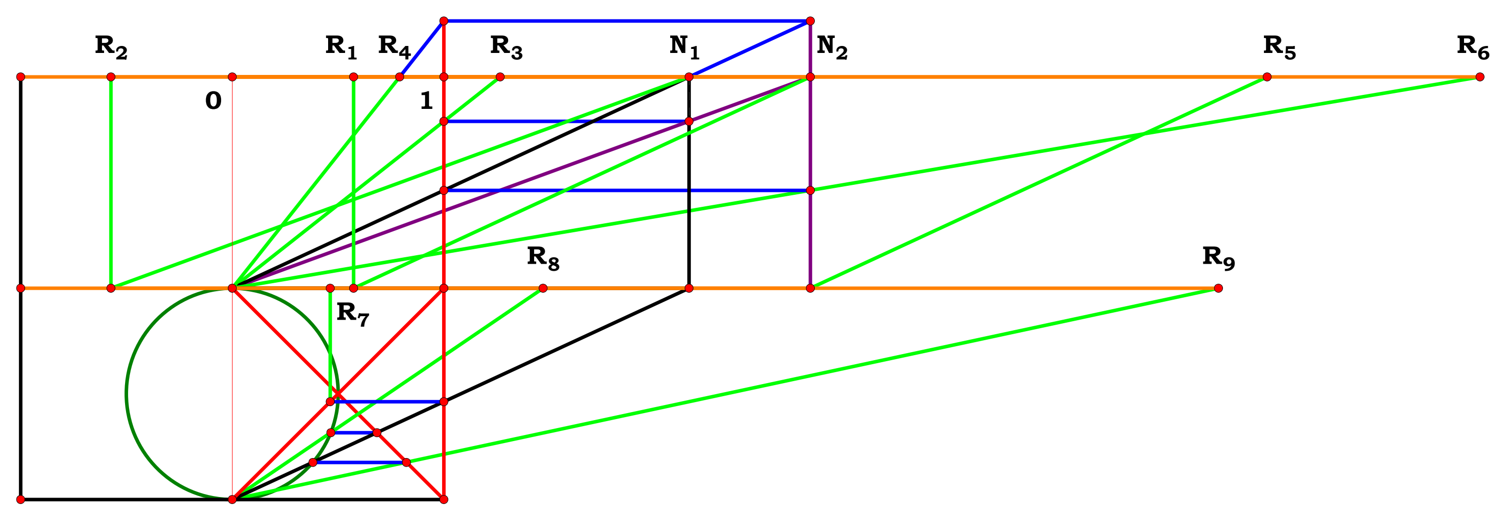
Q. E. D

Basic Analog Mathematics.

Let 0 to 1 be the given Unit and N1 and N2 be any two given differences:



With the given analog (figure), it is required to render the products of these two differences using the paradigms sum, differences, and ratio:--



$N_2 - N_1 - R_1 = 0.00000$

$N_1 - N_2 - R_2 = 0.00000$

$(N_1 + N_2) - R_5 = 0.00000$

$\frac{N_2}{N_1} - R_3 = 0.00000$

$\frac{N_1}{N_2} - R_4 = 0.00000$

$\frac{1}{N_1} - R_7 = 0.00000$

$N_1 \cdot N_2 - R_6 = 0.00000$

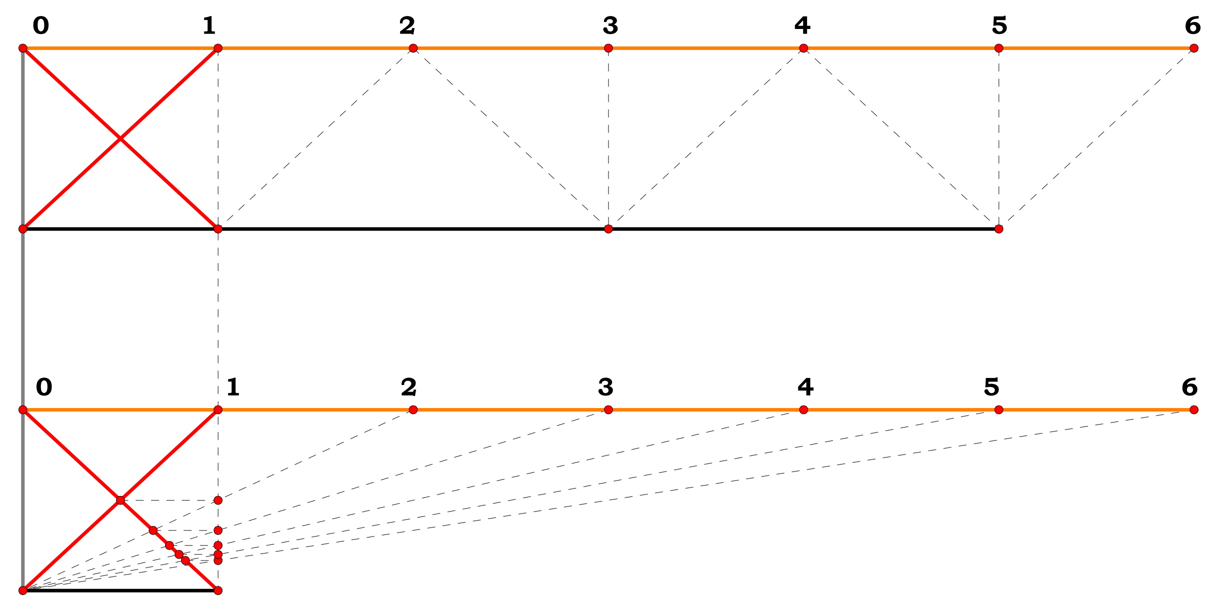
$\sqrt{N_1} - R_8 = 0.00000$

$N_1^2 - R_9 = 0.00000$

It should thus be clear that the so called mathematical paradigms exist *a priori* to Arithmetic and Algebraic Logic, that is--language systems--or in other words, Analogic precedes Logic. Or again, in a metaphor, perception determines conception, conception determines will, or in a more ancient metaphor, The Father, The Son, and the Holy Spirit are One. This fact means that the real Theory of Relativity is no more than the linguistic principles of Analogic.

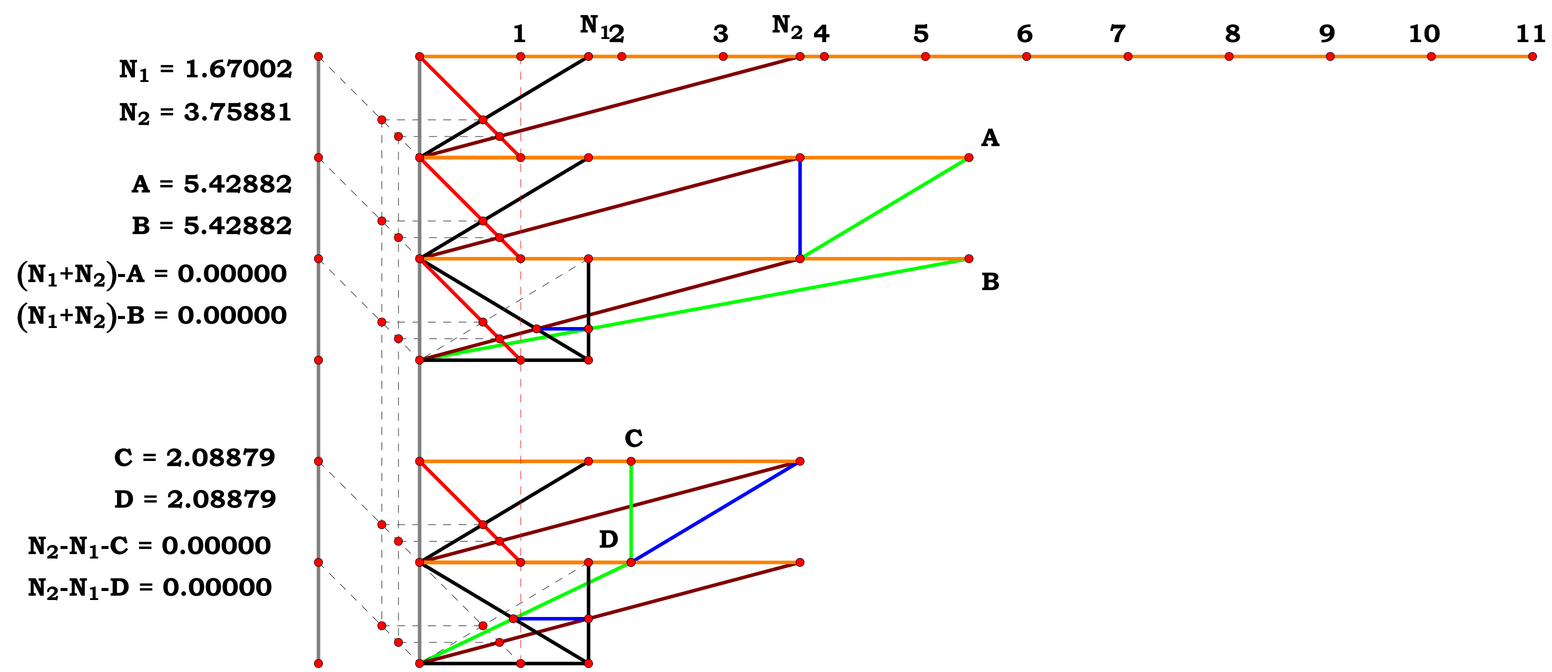
Complete Induction.

or counting.

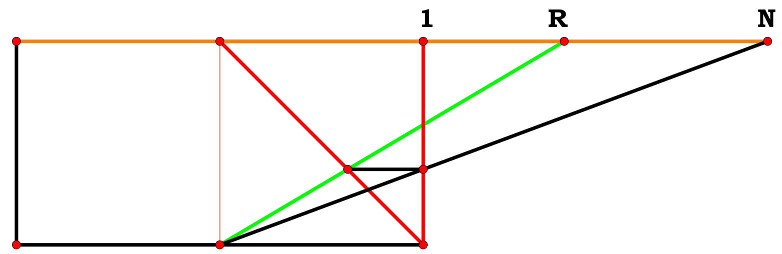


Basic paradigms, equal and unequal, greater and less, sum and difference, ratio.

Addition and Subtraction.

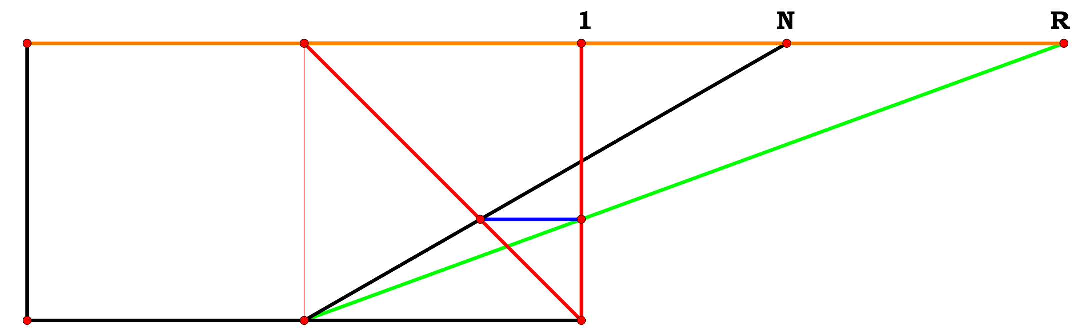


1CST4R1



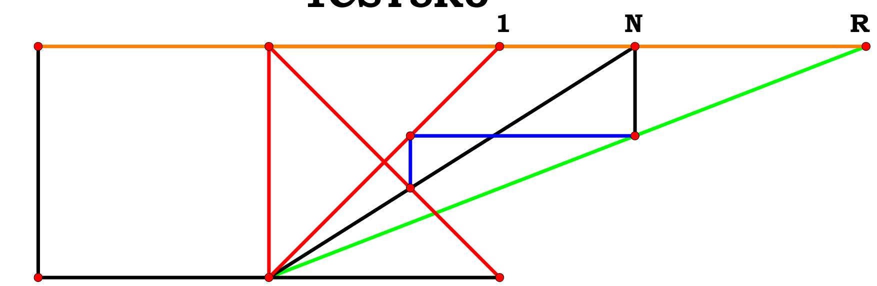
N = 2.69220
N-1-R = 0.00000
R = 1.69220

1CST4R4



N = 1.74038
(N+1)-R = 0.00000
R = 2.74038

1CST5R6



N = 1.58678
(N+1)-R = 0.00000
R = 2.58678

Multiplication and Division

$N_1 = 2.30957$

$N_2 = 4.43080$

A = 10.23326

B = 10.23326

C = 10.23326

D = 10.23326

$N_1 \cdot N_2$-A = 0.00000

$N_1 \cdot N_2$-B = 0.00000

$N_1 \cdot N_2$-C = 0.00000

$N_1 \cdot N_2$-D = 0.00000

E = 0.52125

F = 0.52125

G = 0.52125

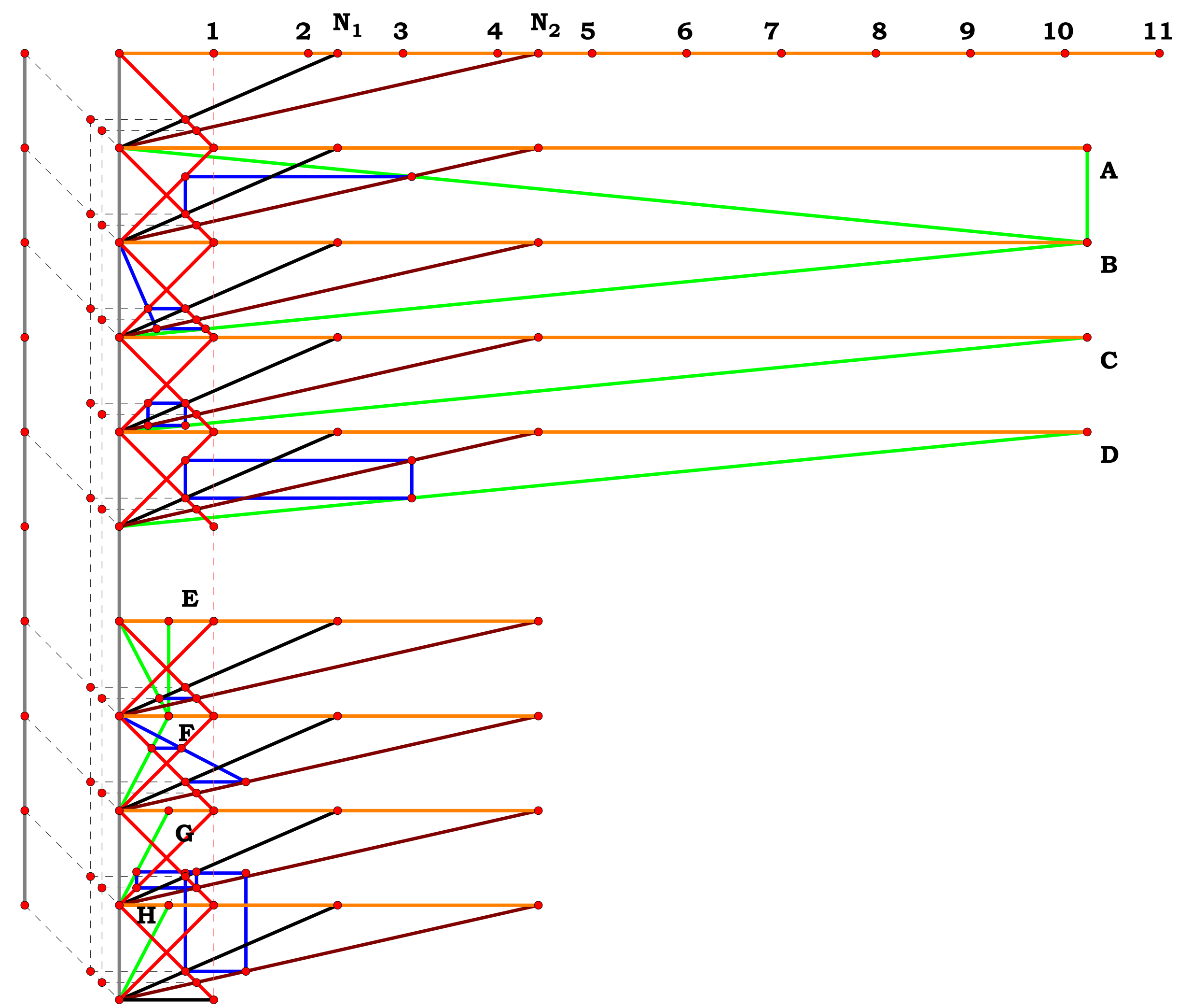
H = 0.52125

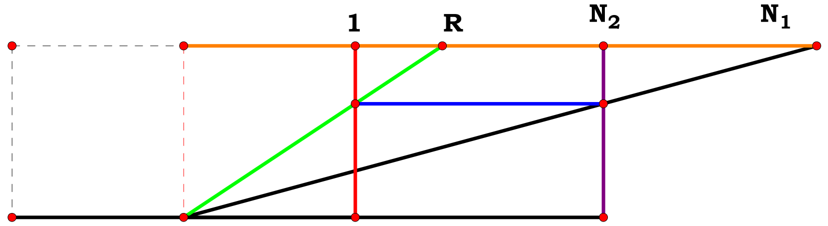
$\frac{N_1}{N_2}$-E = 0.00000

$\frac{N_1}{N_2}$-F = 0.00000

$\frac{N_1}{N_2}$-G = 0.00000

$\frac{N_1}{N_2}$-H = 0.00000



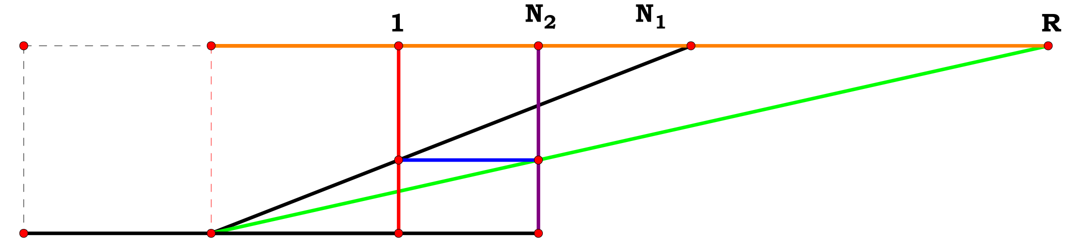


$N_1 = 3.68640$

$N_2 = 2.44547$

$R = 1.50744$

$$\frac{N_1}{N_2} - R = 0.00000$$



$N_1 = 2.55862$

$N_2 = 1.74503$

$R = 4.46487$

$N_1 \cdot N_2 - R = 0.00000$

Square, Root and Reciprocal.

N = 1.68576

A = 2.84179

B = 2.84179

C = 2.84179

D = 2.84179

E = 2.84179

F = 2.84179

G = 1.29837

H = 0.59320

N^2-A = 0.00000

N^2-B = 0.00000

N^2-C = 0.00000

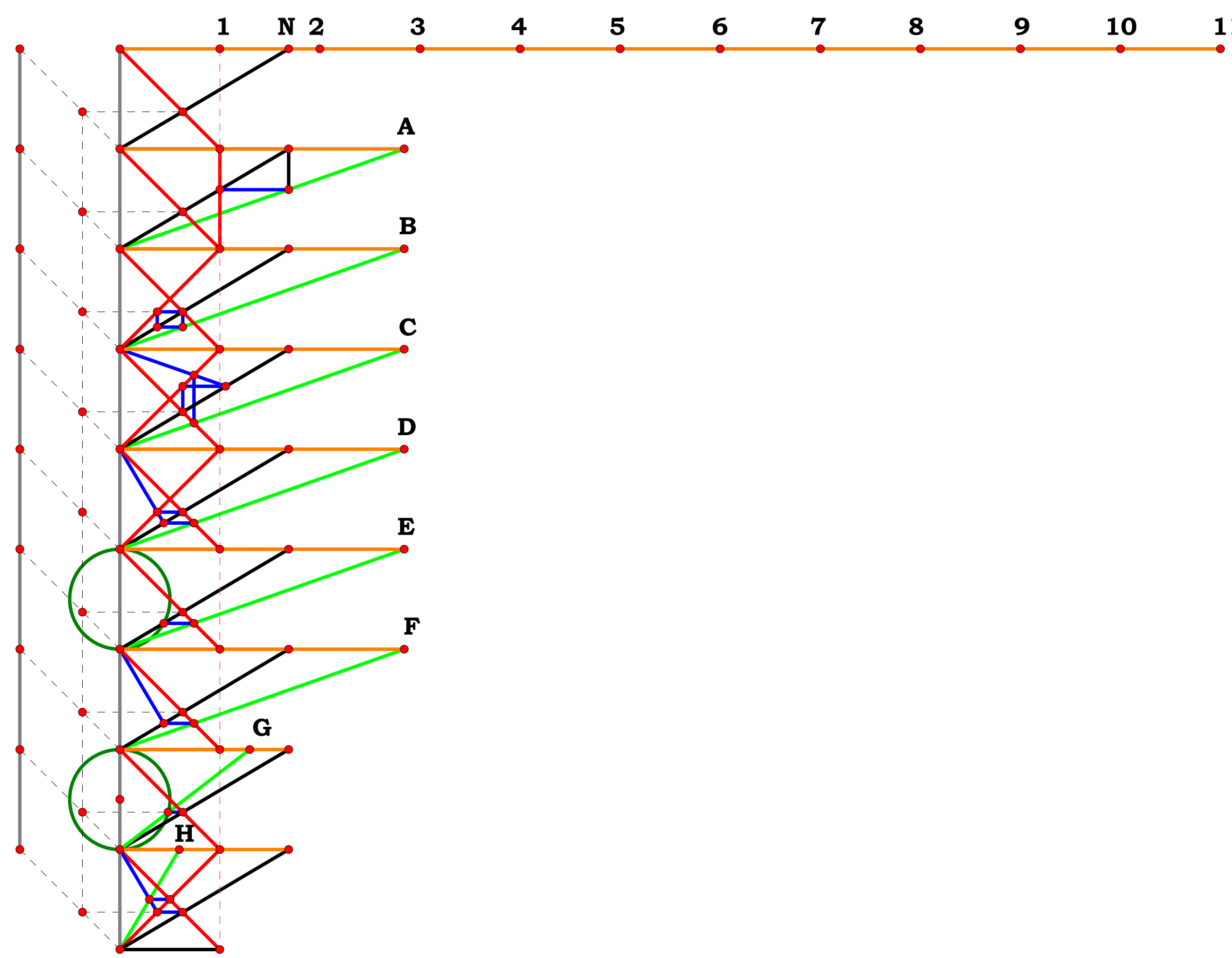
N^2-D = 0.00000

N^2-E = 0.00000

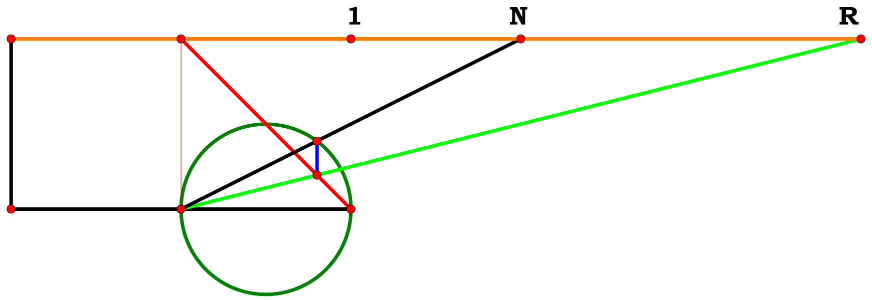
N^2-F = 0.00000

$\sqrt{N}$-G = 0.00000

$\frac{1}{N}$-H = 0.00000



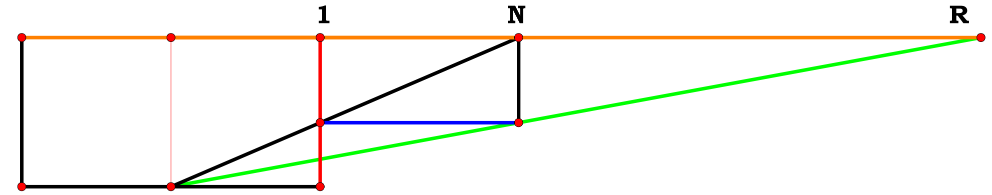
3OBT3R12



N = 2.00000 N^2-R = 0.00000

R = 4.00000

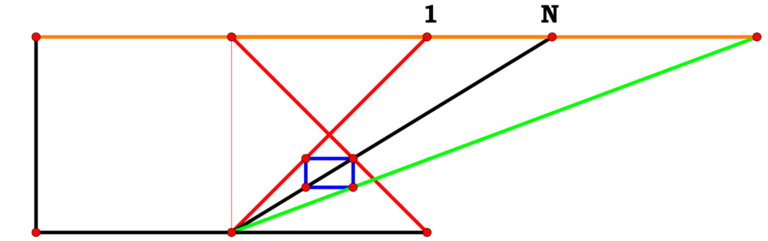
1CST1R7



N = 2.32967 N^2-R = 0.00000

R = 5.42736

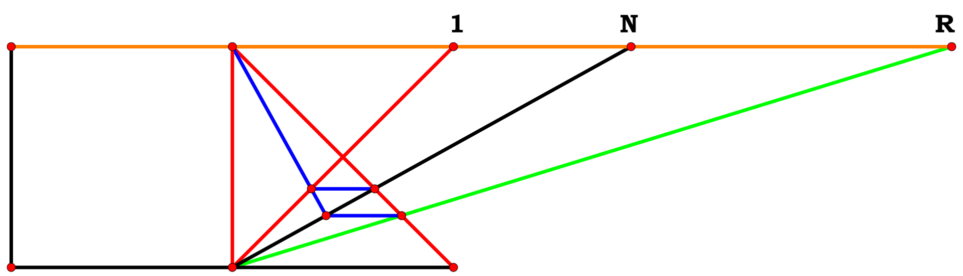
1CST5R5



N = 1.63893 N^2-R = 0.00000

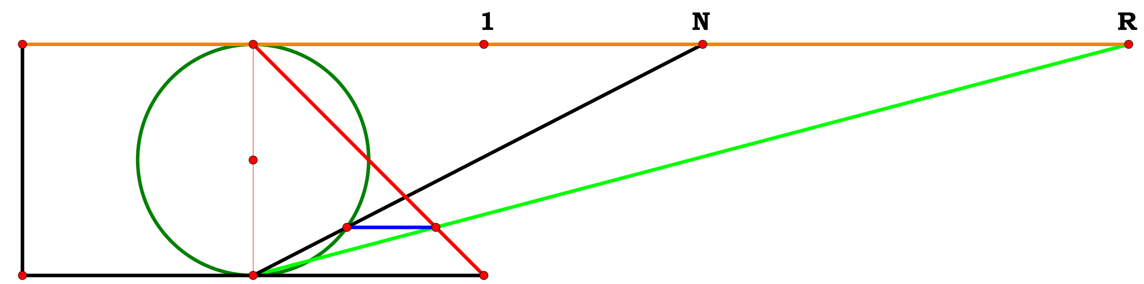
R = 2.68609

1CST5R14



N = 1.80322 N^2-R = 0.00000

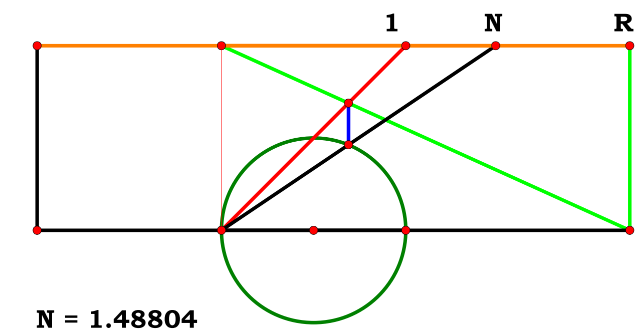
R = 3.25161



N = 1.94754 N^2-R = 0.00000

R = 3.79291

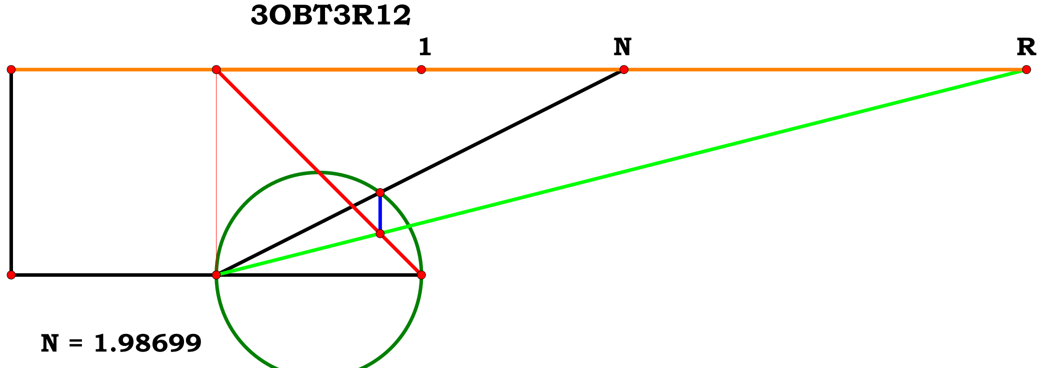
3OBT4R7



N = 1.48804

R = 2.21427 N^2-R = 0.00000

3OBT3R12

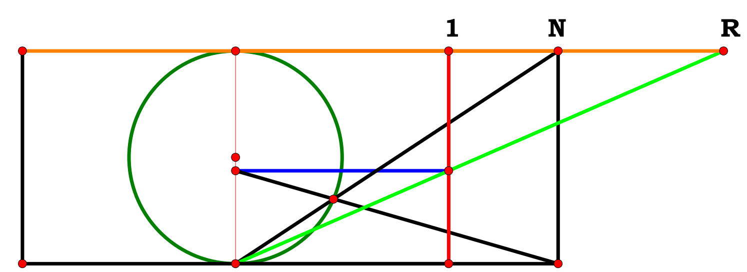


N = 1.98699

R = 3.94815

N^2-R = 0.00000

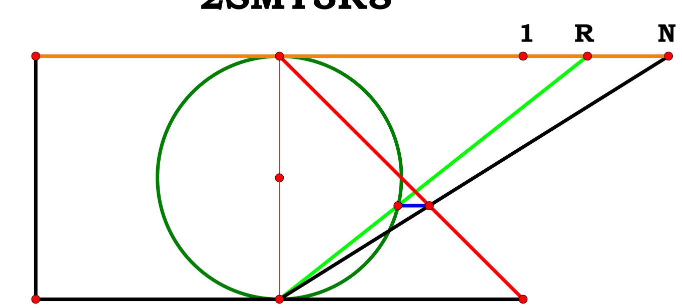
2SMT8R9



N = 1.51285 N^2-R = 0.00000

R = 2.28871

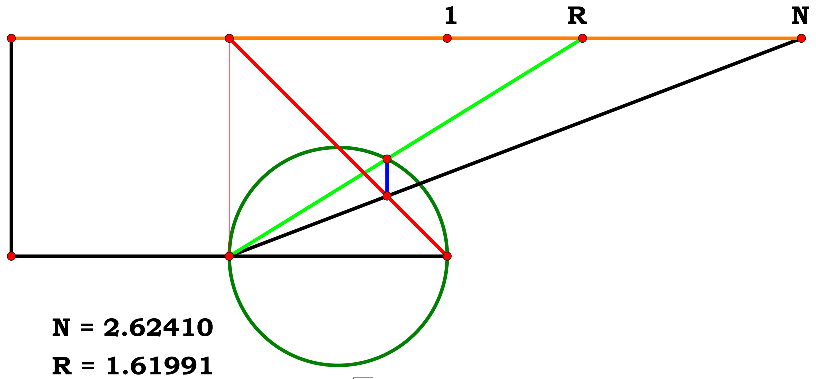
2SMT3R8



N = 1.59579 $\sqrt{N}$-R = 0.00000

R = 1.26325

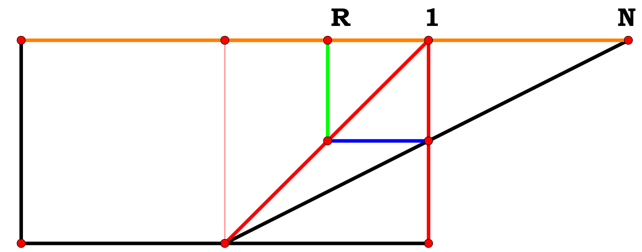
3OBT3R5



N = 2.62410
R = 1.61991

$\sqrt{N}$-R = 0.00000

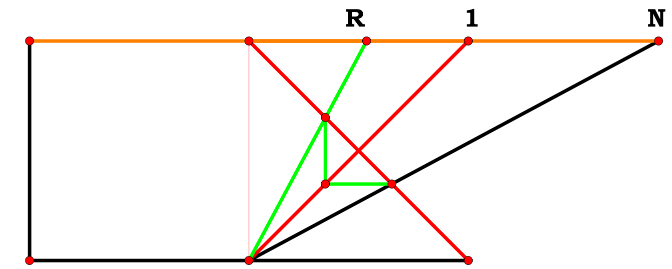
1CST3R0



N = 1.97991
R = 0.50507

$\frac{1}{N}$-R = 0.00000

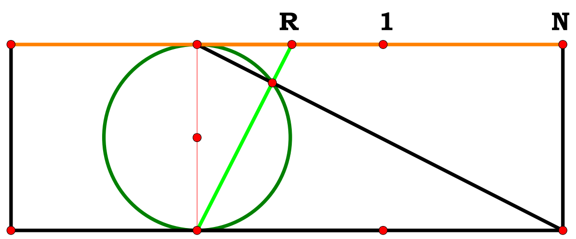
1CST5R2



N = 1.86654
R = 0.53575

$\frac{1}{N}$-R = 0.00000

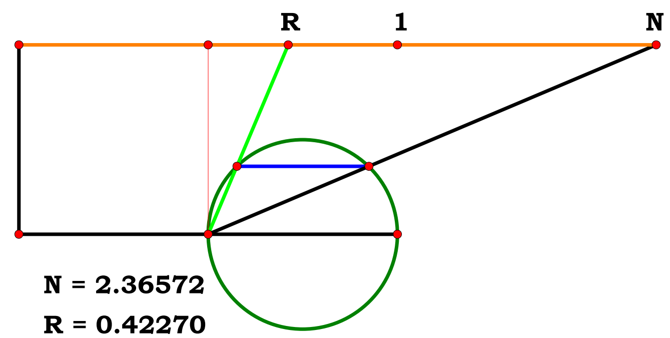
2SMT1R3



N = 1.96457
R = 0.50902

$\frac{1}{N}$-R = 0.00000

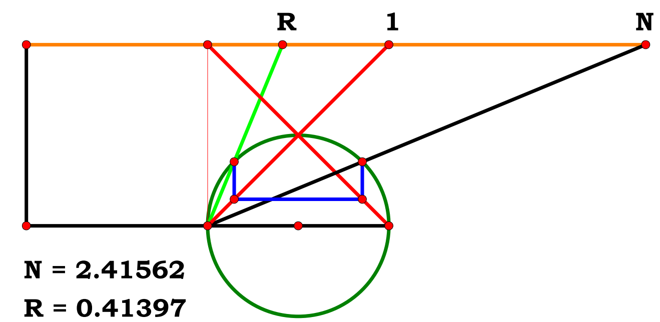
3OBT1R2



N = 2.36572
R = 0.42270

$\frac{1}{N}$-R = 0.00000

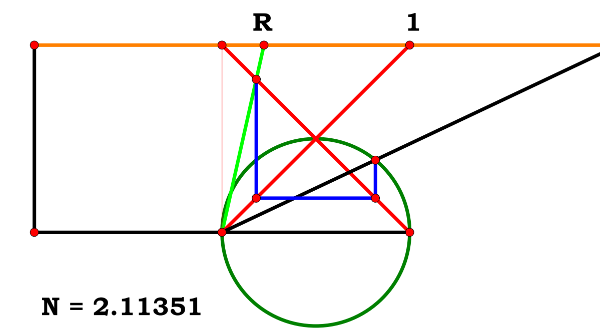
3OBT5R2



N = 2.41562
R = 0.41397

$\frac{1}{N}$-R = 0.00000

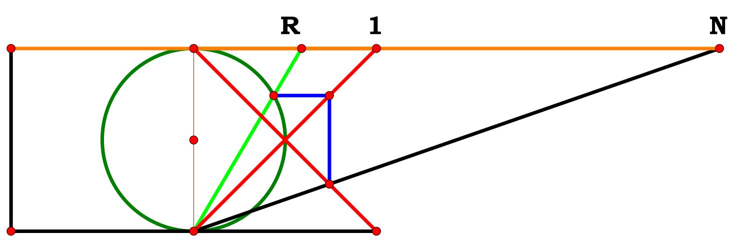
3OBT5R1



N = 2.11351
R = 0.22387

$\frac{1}{N^2}$-R = 0.00000

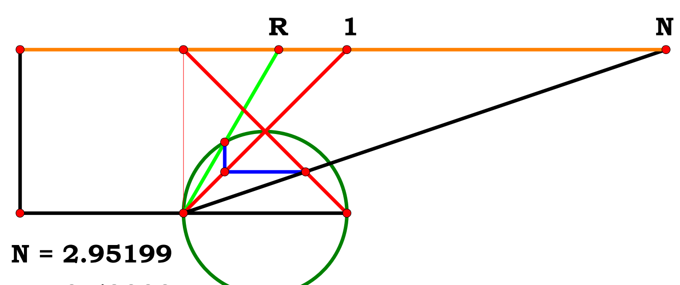
2SMT6R3



N = 2.87663
R = 0.58960

$\frac{1}{\sqrt{N}}$-R = 0.00000

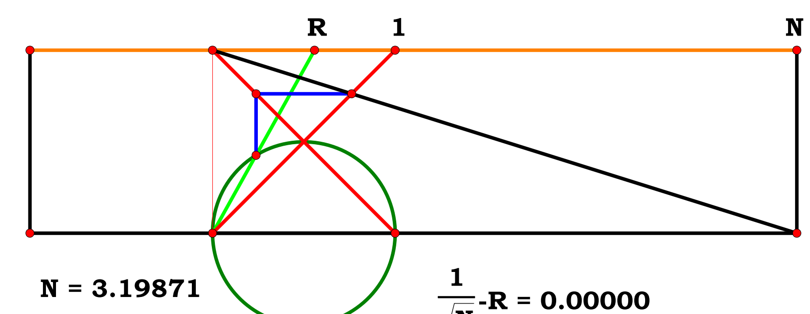
3OBT5R4



N = 2.95199
R = 0.58203

$\frac{1}{\sqrt{N}}$-R = 0.00000

3OBT11R2

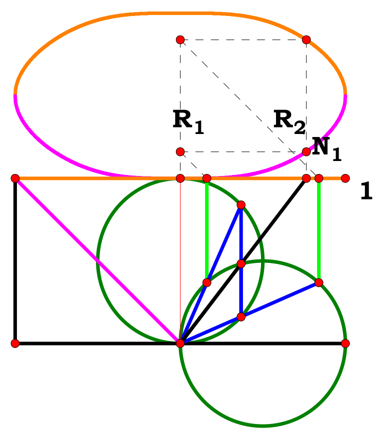


N = 3.19871
R = 0.55913

$\frac{1}{\sqrt{N}}$-R = 0.00000

3OBT10AR0

3OBT10AR3



$$\frac{(N_1^2+1)-\sqrt{(2\cdot N_1^2+1)-3\cdot N_1^4}}{2\cdot N_1^2+2} = 0.16123$$

$$\frac{N_1^2+1+\sqrt{(2\cdot N_1^2-3\cdot N_1^4)+1}}{2\cdot N_1^2+2} = 0.83877$$

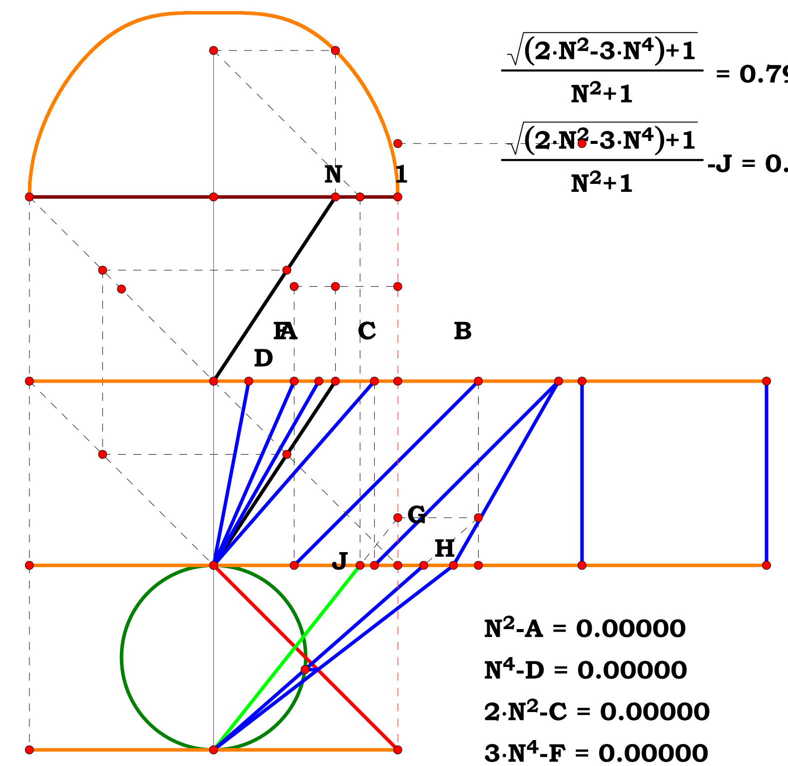
$N_1 = 0.76264$

$R_1 = 0.16123$

$R_2 = 0.83877$

$$\frac{N_1^2+1+\sqrt{(2\cdot N_1^2-3\cdot N_1^4)+1}}{2\cdot N_1^2+2} - \frac{(N_1^2+1)-\sqrt{(2\cdot N_1^2+1)-3\cdot N_1^4}}{2\cdot N_1^2+2} = 0.67755$$

$$\frac{\sqrt{(2\cdot N_1^2-3\cdot N_1^4)+1}}{N_1^2+1} = 0.67755$$



$$\frac{\sqrt{(2\cdot N^2-3\cdot N^4)+1}}{N^2+1} = 0.79409$$

$$\frac{\sqrt{(2\cdot N^2-3\cdot N^4)+1}}{N^2+1}-J = 0.00000$$

$N = 0.66074$

$A = 0.43658$

$B = 1.43658$

$C = 0.87315$

$D = 0.19060$

$F = 0.57180$

$G = 1.30136$

$H = 1.14077$

$J = 0.79409$

$N^2-A = 0.00000$

$N^4-D = 0.00000$

$2\cdot N^2-C = 0.00000$

$3\cdot N^4-F = 0.00000$

$(N^2+1)-B = 0.00000$

$((2\cdot N^2-3\cdot N^4)+1)-G = 0.00000$

$\sqrt{(2\cdot N^2-3\cdot N^4)+1}-H = 0.00000$

Data Piping

7/20/2013

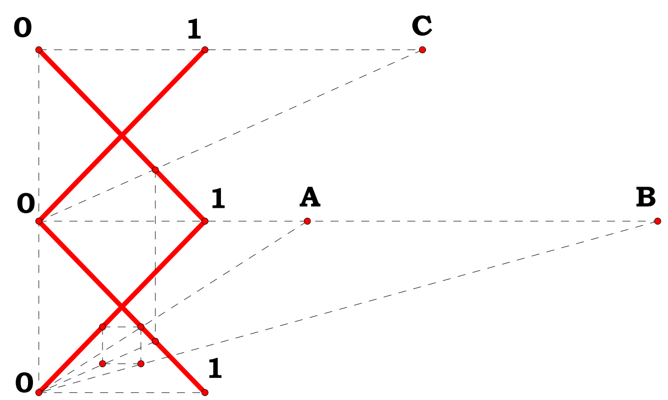
One of the results of my project to provide the constructions for the many propositions in the Elements that do not have them, the lines provided which may nor may not be proportional to each other in the book are supposed results, but not constructions, was to lead me to advance my understanding of figure stacking for Jacob's Ladder.

One of the early recognized benefits of the Jacobs Ladder Geometrical computation figure is stackability so that one can do computations within another figure on another line. The stacking maintains a one-to-one ratio with the given unit over all the figures.

Basic Stacking

01 = 2.20133 cm
0A = 3.55600 cm
0B = 8.19150 cm
0C = 5.07093 cm

A = 1.61538
B = 3.72115
C = 2.30357
$\frac{B}{A}$-C = 0.00000



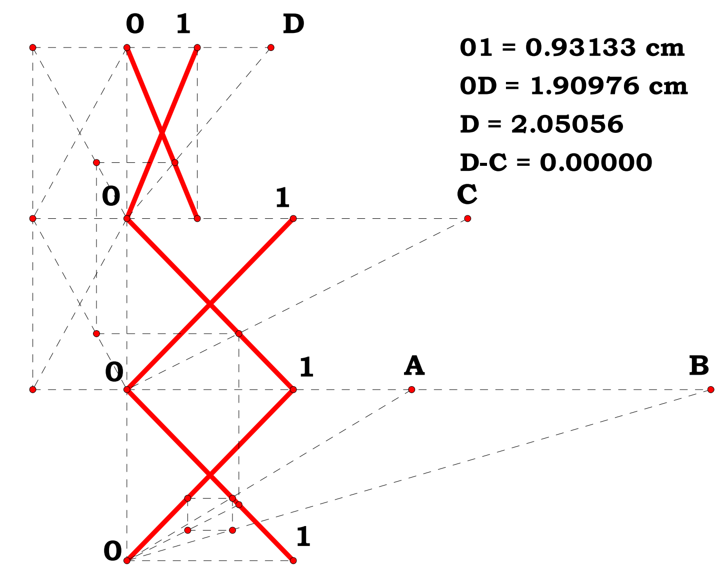
However, to take stackability to a new level, one must recognize that one can pipe results to a staked figure and also, simultaneous choose an independent unit for that line of computation while have no effect on the results. Thus if one desires to project the results for the construction of a figure, they can do so quite independently of units chosen to do the math in Jacob's Ladder.

Proportional Stacking

01 = 2.20133 cm
0A = 3.76767 cm
0B = 7.72583 cm
0C = 4.51397 cm

A = 1.71154
B = 3.50962
C = 2.05056
$\frac{B}{A}$-C = 0.00000

01 = 0.93133 cm
0D = 1.90976 cm
D = 2.05056
D-C = 0.00000



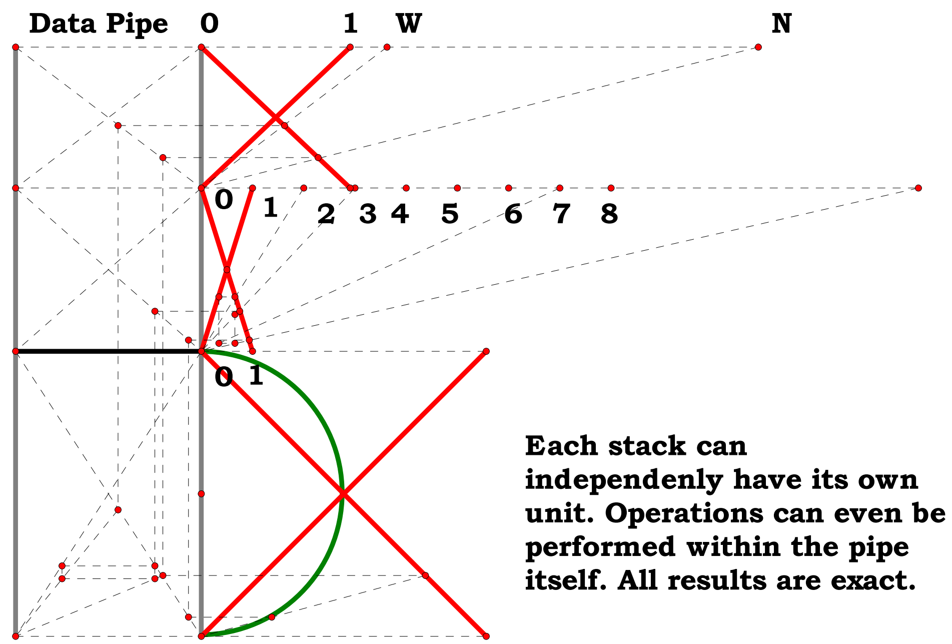
One can see that this aids in building geometric figures which are in fact, very complex computational computers. Thus complex computational analog computers can constructed, which, if I am not mistaken, will eventually lead to the realization of holographic analog computing at speeds unimagined at this time.

Data Piping.

01 = 1.96850 cm   N = 3.74166

0N = 7.36545 cm N^2 = 14.00000

0W = 2.45515 cm   W = 1.24722 $(W \cdot 3)^2$ = 14.00000



All of legitimate mathematics are derived from the physical world—each concept geometrically demonstrable. Thus, all legitimate mathematics are demonstrable in the analogic of geometry—through a simple figure I have called Jacob's Ladder. The information to construct the figure, that of proportion between parallel lines is actually presented in Euclid's Elements Book 1. So, all that is needed to understand mathematics is presented by the end of Book 1, and the results of the implication of proportion is explored throughout the rest of the work.

1CST1

$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_0 = 0$$

$$N_1 - N_2 - R_1 = 0$$

$$\frac{N_1^2 - N_1 \cdot N_2}{N_2} - R_2 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - 2 \cdot N_2} - R_3 = 0$$

$$\frac{N_1^2 - N_1 \cdot N_2}{N_1 - 2 \cdot N_2} - R_4 = 0$$

$$\frac{N_1^2 - 2 \cdot N_1 \cdot N_2}{N_1 - N_2} - R_5 = 0$$

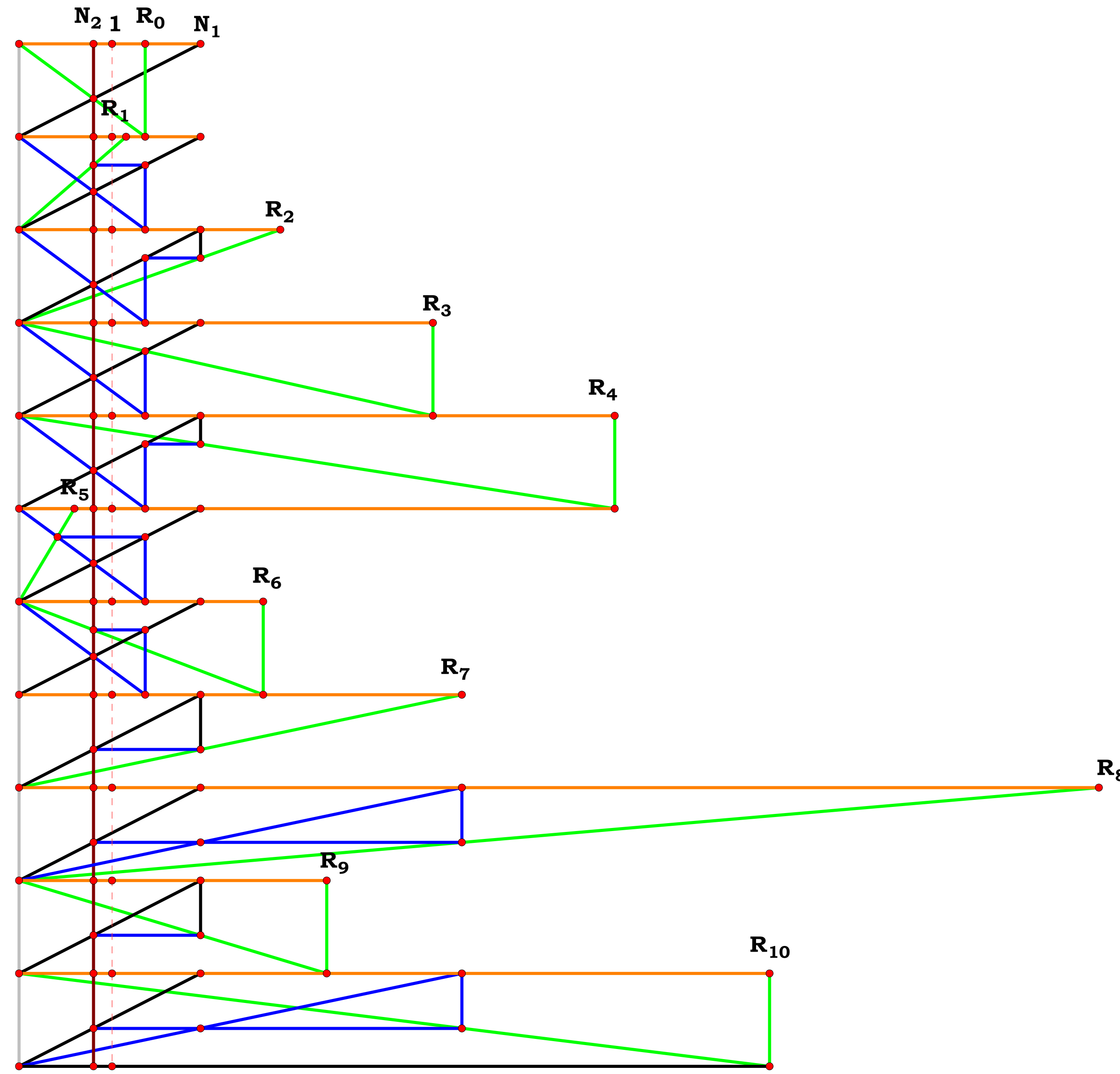
$$\frac{N_1 \cdot N_2 - N_2^2}{N_1 - 2 \cdot N_2} - R_6 = 0$$

$$\frac{N_1^2}{N_2} - R_7 = 0$$

$$\frac{N_1^3}{N_2^2} - R_8 = 0$$

$$\frac{N_1^2}{N_1 - N_2} - R_9 = 0$$

$$\frac{N_1^3}{N_1 \cdot N_2 - N_2^2} - R_{10} = 0$$



1CST1 [1, 0]

$$\frac{N_1}{N_1-1}-R_0 = 0$$

$$N_1-1-R_1 = 0$$

$$N_1^2-N_1-R_2 = 0$$

$$\frac{N_1}{N_1-2}-R_3 = 0$$

$$\frac{N_1^2-N_1}{N_1-2}-R_4 = 0$$

$$\frac{N_1^2-2 \cdot N_1}{N_1-1}-R_5 = 0$$

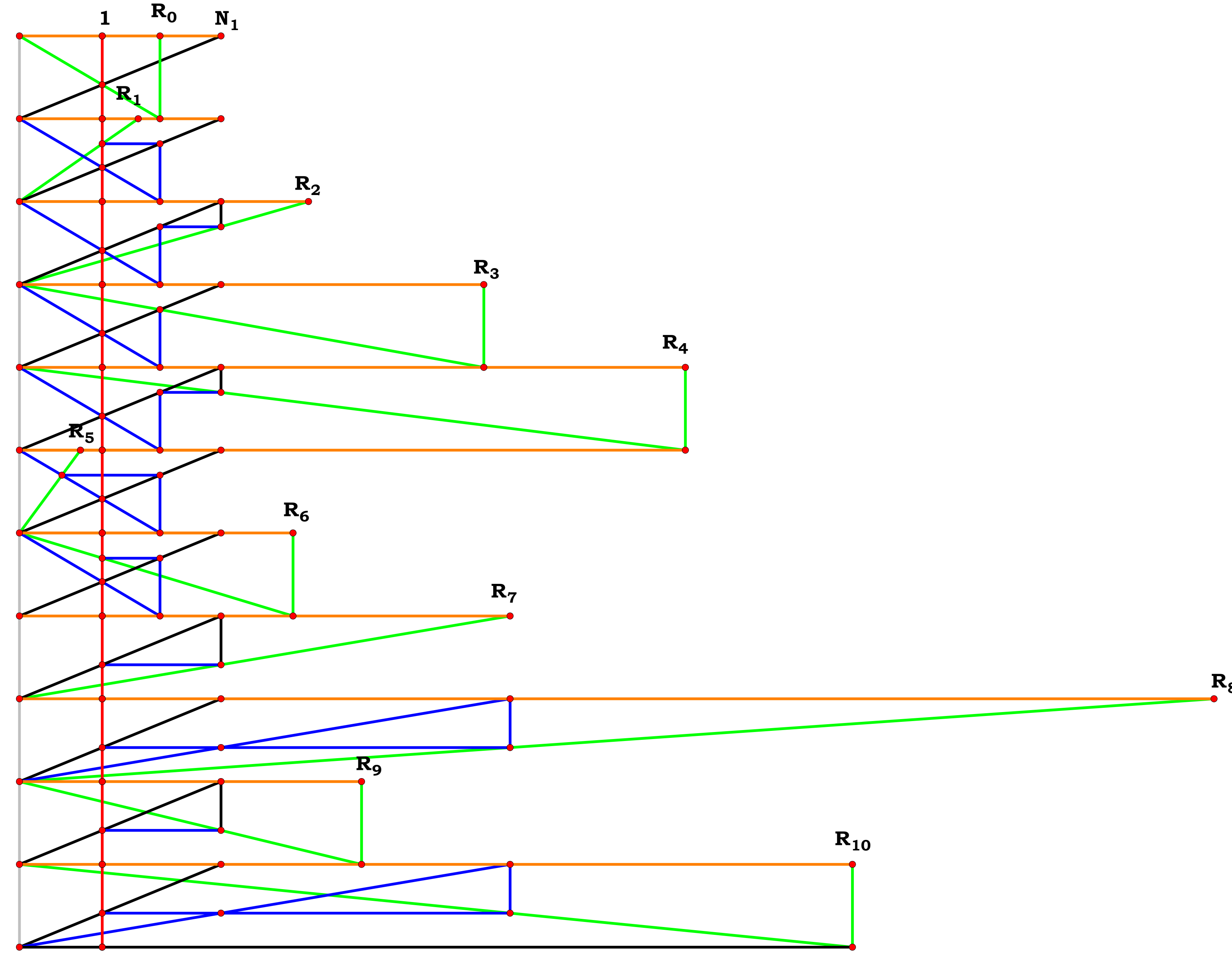
$$\frac{N_1-1}{N_1-2}-R_6 = 0$$

$$N_1^2-R_7 = 0$$

$$N_1^3-R_8 = 0$$

$$\frac{N_1^2}{N_1-1}-R_9 = 0$$

$$\frac{N_1^3}{N_1-1}-R_{10} = 0$$



1CST1 [0, 1]

$$\frac{N_2}{1-N_2}-R_0 = 0$$

$$1-N_2-R_1 = 0$$

$$\frac{1-N_2}{N_2}-R_2 = 0$$

$$\frac{N_2}{1-2\cdot N_2}-R_3 = 0$$

$$\frac{1-N_2}{1-2\cdot N_2}-R_4 = 0$$

$$\frac{1-2\cdot N_2}{1-N_2}-R_5 = 0$$

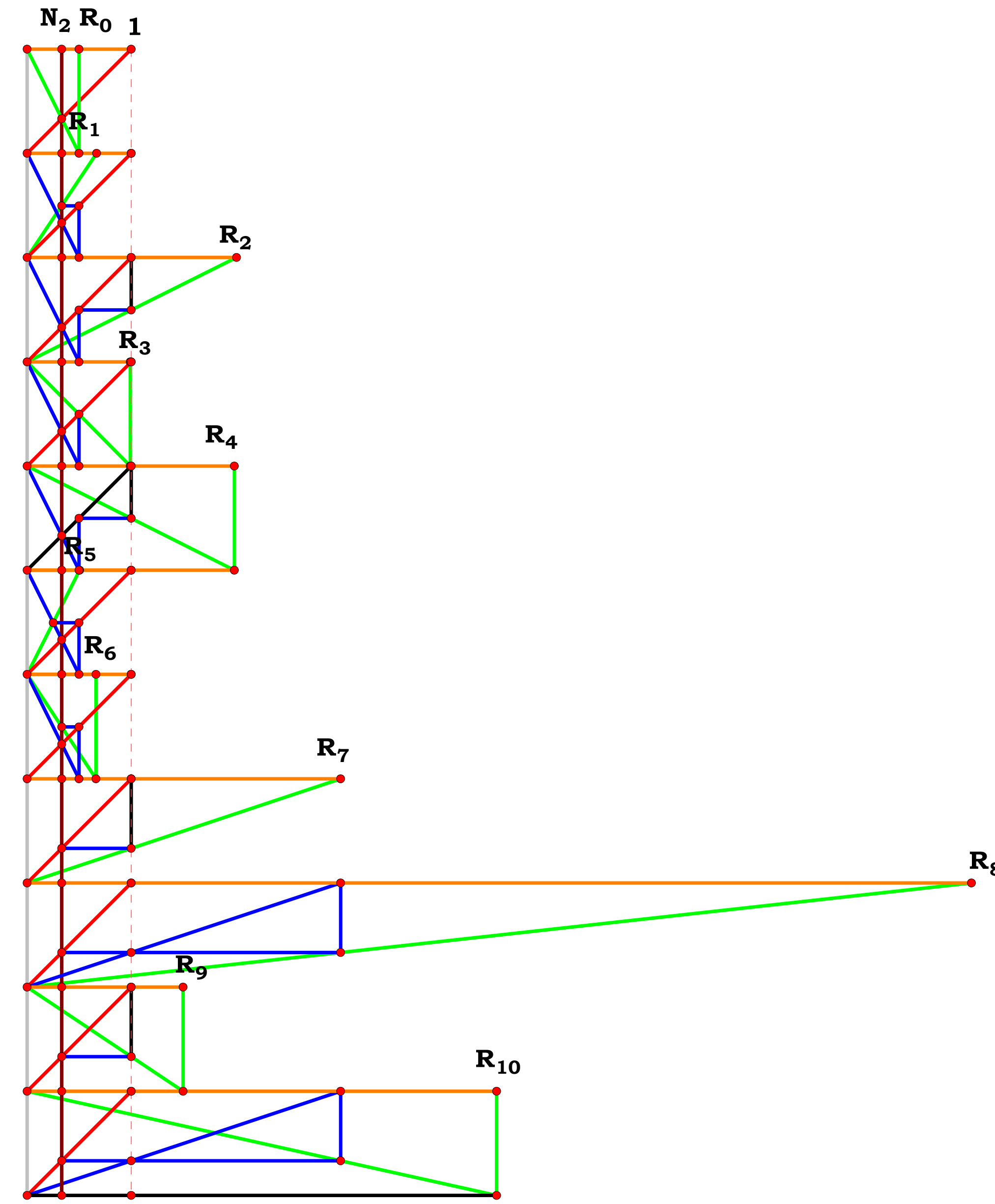
$$\frac{N_2-N_2^2}{1-2\cdot N_2}-R_6 = 0$$

$$\frac{1}{N_2}-R_7 = 0$$

$$\frac{1}{N_2^2}-R_8 = 0$$

$$\frac{1}{1-N_2}-R_9 = 0$$

$$\frac{1}{N_2-N_2^2}-R_{10} = 0$$



1CST2

$$\frac{N_1 \cdot N_2}{N_1+N_2}-R_0 = 0$$

$$\frac{N_1 \cdot N_2}{2 \cdot N_1+N_2}-R_1 = 0$$

$$\frac{2 \cdot N_1^2 \cdot N_2+N_1 \cdot N_2^2}{N_1^2+2 \cdot N_1 \cdot N_2+N_2^2}-R_2 = 0$$

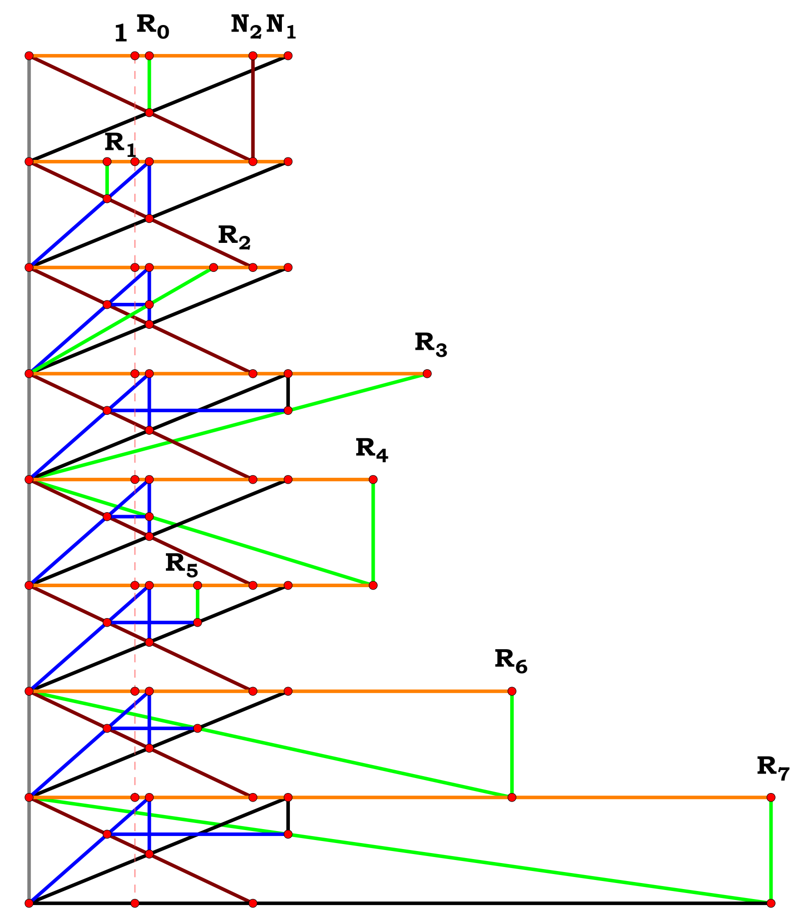
$$\frac{2 \cdot N_1^2+N_1 \cdot N_2}{N_1+N_2}-R_3 = 0$$

$$\frac{2 \cdot N_1 \cdot N_2+N_2^2}{N_1+N_2}-R_4 = 0$$

$$\frac{N_1^2+N_1 \cdot N_2}{2 \cdot N_1+N_2}-R_5 = 0$$

$$(N_1+N_2)-R_6 = 0$$

$$(2 \cdot N_1+N_2)-R_7 = 0$$



1CST2 [1, 0]

$$\frac{N_1}{N_1+1}-R_0=0$$

$$\frac{N_1}{2\cdot N_1+1}-R_1=0$$

$$\frac{2\cdot N_1^2+N_1}{N_1^2+2\cdot N_1+1}-R_2=0$$

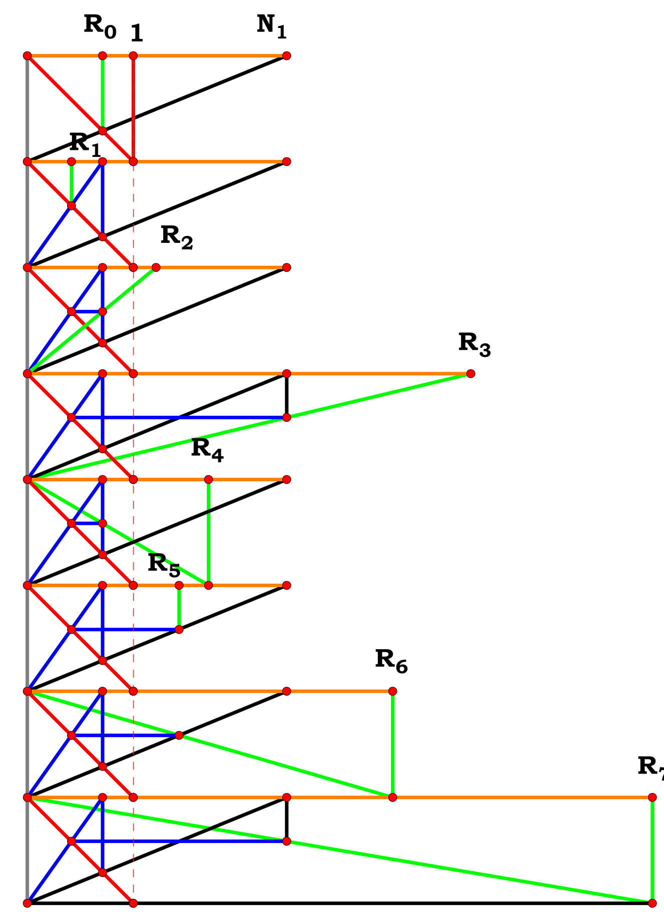
$$\frac{2\cdot N_1^2+N_1}{N_1+1}-R_3=0$$

$$\frac{2\cdot N_1+1}{N_1+1}-R_4=0$$

$$\frac{N_1^2+N_1}{2\cdot N_1+1}-R_5=0$$

$$(N_1+1)-R_6=0$$

$$(2\cdot N_1+1)-R_7=0$$



1CST2 [0, 1]

$$\frac{N_2}{1+N_2}-R_0=0$$

$$\frac{N_2}{2+N_2}-R_1=0$$

$$\frac{2\cdot N_2+N_2^2}{1+2\cdot N_2+N_2^2}-R_2=0$$

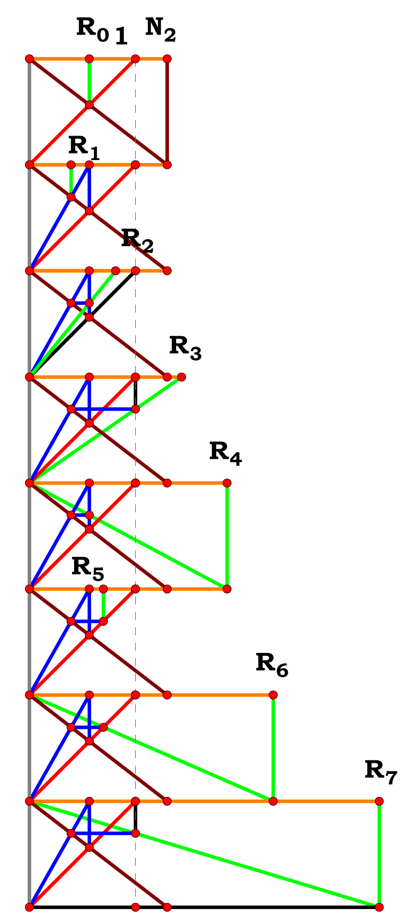
$$\frac{2+N_2}{1+N_2}-R_3=0$$

$$\frac{2\cdot N_2+N_2^2}{1+N_2}-R_4=0$$

$$\frac{1+N_2}{2+N_2}-R_5=0$$

$$(1+N_2)-R_6=0$$

$$(2+N_2)-R_7=0$$



1CST3

$$\frac{N_2 \cdot N_3}{N_1} - R_0 = 0$$

$$\frac{N_2^2 \cdot N_3}{N_1^2} - R_1 = 0$$

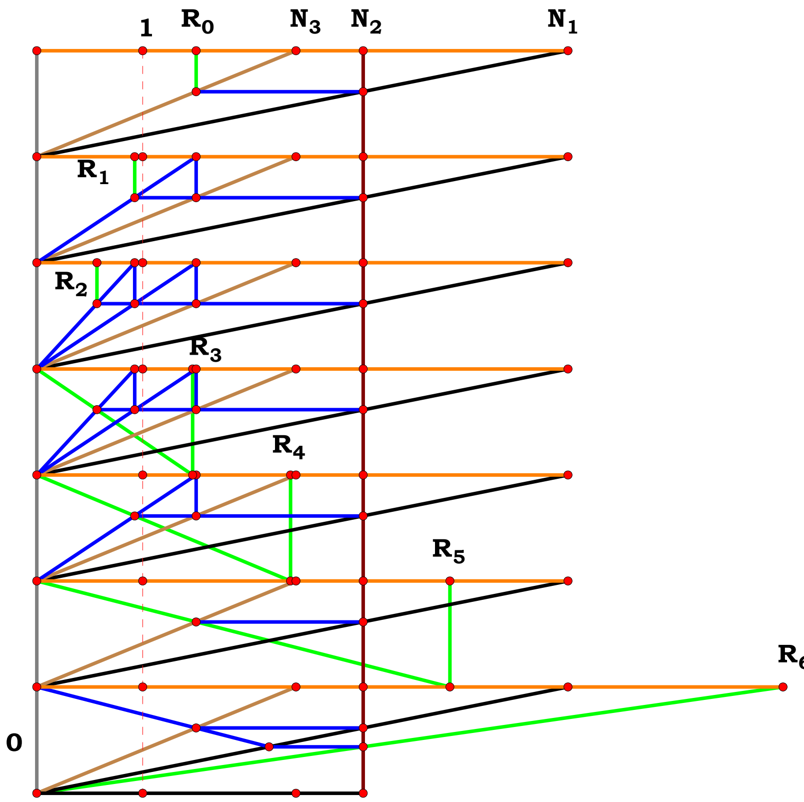
$$\frac{N_2^3 \cdot N_3}{N_1^3} - R_2 = 0$$

$$\frac{N_2^3 \cdot N_3}{N_1^3 - N_1^2 \cdot N_2} - R_3 = 0$$

$$\frac{N_2^2 \cdot N_3}{N_1^2 - N_1 \cdot N_2} - R_4 = 0$$

$$\frac{N_2 \cdot N_3}{N_1 - N_2} - R_5 = 0$$

$$\frac{(N_1^2 + N_2 \cdot N_3) - N_1 \cdot N_2}{N_3} - R_6 = 0$$



1CST3 [1, 0, 0]

$$\frac{1}{N_1} - R_0 = 0$$

$$\frac{1}{N_1^2} - R_1 = 0$$

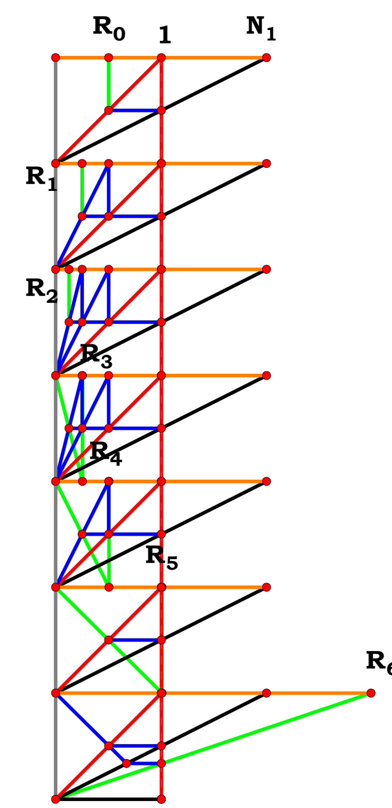
$$\frac{N_2^3 \cdot N_3}{N_1^3} - R_2 = 0$$

$$\frac{1}{N_1^3 - N_1^2} - R_3 = 0$$

$$\frac{1}{N_1^2 - N_1} - R_4 = 0$$

$$\frac{1}{N_1 - 1} - R_5 = 0$$

$$\left(N_1^2 + 1\right) - N_1 - R_6 = 0$$



1CST3 [0, 2, 0]

$N_2 - R_0 = 0$

$N_2^2 - R_1 = 0$

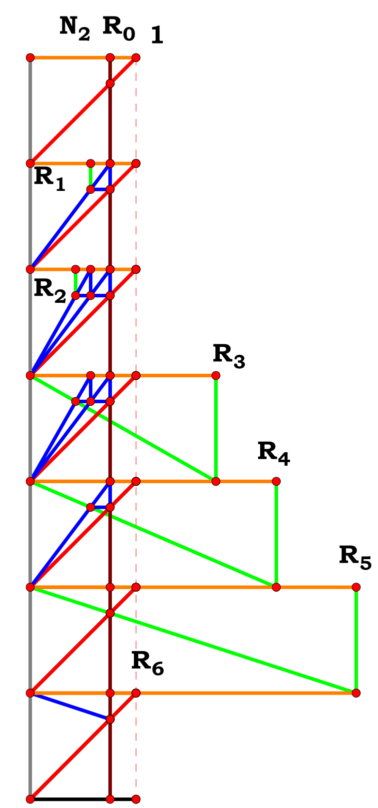
$\frac{N_2^3 \cdot N_3}{N_1^3} - R_2 = 0$

$\frac{N_2^3}{1-N_2} - R_3 = 0$

$\frac{N_2^2}{1-N_2} - R_4 = 0$

$\frac{N_2}{1-N_2} - R_5 = 0$

$1 - R_6 = 0$



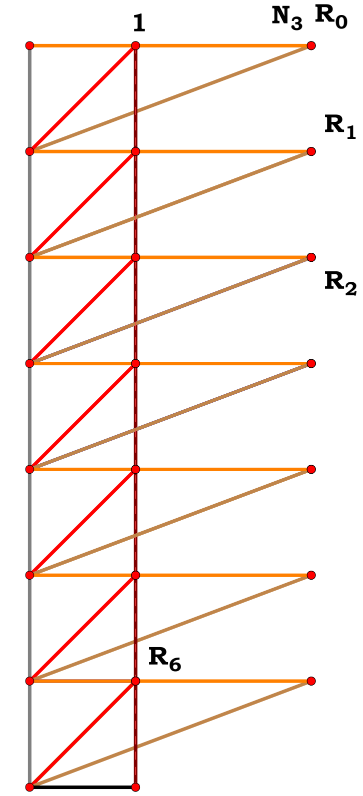
1CST3 [0, 0, 3]

N_3-R_0 = 0

N_3-R_1 = 0

N_3-R_2 = 0

1-R_6 = 0



1CST3 [1, 2, 0]

$$\frac{N_2}{N_1} - R_0 = 0$$

$$\frac{N_2{}^2}{N_1{}^2} - R_1 = 0$$

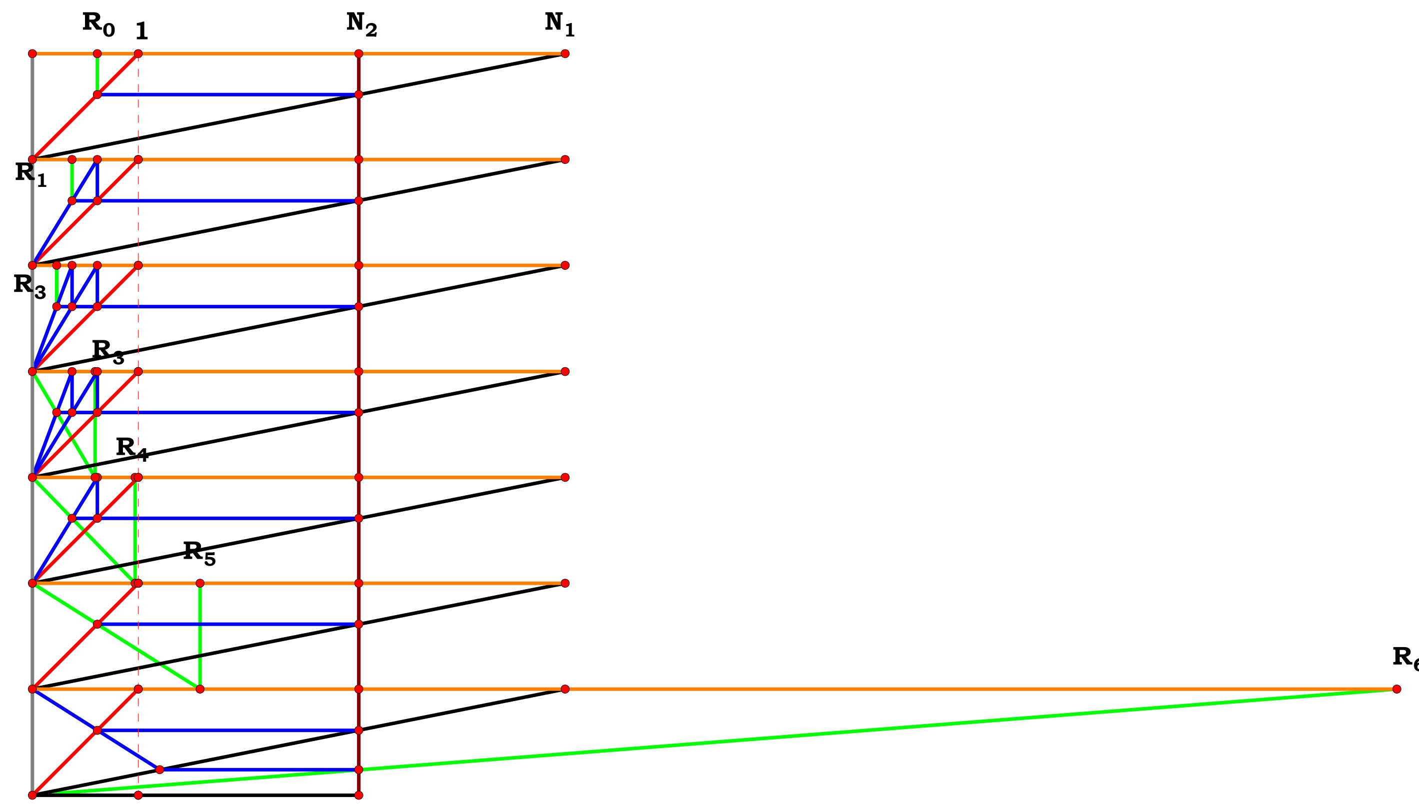
$$\frac{N_2{}^3 \cdot N_3}{N_1{}^3} - R_2 = 0$$

$$\frac{N_2{}^3}{N_1{}^3 - N_1{}^2 \cdot N_2} - R_3 = 0$$

$$\frac{N_2{}^2}{N_1{}^2 - N_1 \cdot N_2} - R_4 = 0$$

$$\frac{N_2}{N_1 - N_2} - R_5 = 0$$

$$\left(N_1{}^2 + N_2\right) - N_1 \cdot N_2 - R_6 = 0$$



1CST3 [1, 0, 3]

$$\frac{N_3}{N_1} - R_0 = 0$$

$$\frac{N_3}{N_1^2} - R_1 = 0$$

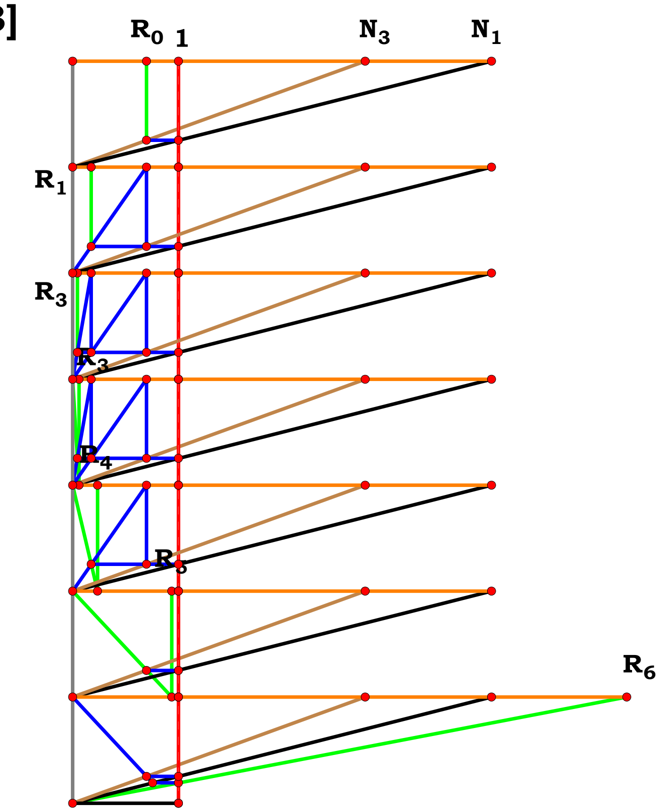
$$\frac{N_2^3 \cdot N_3}{N_1^3} - R_2 = 0$$

$$\frac{N_3}{N_1^3 - N_1^2} - R_3 = 0$$

$$\frac{N_3}{N_1^2 - N_1} - R_4 = 0$$

$$\frac{N_3}{N_1 - 1} - R_5 = 0$$

$$\frac{(N_1^2 + N_3) - N_1}{N_3} - R_6 = 0$$



1CST3 [0, 2, 3]

$N_2 \cdot N_3 - R_0 = 0$

$N_2^2 \cdot N_3 - R_1 = 0$

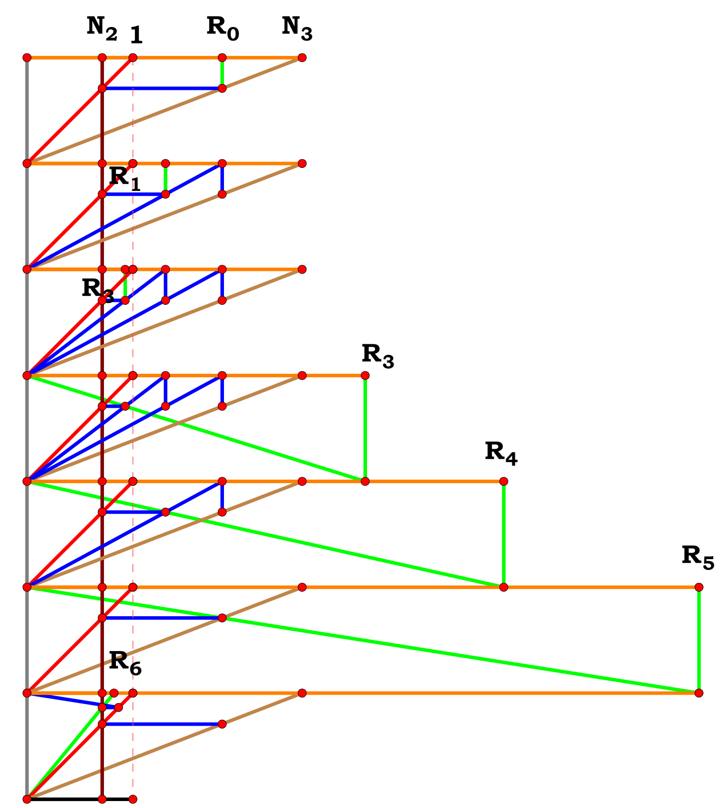
$N_2^3 \cdot N_3 - R_2 = 0$

$\frac{N_2^3 \cdot N_3}{1-N_2} - R_3 = 0$

$\frac{N_2^2 \cdot N_3}{1-N_2} - R_4 = 0$

$\frac{N_2 \cdot N_3}{1-N_2} - R_5 = 0$

$\frac{(1+N_2 \cdot N_3)-N_2}{N_3} - R_6 = 0$



1CST4

$$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} - R_0 = 0$$

$$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_2} - R_1 = 0$$

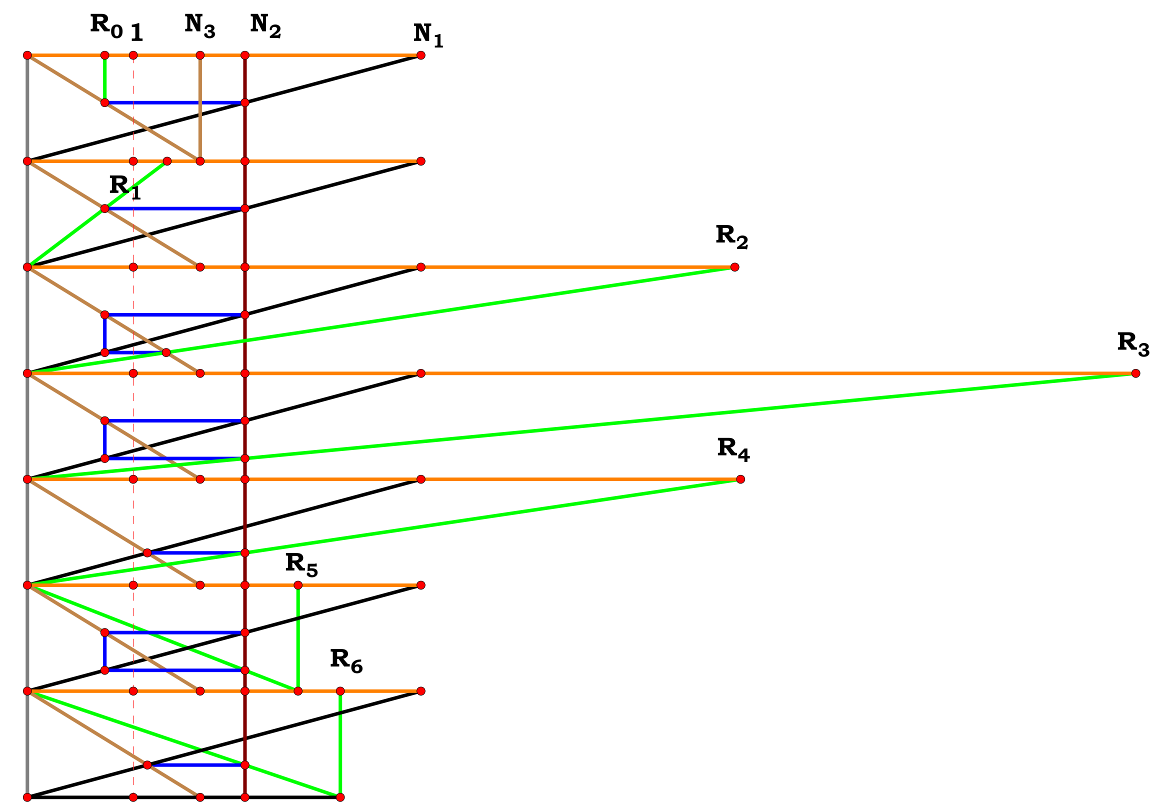
$$\frac{\left(N_1^2 - N_1 \cdot N_3\right) + N_2 \cdot N_3}{N_1 - N_2} - R_2 = 0$$

$$\frac{N_1^2 \cdot N_2}{N_1 \cdot N_3 - N_2 \cdot N_3} - R_3 = 0$$

$$\frac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_3} - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2}{\left(N_1^2 - N_1 \cdot N_3\right) + N_2 \cdot N_3} - R_5 = 0$$

$$\frac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_1} - R_6 = 0$$



1CST4 [1, 0, 0]

$$\frac{N_1-1}{N_1}-R_0 = 0$$

$$N_1-1-R_1 = 0$$

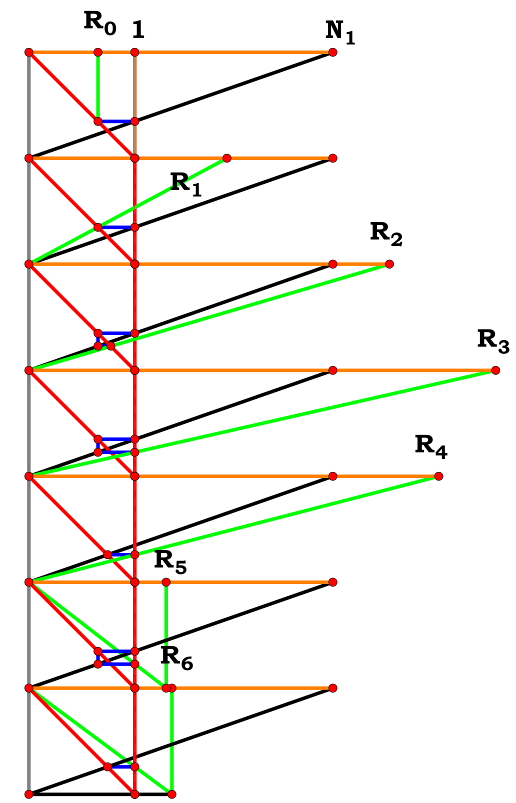
$$\frac{(N_1^2-N_1)+1}{N_1-1}-R_2 = 0$$

$$\frac{N_1^2}{N_1-1}-R_3 = 0$$

$$(N_1+1)-R_4 = 0$$

$$\frac{N_1^2}{(N_1^2-N_1)+1}-R_5 = 0$$

$$\frac{N_1+1}{N_1}-R_6 = 0$$



1CST4 [0, 2, 0]

$1\text{-}N_2\text{-}R_0 = 0$

$\frac{1\text{-}N_2}{N_2}\text{-}R_1 = 0$

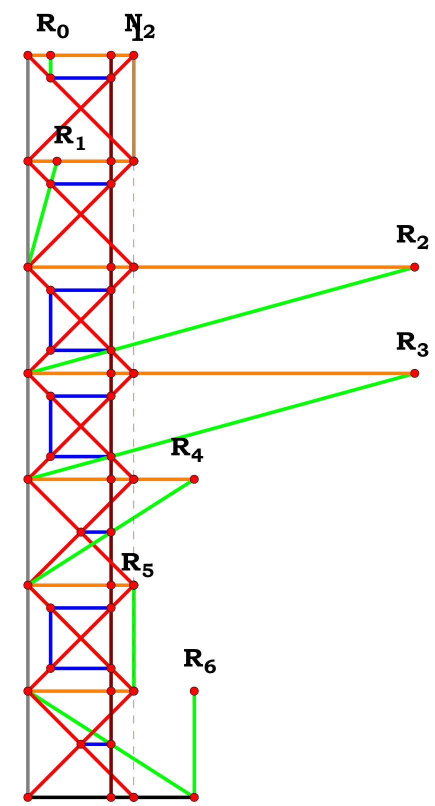
$\frac{N_2}{1\text{-}N_2}\text{-}R_2 = 0$

$\frac{N_2}{1\text{-}N_2}\text{-}R_3 = 0$

$2 \cdot N_2\text{-}R_4 = 0$

$1\text{-}R_5 = 0$

$2 \cdot N_2\text{-}R_6 = 0$



1CST4 [0, 0, 3]

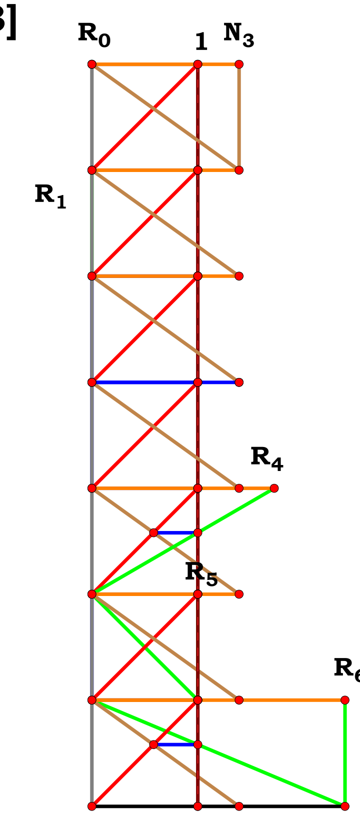
$0\text{-}R_0 = 0$

$0\text{-}R_1 = 0$

$\frac{1+N_3}{N_3}\text{-}R_4 = 0$

$1\text{-}R_5 = 0$

$(1+N_3)\text{-}R_6 = 0$



1CST4 [1, 2, 0]

$$\frac{N_1 - N_2}{N_1} - R_0 = 0$$

$$\frac{N_1 - N_2}{N_2} - R_1 = 0$$

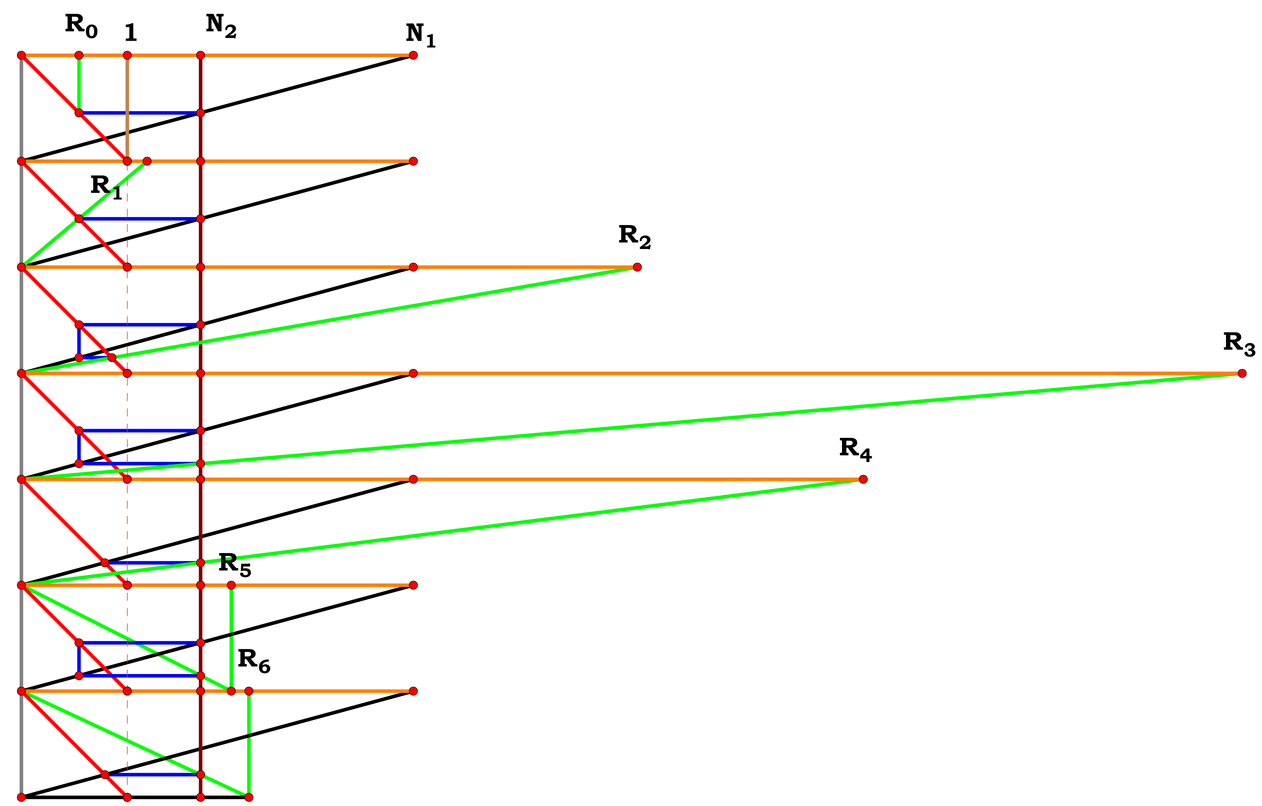
$$\frac{(N_1^2 - N_1) + N_2}{N_1 - N_2} - R_2 = 0$$

$$\frac{N_1^2 \cdot N_2}{N_1 - N_2} - R_3 = 0$$

$$(N_1 \cdot N_2 + N_2) - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1) + N_2} - R_5 = 0$$

$$\frac{N_1 \cdot N_2 + N_2}{N_1} - R_6 = 0$$



1CST4 [1, 0, 3]

$$\frac{N_1 \cdot N_3 - N_3}{N_1} - R_0 = 0$$

$$N_1 \cdot N_3 - N_3 - R_1 = 0$$

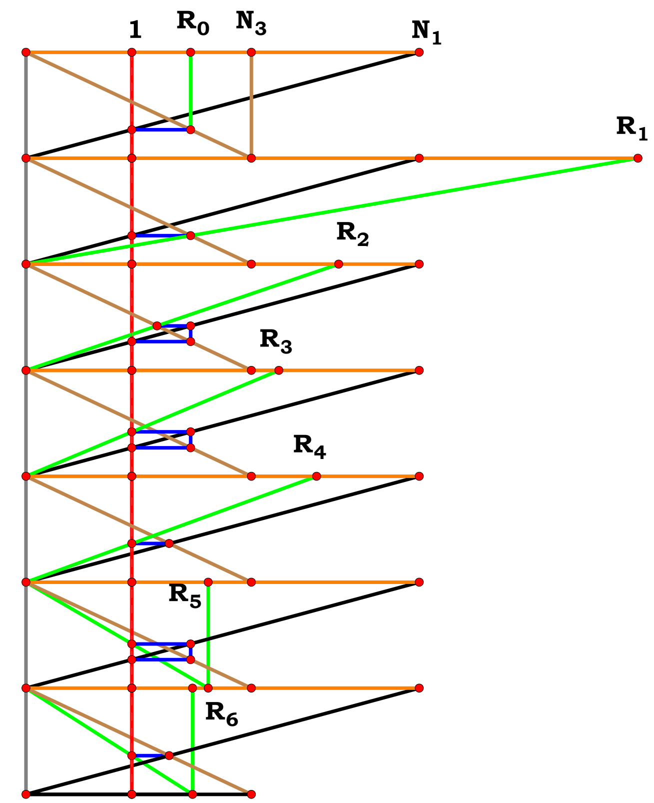
$$\frac{\left(N_1^2 - N_1 \cdot N_3\right) + N_3}{N_1 - 1} - R_2 = 0$$

$$\frac{N_1^2}{N_1 \cdot N_3 - N_3} - R_3 = 0$$

$$\frac{N_1 + N_3}{N_3} - R_4 = 0$$

$$\frac{N_1^2}{\left(N_1^2 - N_1 \cdot N_3\right) + N_3} - R_5 = 0$$

$$\frac{N_1 + N_3}{N_1} - R_6 = 0$$



1CST4 [0, 2, 3]

$$N_3-N_2 \cdot N_3-R_0 = 0$$

$$\frac{N_3-N_2 \cdot N_3}{N_2}-R_1 = 0$$

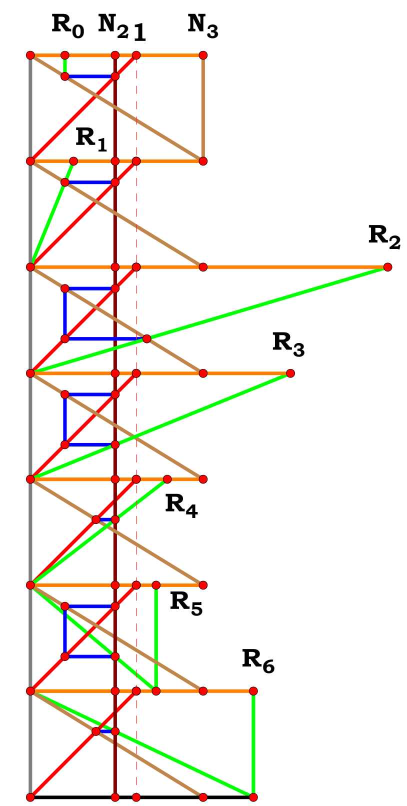
$$\frac{(1-N_3)+N_2 \cdot N_3}{1-N_2}-R_2 = 0$$

$$\frac{N_2}{N_3-N_2 \cdot N_3}-R_3 = 0$$

$$\frac{N_2+N_2 \cdot N_3}{N_3}-R_4 = 0$$

$$\frac{N_2}{(1-N_3)+N_2 \cdot N_3}-R_5 = 0$$

$$(N_2+N_2 \cdot N_3)-R_6 = 0$$



1CST5

$$\frac{N_2^2 \cdot N_3}{(N_1^2+N_1 \cdot N_3)-N_2 \cdot N_3}-R_0 = 0$$

$$\frac{N_2 \cdot N_3}{N_1+N_3}-R_1 = 0$$

$$\frac{N_2 \cdot N_3}{(N_1-N_2)+N_3}-R_2 = 0$$

$$\frac{N_1^2 \cdot N_3}{N_1 \cdot N_2+N_2 \cdot N_3}-R_3 = 0$$

$$\frac{((N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_2 \cdot N_3)+N_2^2 \cdot N_3}{N_1 \cdot N_2+N_2 \cdot N_3}-R_4 = 0$$

$$\frac{N_1^2}{N_2}-R_5 = 0$$

$$\frac{N_1 \cdot N_2+N_2 \cdot N_3}{N_3}-R_6 = 0$$

$$\frac{(N_1^2+N_1 \cdot N_3)-N_2 \cdot N_3}{N_2}-R_7 = 0$$

$$\frac{((N_1^4+N_1^3 \cdot N_3)-N_1^2 \cdot N_2 \cdot N_3)+N_1 \cdot N_2^2 \cdot N_3}{N_1 \cdot N_2^2+N_2^2 \cdot N_3}-R_8 = 0$$

$$\frac{(N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_2 \cdot N_3}{N_2^2}-R_9 = 0$$

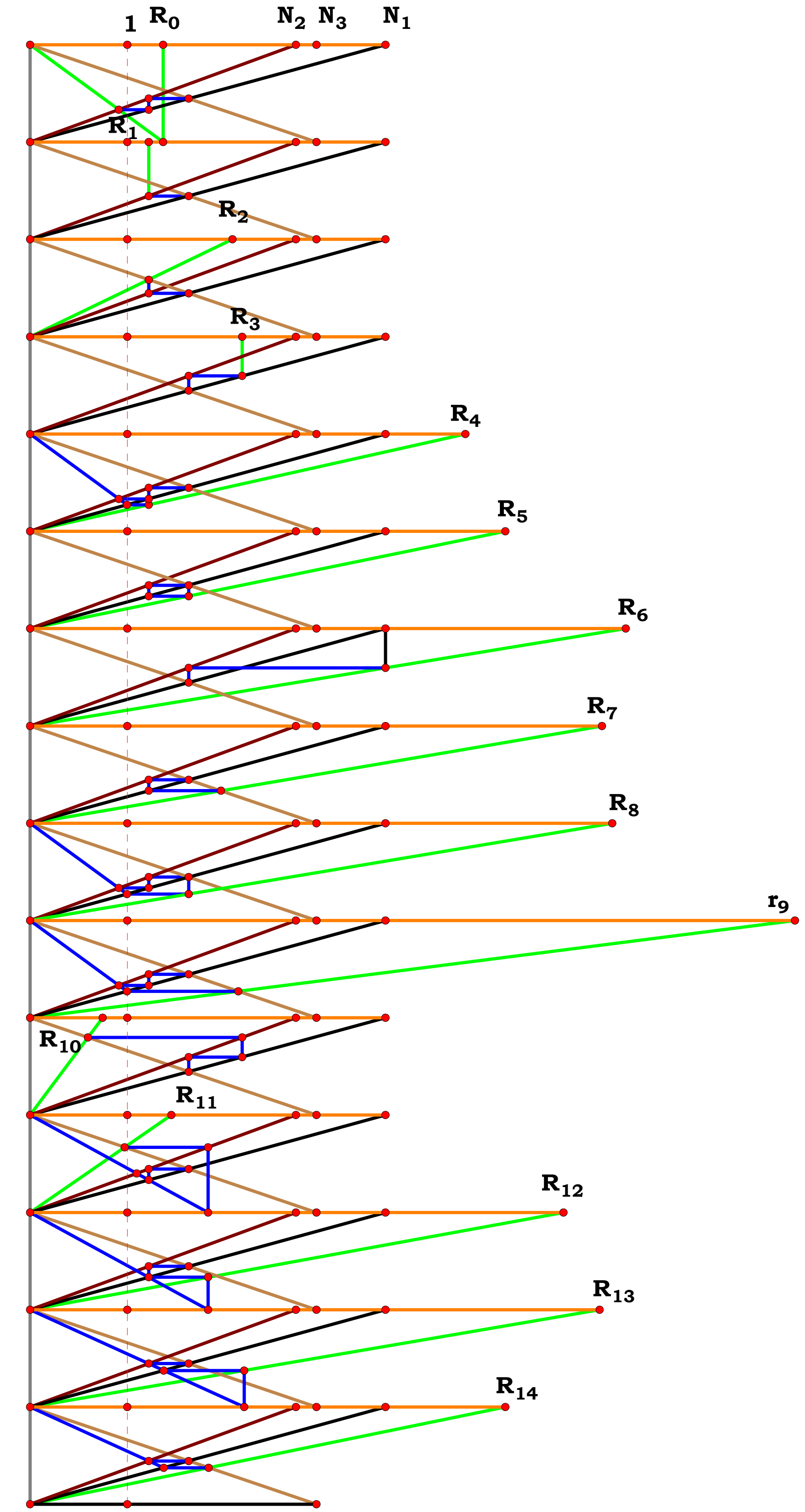
$$\frac{(N_1 \cdot N_2^2+N_2^2 \cdot N_3)-N_1^2 \cdot N_3}{N_1^2}-R_{10} = 0$$

$$\frac{N_1^2-N_2 \cdot N_3}{N_1}-R_{11} = 0$$

$$\frac{N_1^3+N_1^2 \cdot N_3}{(N_1^2+N_1 \cdot N_3)-N_2 \cdot N_3}-R_{12} = 0$$

$$\frac{N_1^2+N_2 \cdot N_3}{N_1}-R_{13} = 0$$

$$\frac{N_1^2}{N_2}-R_{14} = 0$$



1CST5 [1, 0, 0]

$$\frac{1}{(N_1^2+N_1)-1}-R_0 = 0$$

$$\frac{1}{N_1+1}-R_1 = 0$$

$$\frac{1}{N_1}-R_2 = 0$$

$$\frac{N_1^2}{N_1+1}-R_3 = 0$$

$$\frac{((N_1^3+N_1^2)-N_1)+1}{N_1+1}-R_4 = 0$$

$$N_1^2-R_5 = 0$$

$$(N_1+1)-R_6 = 0$$

$$(N_1^2+N_1)-1-R_7 = 0$$

$$\frac{((N_1^4+N_1^3)-N_1^2)+N_1}{N_1+1}-R_8 = 0$$

$$(N_1^3+N_1^2)-N_1-R_9 = 0$$

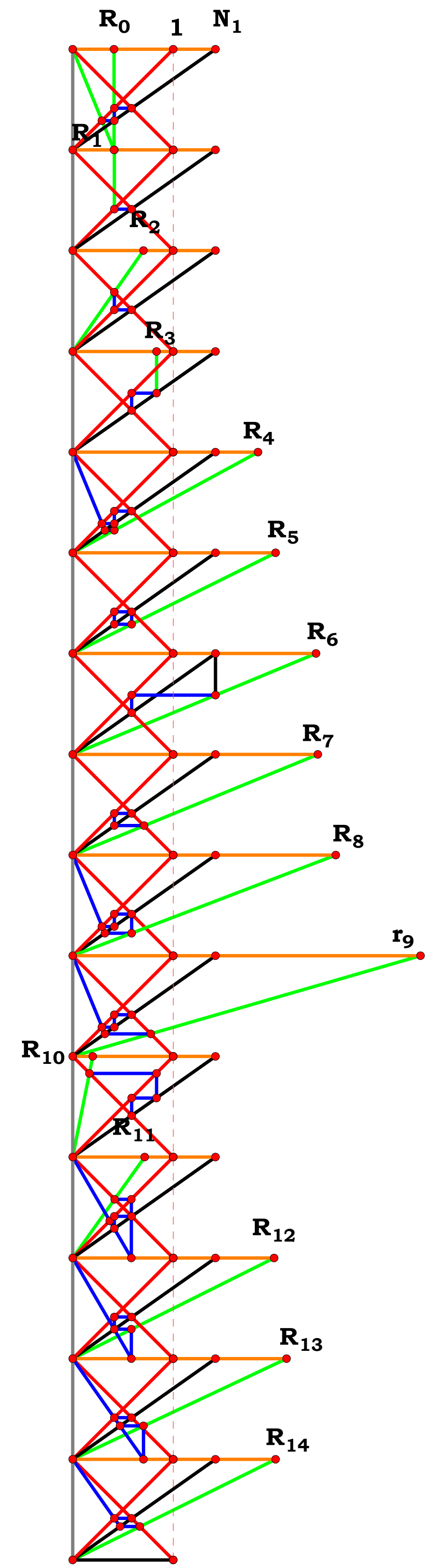
$$\frac{(N_1+1)-N_1^2}{N_1^2}-R_{10} = 0$$

$$\frac{N_1^2-1}{N_1}-R_{11} = 0$$

$$\frac{N_1^3+N_1^2}{(N_1^2+N_1)-1}-R_{12} = 0$$

$$\frac{N_1^2+1}{N_1}-R_{13} = 0$$

$$N_1^2-R_{14} = 0$$



1CST5 [0, 2, 0]

$$\frac{N_2^2}{2-N_2}-R_0 = 0$$

$$\frac{N_2}{2}-R_1 = 0$$

$$\frac{N_2}{2-N_2}-R_2 = 0$$

$$\frac{1}{2 \cdot N_2}-R_3 = 0$$

$$\frac{(2-N_2)+N_2^2 \cdot N_3}{2 \cdot N_2}-R_4 = 0$$

$$\frac{1}{N_2}-R_5 = 0$$

$$2 \cdot N_2-R_6 = 0$$

$$\frac{2-N_2}{N_2}-R_7 = 0$$

$$\frac{(2-N_2)+N_2^2}{2 \cdot N_2^2}-R_8 = 0$$

$$\frac{2-N_2}{N_2^2}-R_9 = 0$$

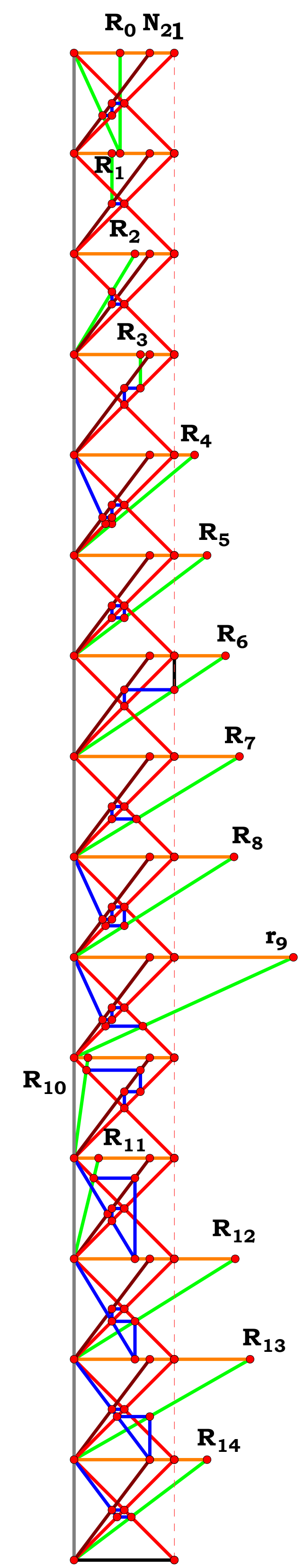
$$2 \cdot N_2^2-1-R_{10} = 0$$

$$1-N_2-R_{11} = 0$$

$$\frac{2}{2-N_2}-R_{12} = 0$$

$$(1+N_2)-R_{13} = 0$$

$$\frac{1}{N_2}-R_{14} = 0$$



1CST5 [0, 0, 3]

$N_3\text{-}R_0 = 0$

$\frac{N_3}{1+N_3}\text{-}R_1 = 0$

$1\text{-}R_2 = 0$

$\frac{N_3}{1+N_3}\text{-}R_3 = 0$

$1\text{-}R_4 = 0$

$1\text{-}R_5 = 0$

$\frac{1+N_3}{N_3}\text{-}R_6 = 0$

$1\text{-}R_7 = 0$

$1\text{-}R_8 = 0$

$1\text{-}R_9 = 0$

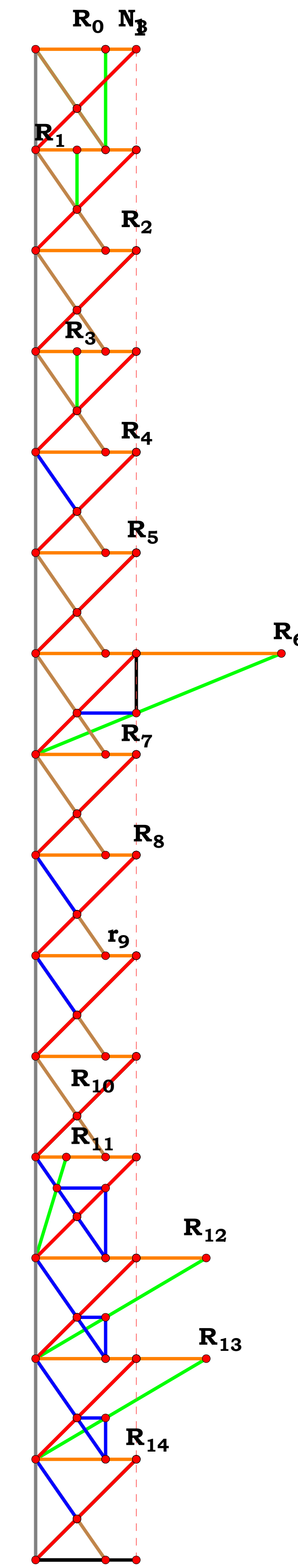
$1\text{-}R_{10} = 0$

$1\text{-}N_3\text{-}R_{11} = 0$

$(1+N_3)\text{-}R_{12} = 0$

$(1+N_3)\text{-}R_{13} = 0$

$1\text{-}R_{14} = 0$



1CST5 [1, 2, 0]

$$\frac{N_2^2}{(N_1^2+N_1)-N_2}-R_0 = 0$$

$$\frac{N_2}{N_1+1}-R_1 = 0$$

$$\frac{N_2}{(N_1-N_2)+1}-R_2 = 0$$

$$\frac{N_1^2}{N_1 \cdot N_2+N_2}-R_3 = 0$$

$$\frac{((N_1^3+N_1^2)-N_1 \cdot N_2)+N_2^2}{N_1 \cdot N_2+N_2}-R_4 = 0$$

$$\frac{N_1^2}{N_2}-R_5 = 0$$

$$(N_1 \cdot N_2+N_2)-R_6 = 0$$

$$\frac{(N_1^2+N_1)-N_2}{N_2}-R_7 = 0$$

$$\frac{((N_1^4+N_1^3)-N_1^2 \cdot N_2)+N_1 \cdot N_2^2}{N_1 \cdot N_2^2+N_2^2}-R_8 = 0$$

$$\frac{(N_1^3+N_1^2)-N_1 \cdot N_2}{N_2^2}-R_9 = 0$$

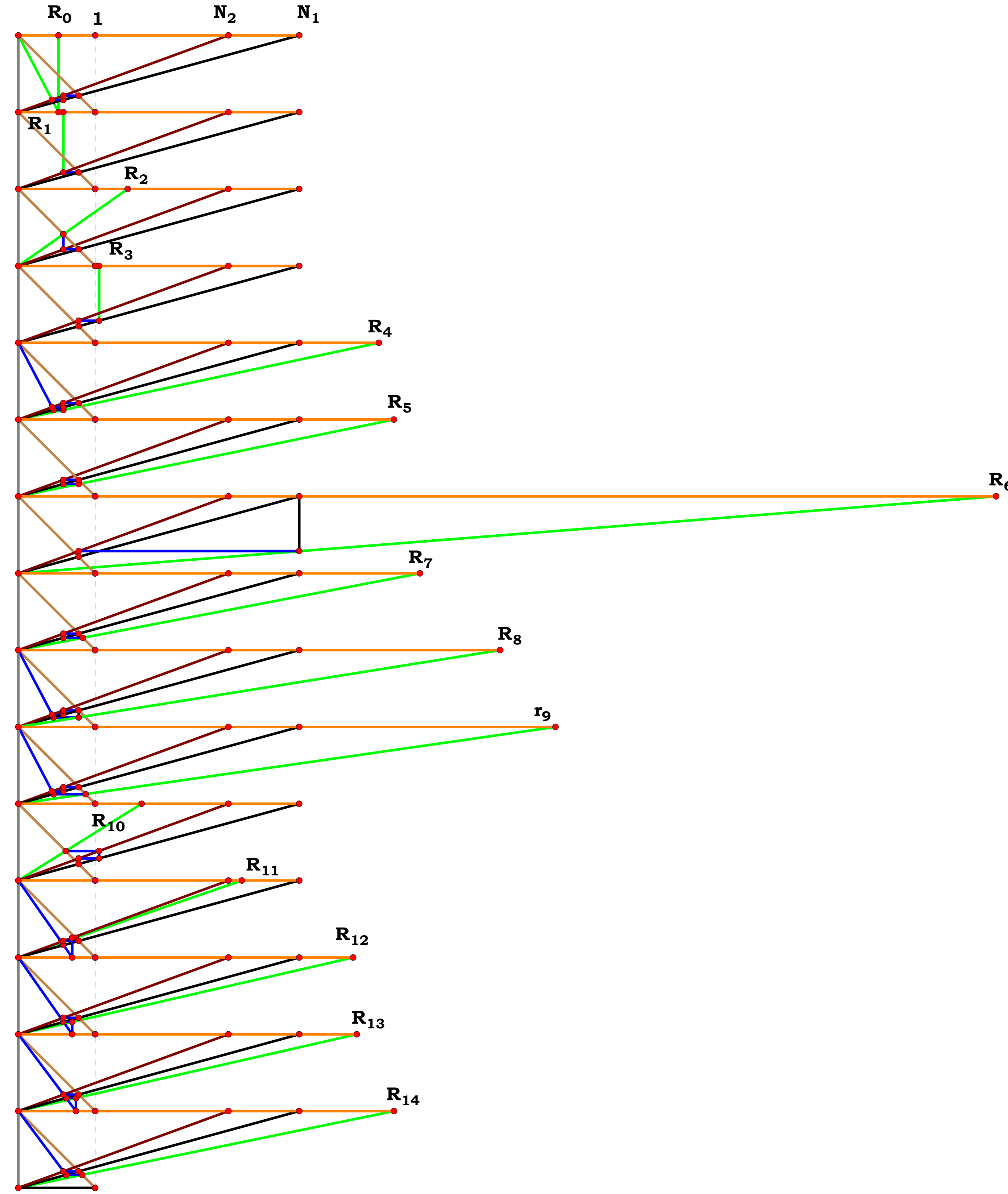
$$\frac{(N_1 \cdot N_2^2+N_2^2)-N_1^2}{N_1^2}-R_{10} = 0$$

$$\frac{N_1^2-N_2}{N_1}-R_{11} = 0$$

$$\frac{N_1^3+N_1^2}{(N_1^2+N_1)-N_2}-R_{12} = 0$$

$$\frac{N_1^2+N_2}{N_1}-R_{13} = 0$$

$$\frac{N_1^2}{N_2}-R_{14} = 0$$



1CST5 [1, 0, 3]

$$\frac{N_3}{(N_1^2+N_1 \cdot N_3)-N_3}-R_0 = 0$$

$$\frac{N_3}{N_1+N_3}-R_1 = 0$$

$$\frac{N_3}{(N_1-1)+N_3}-R_2 = 0$$

$$\frac{N_1^2 \cdot N_3}{N_1+N_3}-R_3 = 0$$

$$\frac{((N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_3)+N_3}{N_1+N_3}-R_4 = 0$$

$$N_1^2-R_5 = 0$$

$$\frac{N_1+N_3}{N_3}-R_6 = 0$$

$$(N_1^2+N_1 \cdot N_3)-N_3-R_7 = 0$$

$$\frac{((N_1^4+N_1^3 \cdot N_3)-N_1^2 \cdot N_3)+N_1 \cdot N_3}{N_1+N_3}-R_8 = 0$$

$$(N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_3-R_9 = 0$$

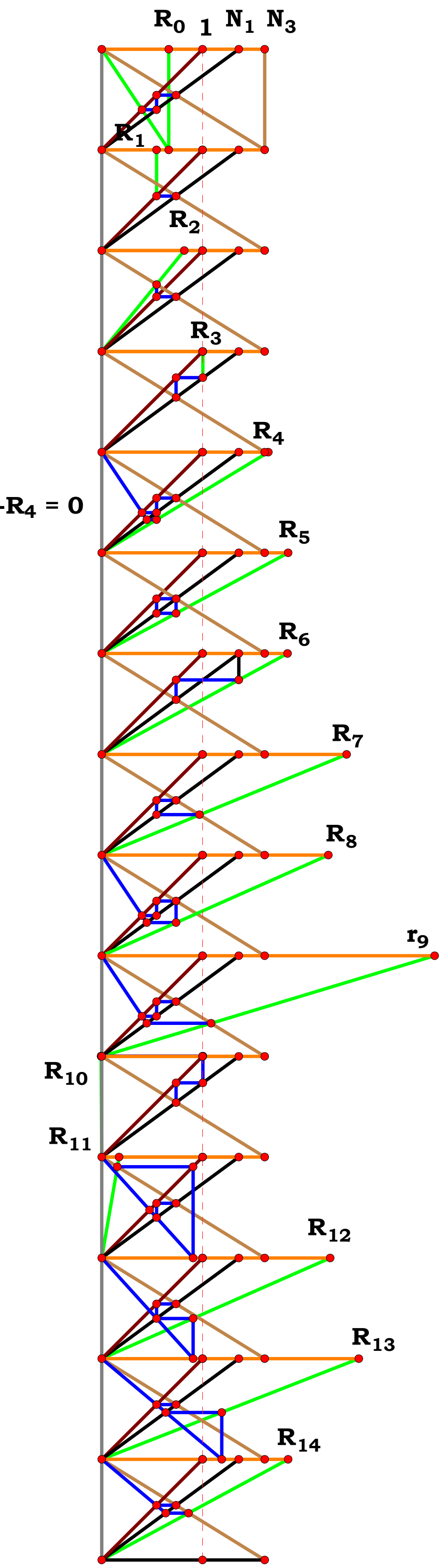
$$\frac{(N_1+N_3)-N_1^2 \cdot N_3}{N_1^2}-R_{10} = 0$$

$$\frac{N_1^2-N_3}{N_1}-R_{11} = 0$$

$$\frac{N_1^3+N_1^2 \cdot N_3}{(N_1^2+N_1 \cdot N_3)-N_3}-R_{12} = 0$$

$$\frac{N_1^2+N_3}{N_1}-R_{13} = 0$$

$$N_1^2-R_{14} = 0$$



1CST5 [0, 2, 3]

$$\frac{N_2{}^2 \cdot N_3}{(1+N_3)-N_2 \cdot N_3}-R_0 = 0$$

$$\frac{N_2 \cdot N_3}{1+N_3}-R_1 = 0$$

$$\frac{N_2 \cdot N_3}{(1-N_2)+N_3}-R_2 = 0$$

$$\frac{N_3}{N_2+N_2 \cdot N_3}-R_3 = 0$$

$$\frac{((1+N_3)-N_2 \cdot N_3)+N_2{}^2 \cdot N_3}{N_2+N_2 \cdot N_3}-R_4 = 0$$

$$N_1{}^2-R_5 = 0$$

$$\frac{N_2+N_2 \cdot N_3}{N_3}-R_6 = 0$$

$$\frac{(1+N_3)-N_2 \cdot N_3}{N_2}-R_7 = 0$$

$$\frac{((1+N_3)-N_2 \cdot N_3)+N_2{}^2 \cdot N_3}{N_2{}^2+N_2{}^2 \cdot N_3}-R_8 = 0$$

$$\frac{(1+N_3)-N_2 \cdot N_3}{N_2{}^2}-R_9 = 0$$

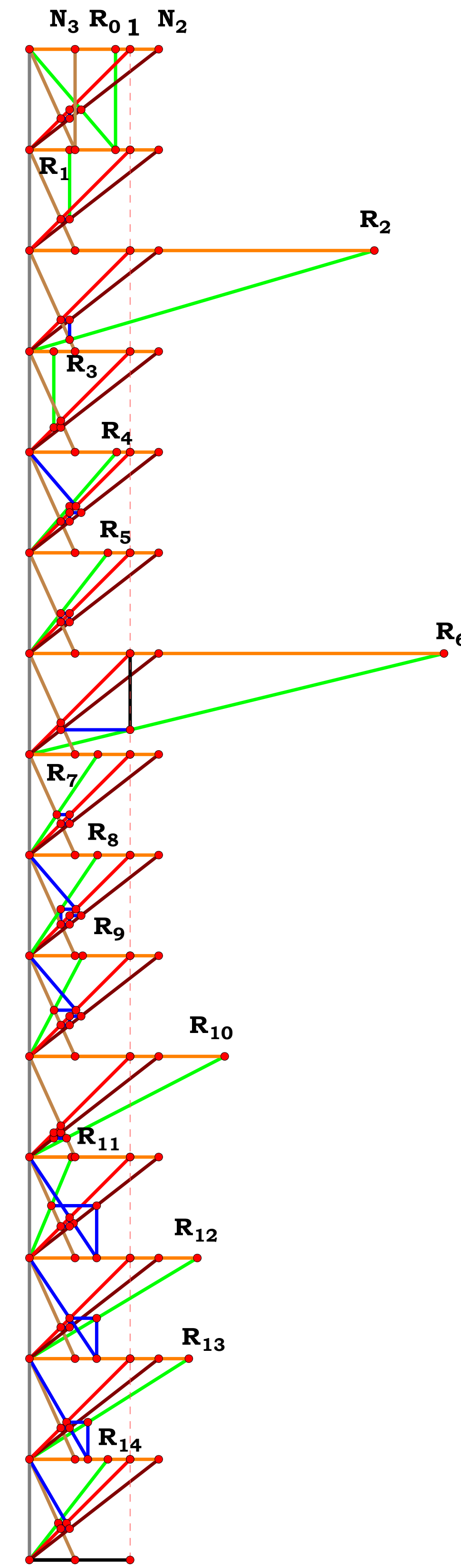
$$(N_2{}^2+N_2{}^2 \cdot N_3)-N_3-R_{10} = 0$$

$$1-N_2 \cdot N_3-R_{11} = 0$$

$$\frac{1+N_3}{(1+N_1 \cdot N_3)-N_2 \cdot N_3}-R_{12} = 0$$

$$(1+N_2 \cdot N_3)-R_{13} = 0$$

$$\frac{1}{N_2}-R_{14} = 0$$



1CST6

$$\frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2 \cdot N_3} - R_0 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_3^2 + N_2 \cdot N_3^2 \cdot N_4}{N_1^2 \cdot N_4} - R_1 = 0$$

$$\frac{\left(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4\right) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} - R_2 = 0$$

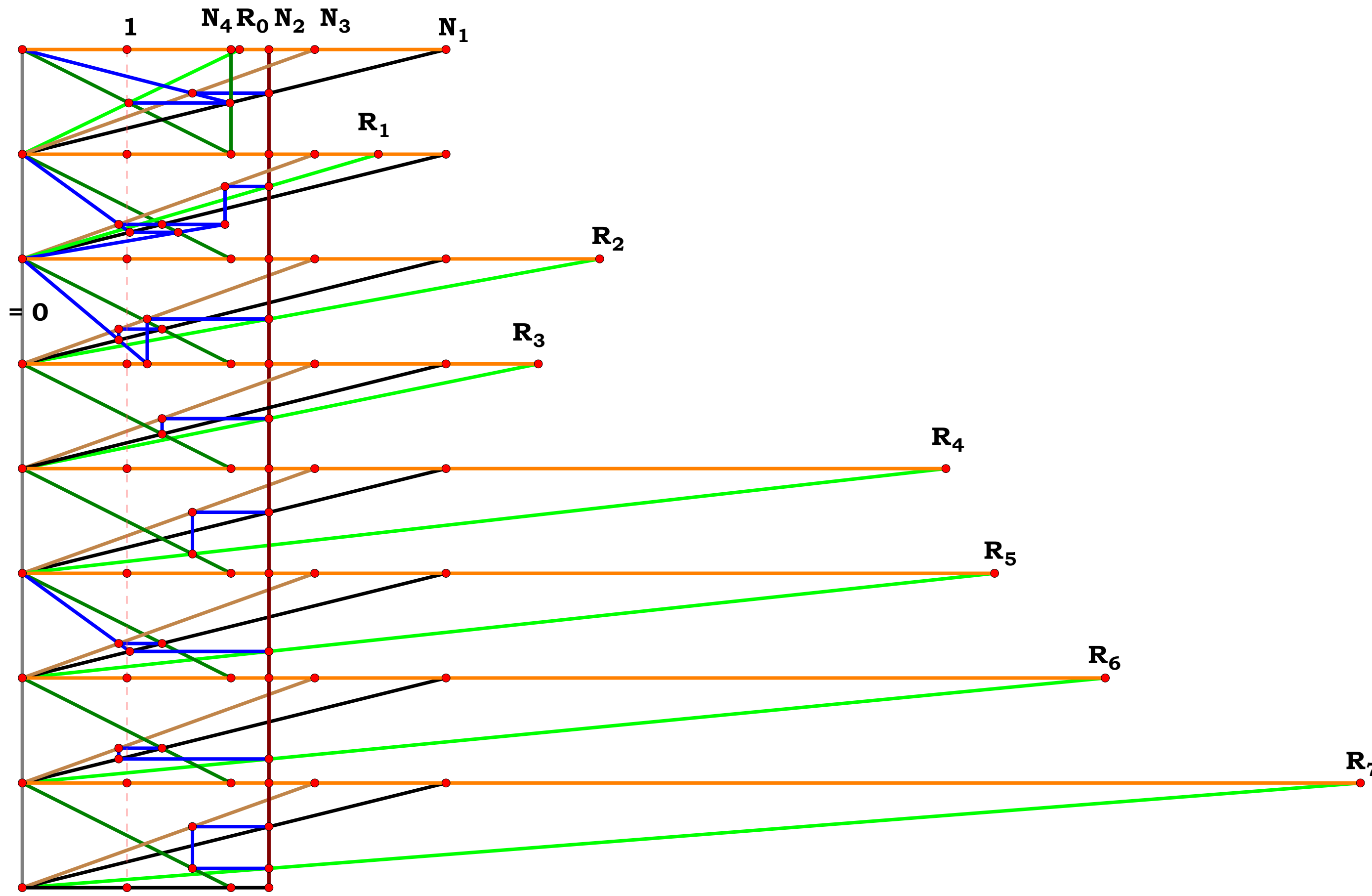
$$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} - R_3 = 0$$

$$\frac{N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_2 \cdot N_3 \cdot N_4}{N_3 \cdot N_4} - R_5 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4}{N_3 \cdot N_4} - R_6 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} - R_7 = 0$$



1CST6 [1, 0, 0, 0]

$N_1{}^2 - N_1 - R_0 = 0$

$\frac{N_1+1}{N_1{}^2} - R_1 = 0$

$\frac{(N_1{}^2+N_1)-1}{N_1} - R_2 = 0$

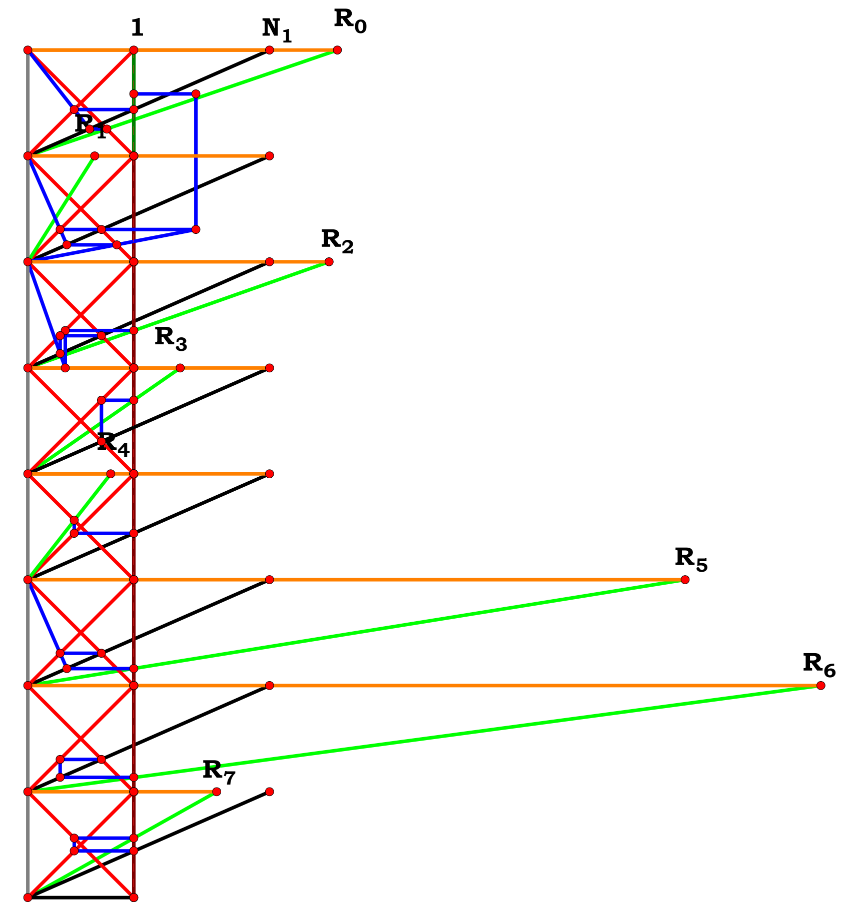
$\frac{N_1+1}{N_1} - R_3 = 0$

$\frac{1}{N_1-1} - R_4 = 0$

$(N_1{}^2+1) - R_5 = 0$

$(N_1{}^2+N_1) - R_6 = 0$

$\frac{N_1}{N_1-1} - R_7 = 0$



1CST6 [0, 2, 0, 0]

$$\frac{1-N_2}{N_2}-R_0 = 0$$

$$2 \cdot N_2-R_1 = 0$$

$$N_2-R_2 = 0$$

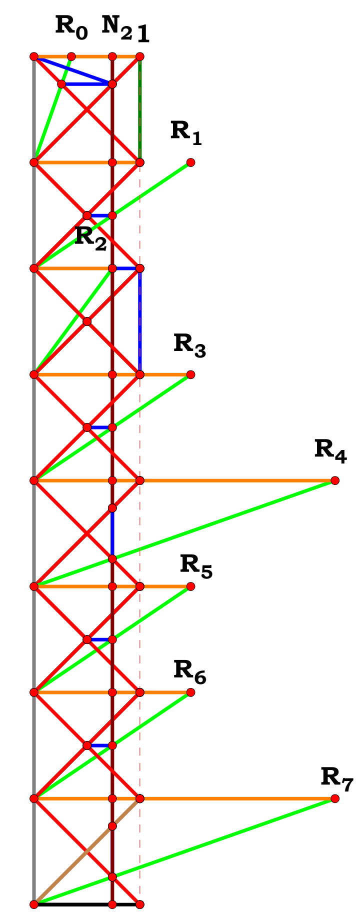
$$2 \cdot N_2-R_3 = 0$$

$$\frac{N_2}{1-N_2}-R_4 = 0$$

$$2 \cdot N_2-R_5 = 0$$

$$2 \cdot N_2-R_6 = 0$$

$$\frac{N_2}{1-N_2}-R_7 = 0$$



1CST6 [0, 0, 3, 0]

$0\text{-}R_0 = 0$

$2\cdot N_3^{2}\text{-}R_1 = 0$

$2\text{-}N_3\text{-}R_2 = 0$

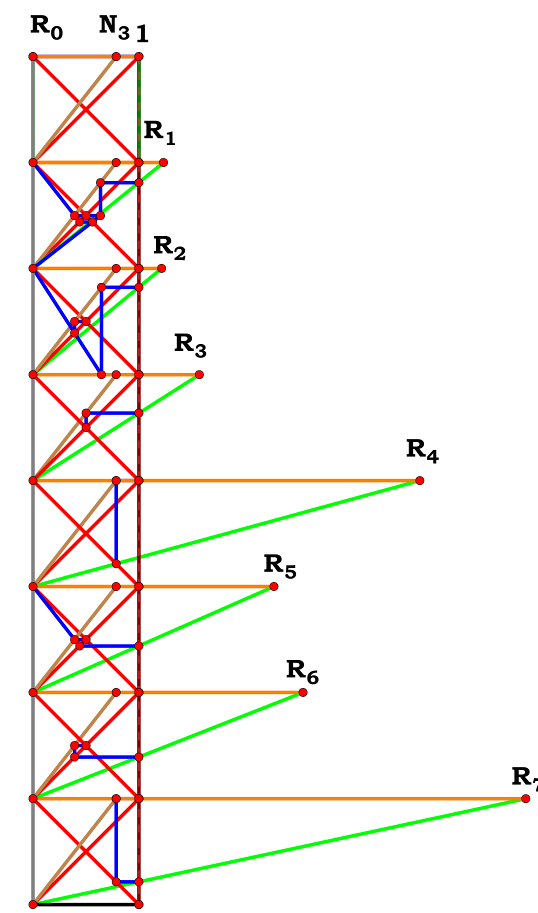
$2\cdot N_3\text{-}R_3 = 0$

$\frac{N_3}{1\text{-}N_3}\text{-}R_4 = 0$

$\frac{1+N_3}{N_3}\text{-}R_5 = 0$

$\frac{2}{N_3}\text{-}R_6 = 0$

$\frac{1}{1\text{-}N_3}\text{-}R_7 = 0$



1CST6 [0, 0, 0, 4]

$0\text{-}R_0 = 0$

$\frac{1+N_4}{N_4}\text{-}R_1 = 0$

$\frac{1}{N_4}\text{-}R_2 = 0$

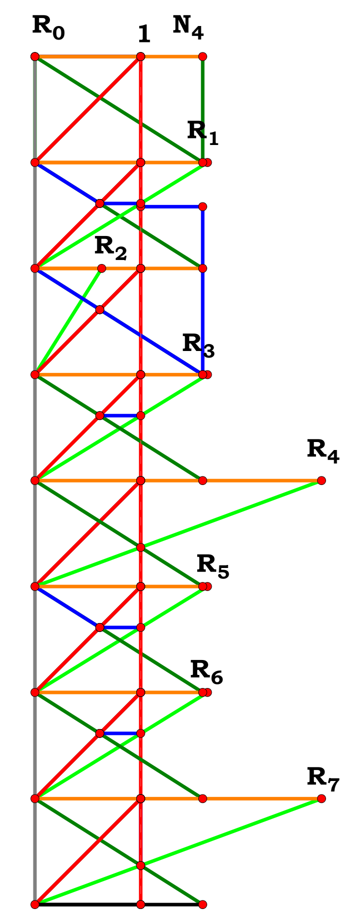
$\frac{1+N_4}{N_4}\text{-}R_3 = 0$

$\frac{N_4}{N_4\text{-}1}\text{-}R_4 = 0$

$\frac{1+N_4}{N_4}\text{-}R_5 = 0$

$\frac{1+N_4}{N_4}\text{-}R_6 = 0$

$\frac{N_4}{N_4\text{-}1}\text{-}R_7 = 0$



1CST6 [1, 2, 0, 0]

$$\frac{N_1^2 - N_1 \cdot N_2}{N_2} - R_0 = 0$$

$$\frac{N_1 \cdot N_2 + N_2}{N_1^2} - R_1 = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2) - N_2}{N_1} - R_2 = 0$$

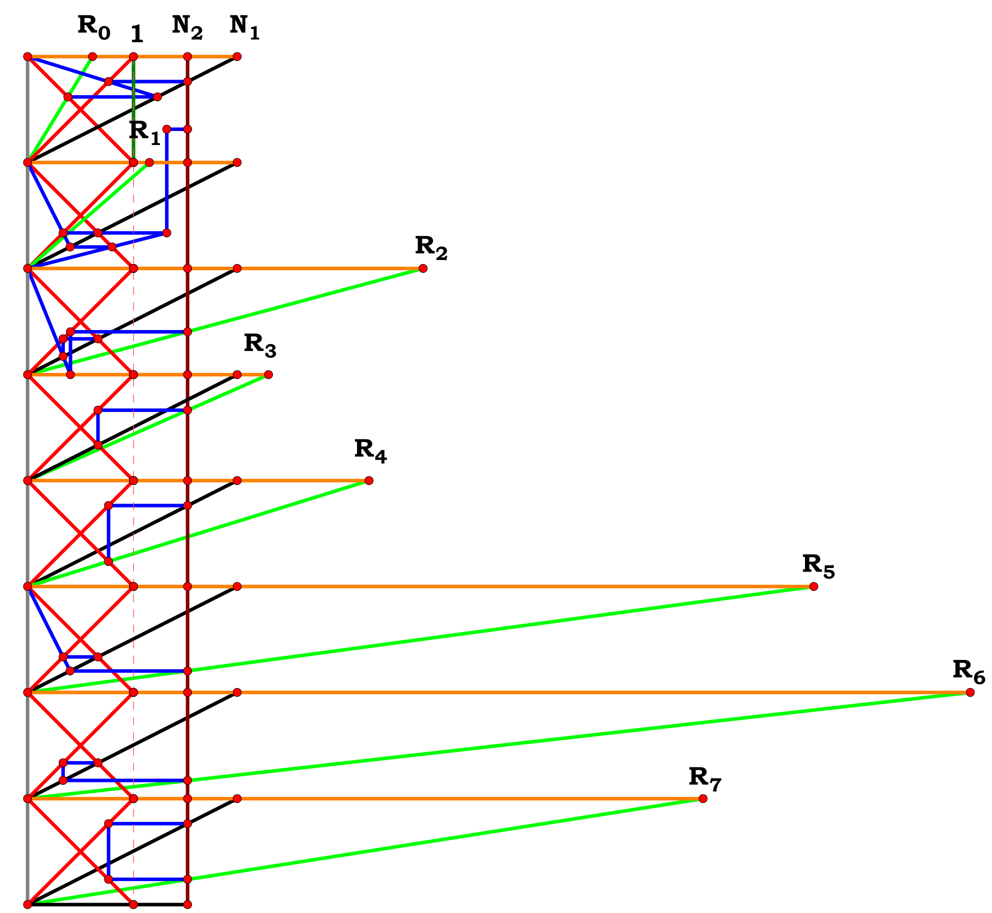
$$\frac{N_1 \cdot N_2 + N_2}{N_1} - R_3 = 0$$

$$\frac{N_2}{N_1 - N_2} - R_4 = 0$$

$$(N_1^2 \cdot N_2 + N_2) - R_5 = 0$$

$$(N_1^2 \cdot N_2 + N_1 \cdot N_2) - R_6 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_7 = 0$$



1CST6 [1, 0, 3, 0]

$$\frac{N_1^2 - N_1}{N_3} - R_0 = 0$$

$$\frac{N_1 \cdot N_3^2 + N_3^2}{N_1^2} - R_1 = 0$$

$$\frac{(N_1^2 + N_1) - N_3}{N_1} - R_2 = 0$$

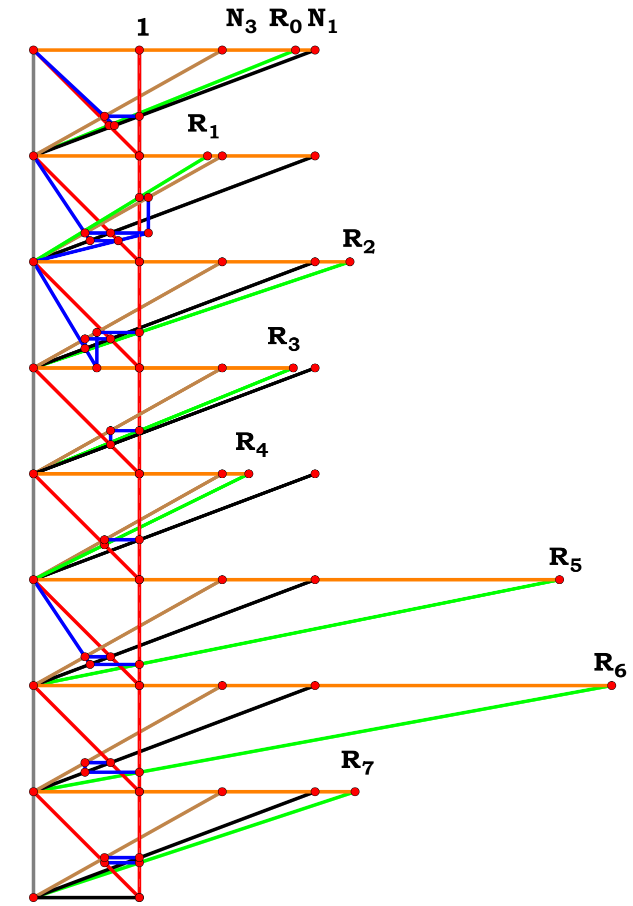
$$\frac{N_1 \cdot N_3 + N_3}{N_1} - R_3 = 0$$

$$\frac{N_3}{N_1 - N_3} - R_4 = 0$$

$$\frac{N_1^2 + N_3}{N_3} - R_5 = 0$$

$$\frac{N_1^2 + N_1}{N_3} - R_6 = 0$$

$$\frac{N_1}{N_1 - N_3} - R_7 = 0$$



1CST6 [1, 0, 0, 4]

$N_1^2 \cdot N_4 - N_1 \cdot N_4 - R_0 = 0$

$\frac{N_1+N_4}{N_1^2 \cdot N_4} - R_1 = 0$

$\frac{(N_1^2+N_1 \cdot N_4)-N_4}{N_1 \cdot N_4} - R_2 = 0$

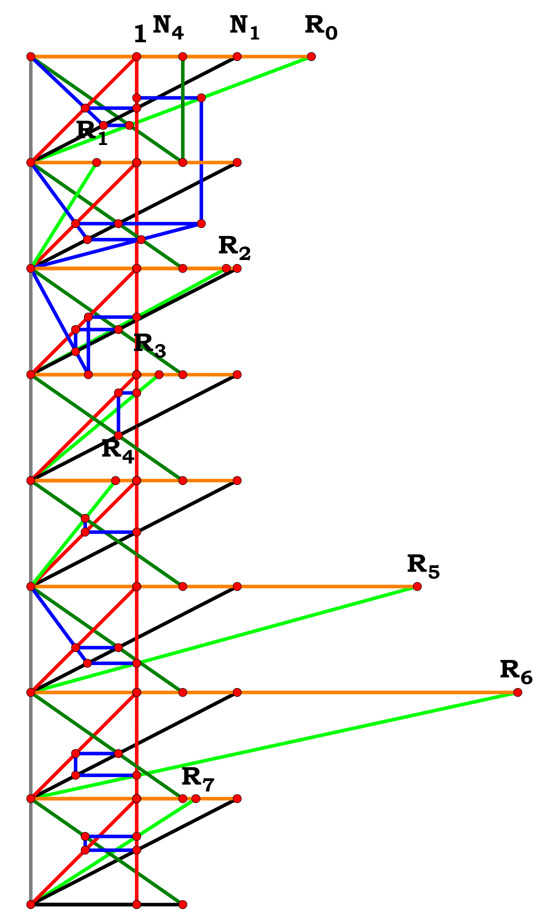
$\frac{N_1+N_4}{N_1 \cdot N_4} - R_3 = 0$

$\frac{N_4}{N_1 \cdot N_4 - 1} - R_4 = 0$

$\frac{N_1^2+N_4}{N_4} - R_5 = 0$

$\frac{N_1^2+N_1 \cdot N_4}{N_4} - R_6 = 0$

$\frac{N_1 \cdot N_4}{N_1 \cdot N_4 - 1} - R_7 = 0$



1CST6 [0, 2, 3, 0]

$$\frac{1-N_2}{N_2 \cdot N_3} - R_0 = 0$$

$$2 \cdot N_2 \cdot N_3^2 - R_1 = 0$$

$$2 \cdot N_2 - N_2 \cdot N_3 - R_2 = 0$$

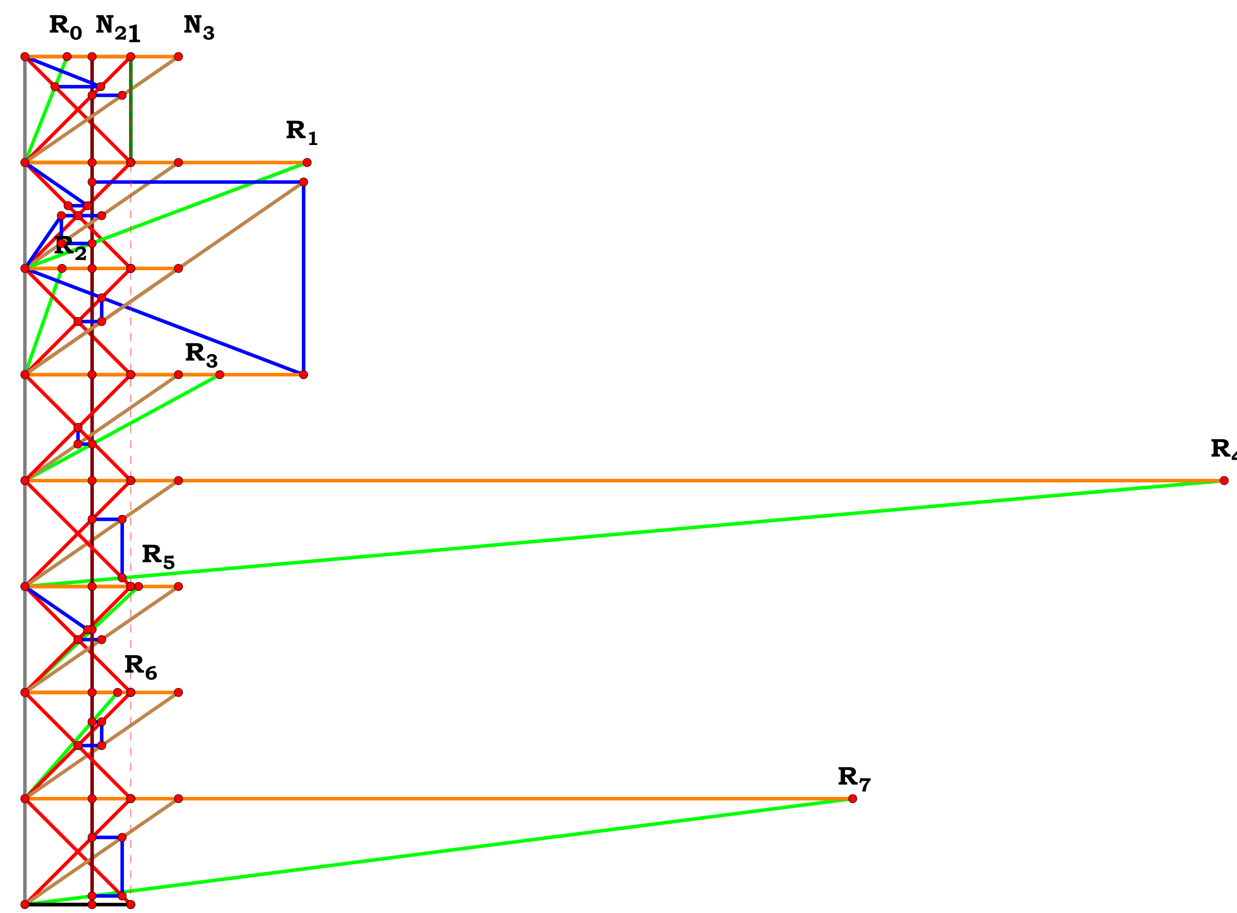
$$2 \cdot N_2 \cdot N_3 - R_3 = 0$$

$$\frac{N_2 \cdot N_3}{1 - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{N_2 + N_2 \cdot N_3}{N_3} - R_5 = 0$$

$$\frac{N_2 + N_2}{N_3} - R_6 = 0$$

$$\frac{N_2}{1 - N_2 \cdot N_3} - R_7 = 0$$



1CST6 [0, 2, 0, 4]

$$\frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2 \cdot N_3} - R_0 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_3^2 + N_2 \cdot N_3^2 \cdot N_4}{N_1^2 \cdot N_4} - R_1 = 0$$

$$\frac{\left(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4\right) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} - R_2 = 0$$

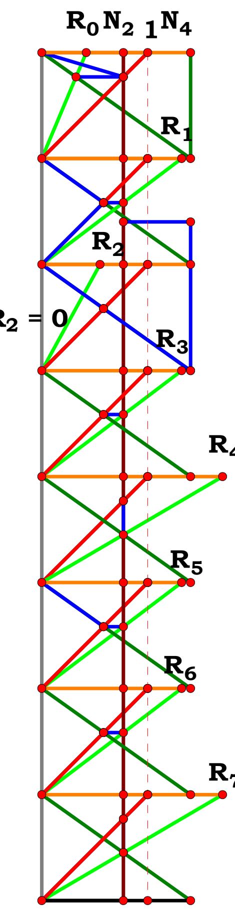
$$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} - R_3 = 0$$

$$\frac{N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_2 \cdot N_3 \cdot N_4}{N_3 \cdot N_4} - R_5 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4}{N_3 \cdot N_4} - R_6 = 0$$

$$\frac{N_2 \cdot N_4}{N_4 - N_2} - R_7 = 0$$



1CST6 [0, 0, 3, 4]

$0 - R_0 = 0$

$\frac{N_3^2 + N_3^2 \cdot N_4}{N_4} - R_1 = 0$

$\frac{(1+N_4) - N_3 \cdot N_4}{N_4} - R_2 = 0$

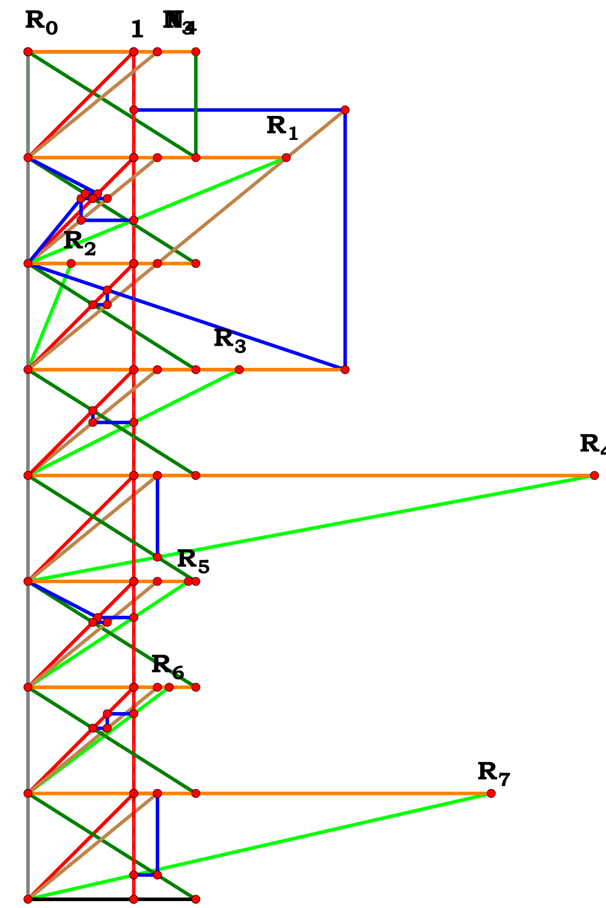
$\frac{N_3 + N_3 \cdot N_4}{N_4} - R_3 = 0$

$\frac{N_3 \cdot N_4}{N_4 - N_3} - R_4 = 0$

$\frac{1 + N_3 \cdot N_4}{N_3 \cdot N_4} - R_5 = 0$

$\frac{1 + N_4}{N_3 \cdot N_4} - R_6 = 0$

$\frac{N_4}{N_4 - N_3} - R_7 = 0$



1CST6 [1, 2, 3, 0]

$$\frac{N_1^2 - N_1 \cdot N_2}{N_2 \cdot N_3} - R_0 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_3^2 + N_2 \cdot N_3^2}{N_1^2} - R_1 = 0$$

$$\frac{\left(N_1^2 \cdot N_2 + N_1 \cdot N_2\right) - N_2 \cdot N_3}{N_1} - R_2 = 0$$

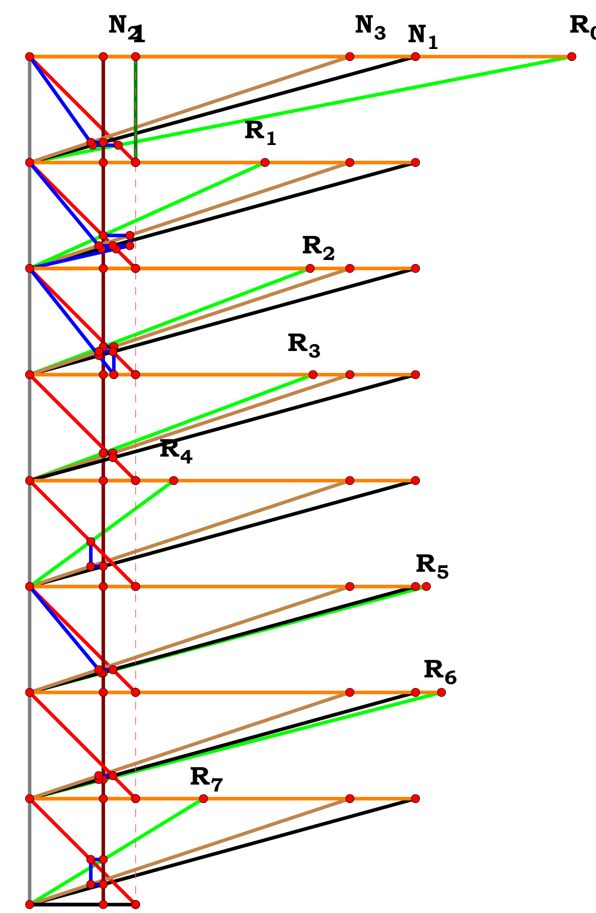
$$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3}{N_4} - R_3 = 7$$

$$\frac{N_2 \cdot N_3}{N_1 - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_2 \cdot N_3}{N_3} - R_5 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2}{N_3} - R_6 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - N_2 \cdot N_3} - R_7 = 0$$



1CST6 [1, 0, 3, 4]

$$\frac{N_1^2 \cdot N_4 - N_1 \cdot N_4}{N_3} - R_0 = 0$$

$$\frac{N_1 \cdot N_3^2 + N_3^2 \cdot N_4}{N_1^2 \cdot N_4} - R_1 = 0$$

$$\frac{(N_1^2 + N_1 \cdot N_4) - N_3 \cdot N_4}{N_1 \cdot N_4} - R_2 = 0$$

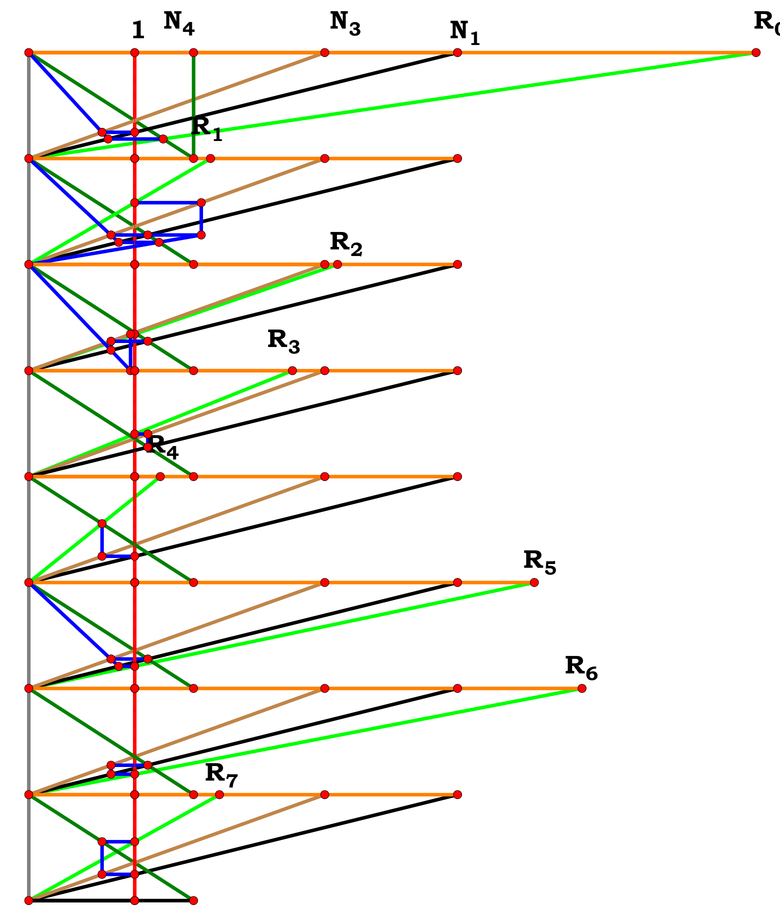
$$\frac{N_1 \cdot N_3 + N_3 \cdot N_4}{N_1 \cdot N_4} - R_3 = 0$$

$$\frac{N_3 \cdot N_4}{N_1 \cdot N_4 - N_3} - R_4 = 0$$

$$\frac{N_1^2 + N_3 \cdot N_4}{N_3 \cdot N_4} - R_5 = 0$$

$$\frac{N_1^2 + N_1 \cdot N_4}{N_3 \cdot N_4} - R_6 = 0$$

$$\frac{N_1 \cdot N_4}{N_1 \cdot N_4 - N_3} - R_7 = 0$$



1CST6 [0, 2, 3, 4]

$$\frac{N_4 - N_2 \cdot N_4}{N_2 \cdot N_3} - R_0 = 0$$

$$\frac{N_2 \cdot N_3^2 + N_2 \cdot N_3^2 \cdot N_4}{N_4} - R_1 = 0$$

$$\frac{(N_2 + N_2 \cdot N_4) - N_2 \cdot N_3 \cdot N_4}{N_4} - R_2 = 0$$

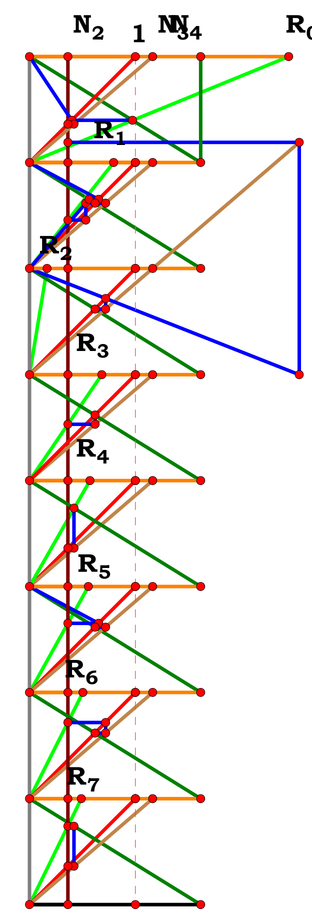
$$\frac{N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_4} - R_3 = 0$$

$$\frac{N_2 \cdot N_3 \cdot N_4}{N_4 - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{N_2 + N_2 \cdot N_3 \cdot N_4}{N_3 \cdot N_4} - R_5 = 0$$

$$\frac{N_2 + N_2 \cdot N_4}{N_3 \cdot N_4} - R_6 = 0$$

$$\frac{N_2 \cdot N_4}{N_4 - N_2 \cdot N_3} - R_7 = 0$$



1CST6 [1, 2, 0, 4]

$$\frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2} - R_0 = 0$$

$$\frac{N_1 \cdot N_2 + N_2 \cdot N_4}{N_1^2 \cdot N_4} - R_1 = 0$$

$$\frac{\left(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4\right) - N_2 \cdot N_4}{N_1 \cdot N_4} - R_2 = 0$$

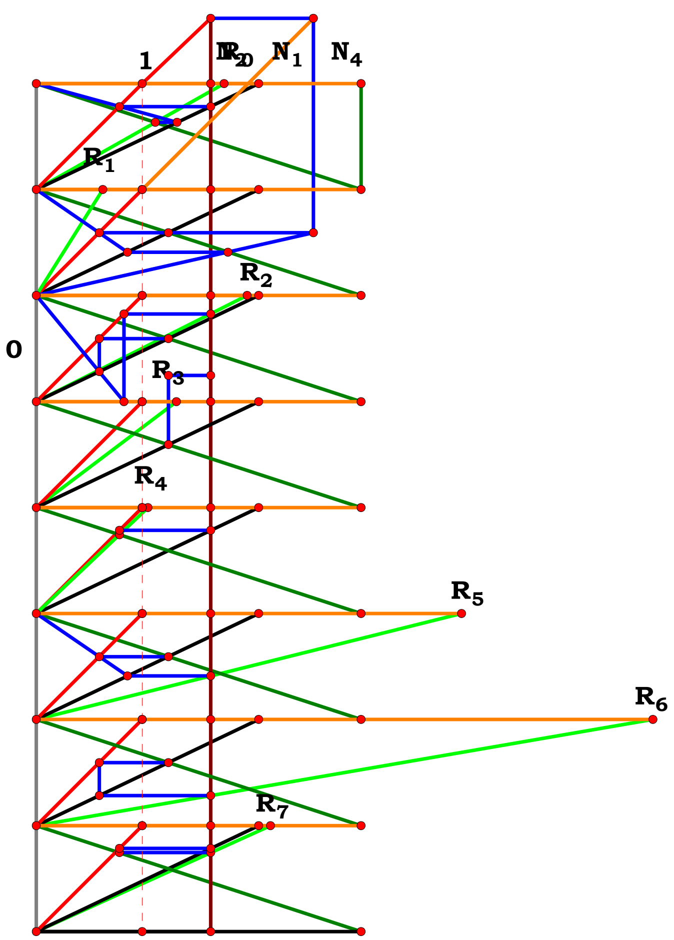
$$\frac{N_1 \cdot N_2 + N_2 \cdot N_4}{N_1 \cdot N_4} - R_3 = 0$$

$$\frac{N_2 \cdot N_4}{N_1 \cdot N_4 - N_2} - R_4 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_2 \cdot N_4}{N_4} - R_5 = 0$$

$$\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4}{N_4} - R_6 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_2} - R_7 = 0$$



1CST7

$$\frac{N_1 \cdot N_2 \cdot N_3 - N_2^2 \cdot N_3}{N_1^2} - R_0 = 0$$

$$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_3}{(N_1 \cdot N_2 + N_1 \cdot N_3) - N_2 \cdot N_3} - R_2 = 0$$

$$\frac{(N_1 \cdot N_2 \cdot N_4^2 + N_1 \cdot N_2 \cdot N_3 \cdot N_4) - N_2 \cdot N_3 \cdot N_4^2}{(((N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4) + N_1 \cdot N_4^2) - N_3 \cdot N_4^2} - R_3 = 0$$

$$\frac{N_1^2 \cdot N_3}{(N_1^2 + N_1 \cdot N_3) - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{(N_1^2 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4^2) - N_2 \cdot N_3 \cdot N_4^2}{((N_1 \cdot N_4^2 - N_3 \cdot N_4^2) + N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_3 \cdot N_4} - R_5 = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_3 \cdot N_4}{N_1^2} - R_6 = 0$$

$$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_2) + N_2 \cdot N_3} - R_7 = 0$$

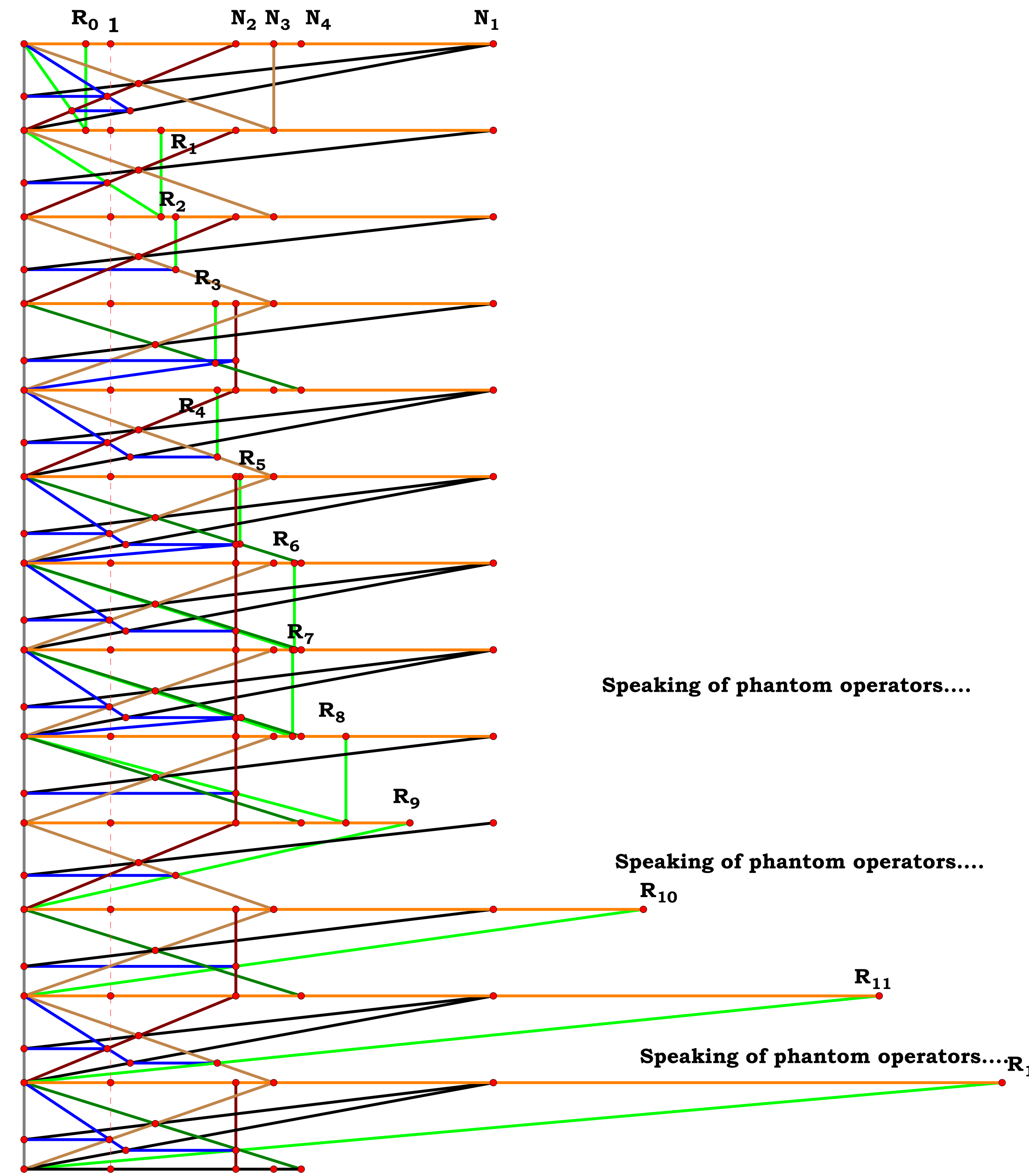
$$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_3} - R_8 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_9 = 0$$

$$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} - R_{10} = 0$$

$$\frac{N_1^2}{N_1 - N_2} - R_{11} = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} - R_{12} = 0$$



Speaking of phantom operators....

Speaking of phantom operators....

Speaking of phantom operators....

1CST7 [1, 0, 0, 0]

$$\frac{N_1-1}{N_1^2}-R_0 = 0$$

$$\frac{N_1-1}{N_1}-R_1 = 0$$

$$\frac{N_1}{2\cdot N_1-1}-R_2 = 0$$

$$\frac{2\cdot N_1-1}{3\cdot N_1-2}-R_3 = 0$$

$$\frac{N_1^2}{(N_1^2+N_1)-1}-R_4 = 0$$

$$\frac{(N_1^2+N_1)-1}{(N_1^2+2\cdot N_1)-2}-R_5 = 0$$

$$\frac{(N_1^2+N_1)-1}{N_1^2}-R_6 = 0$$

$$\frac{N_1^2}{(N_1^2-N_1)+1}-R_7 = 0$$

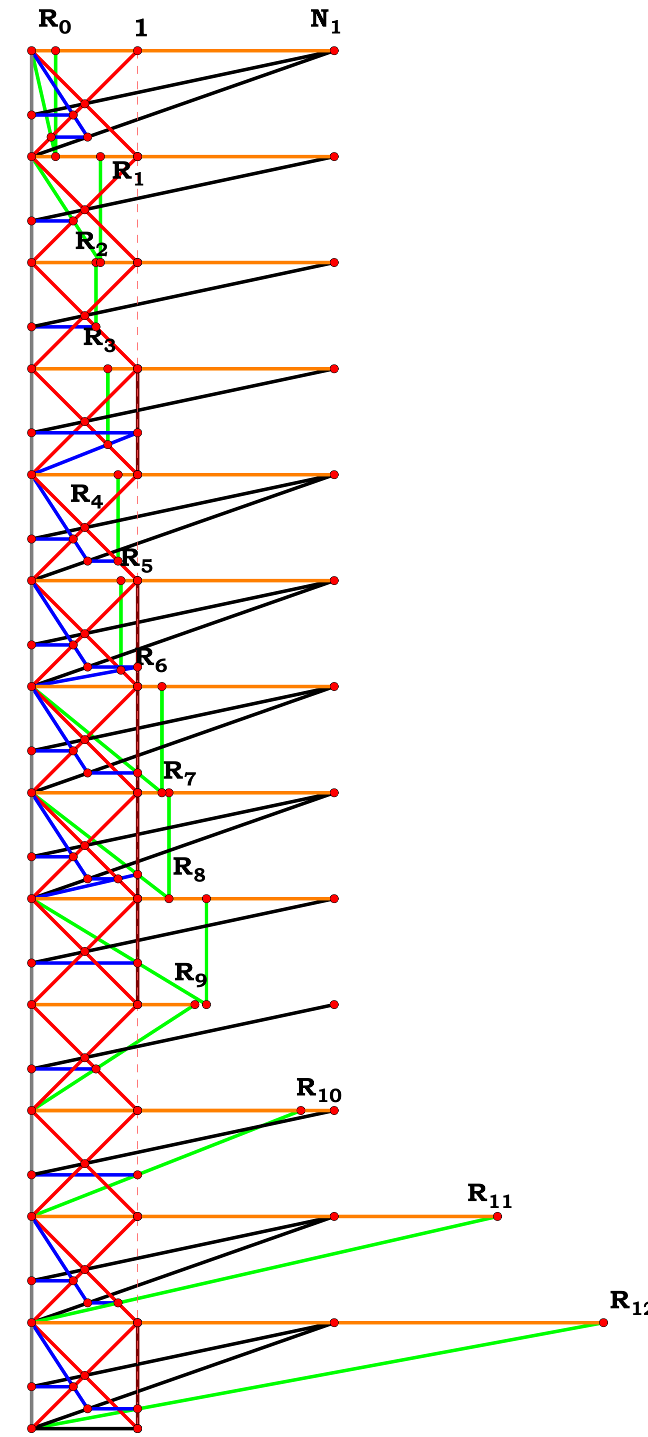
$$\frac{2\cdot N_1-1}{N_1}-R_8 = 0$$

$$\frac{N_1}{N_1-1}-R_9 = 0$$

$$\frac{2\cdot N_1-1}{N_1-1}-R_{10} = 0$$

$$\frac{N_1^2}{N_1-1}-R_{11} = 0$$

$$\frac{(N_1^2+N_1)-1}{N_1-1}-R_{12} = 0$$



1CST7 [0, 2, 0, 0]

$N_2 - N_2^2 - R_0 = 0$

$1 - N_2 - R_1 = 0$

$N_2 - R_2 = 0$

$1 - R_3 = 0$

$\frac{1}{2 - N_2} - R_4 = 0$

$1 - R_5 = 0$

$N_2 - R_6 = 0$

$N_2 - R_7 = 0$

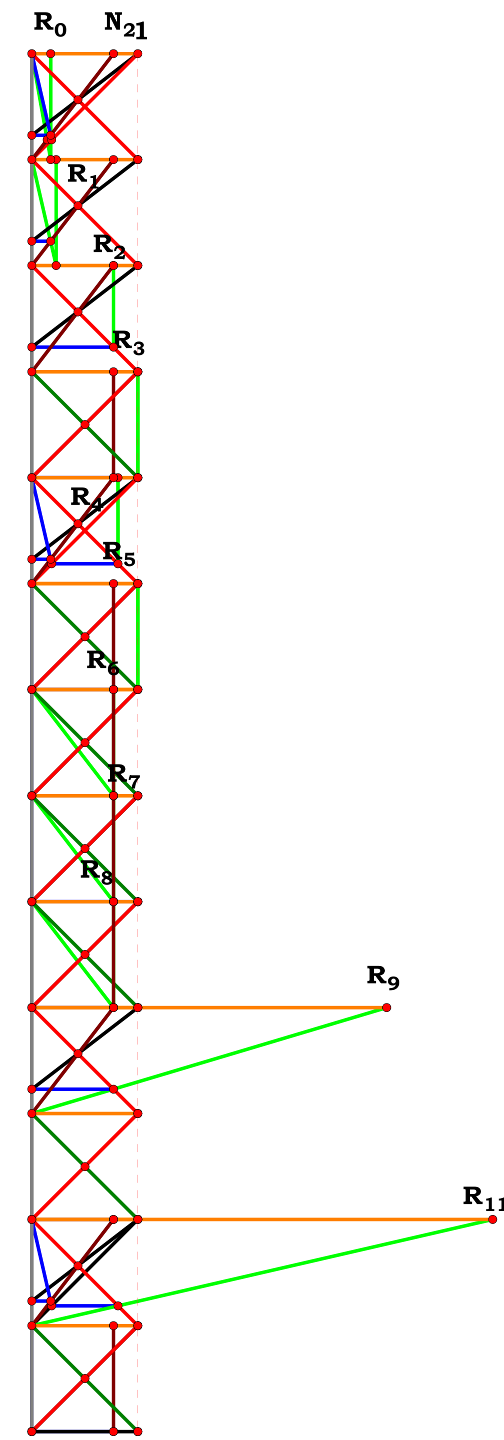
$N_2 - R_8 = 0$

$\frac{N_2}{1 - N_2} - R_9 = 0$

undefined

$\frac{1}{1 - N_2} - R_{11} = 0$

undefined



1CST7 [0, 0, 3, 0]

$0\text{-}R_0 = 0$

$0\text{-}R_1 = 0$

$N_3\text{-}R_2 = 0$

$\frac{1}{2\text{-}N_3}\text{-}R_3 = 0$

$N_3\text{-}R_4 = 0$

$\frac{N_3\text{-}2}{2 \cdot N_3\text{-}3}\text{-}R_5 = 0$

$2\text{-}N_3\text{-}R_6 = 0$

$\frac{1}{N_3}\text{-}R_7 = 0$

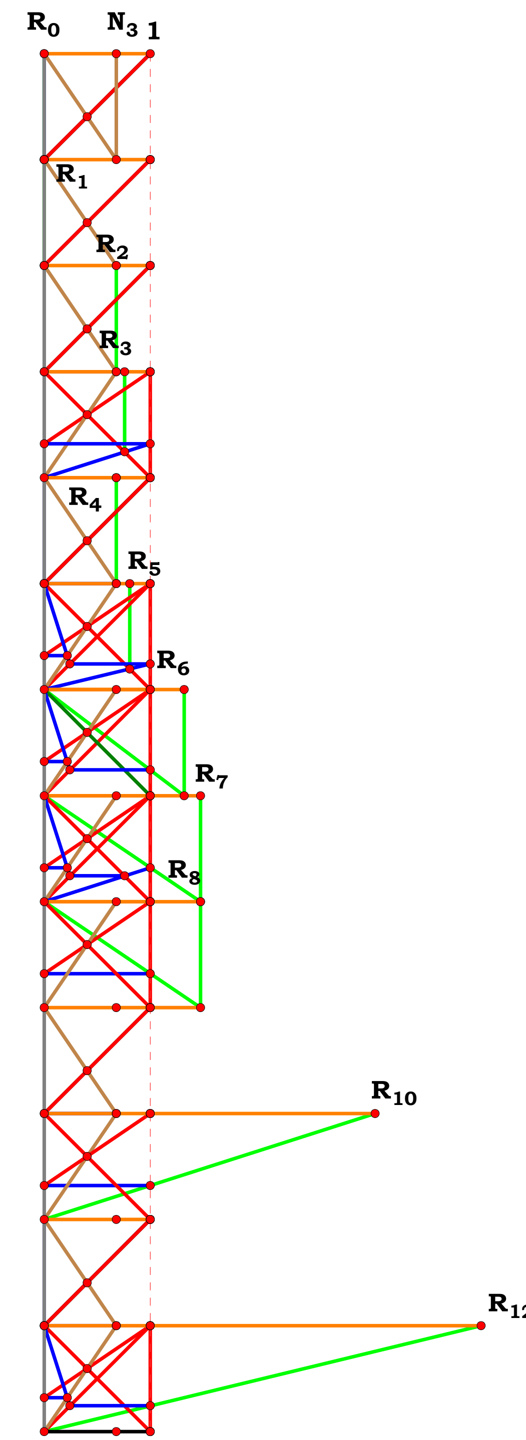
$\frac{1}{N_3}\text{-}R_8 = 0$

undefined

$\frac{1}{1\text{-}N_3}\text{-}R_{10} = 0$

undefined

$\frac{N_3\text{-}2}{N_3\text{-}1}\text{-}R_{12} = 0$



1CST7 [0, 0, 0, 4]

$\frac{0}{1}$-R_0 = 0

$\frac{0}{1}$-R_1 = 0

1-R_2 = 0

N_4-R_3 = 0

undefined

N_4-R_5 = 0

1-R_6 = 0

1-R_7 = 0

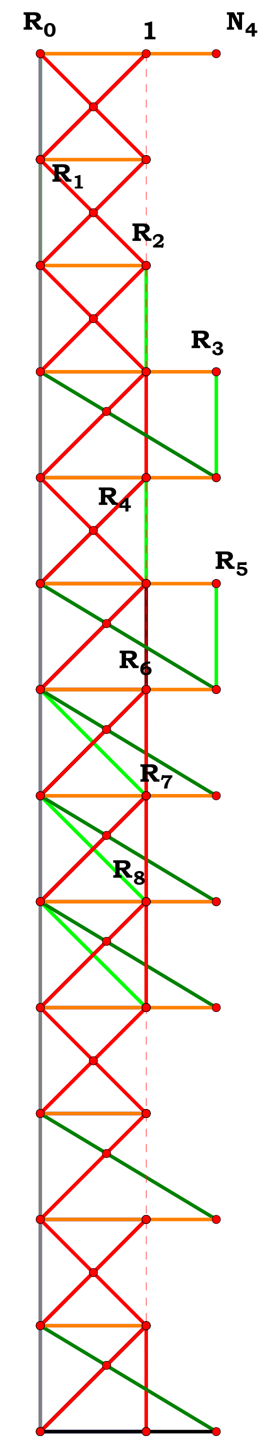
1-R_8 = 0

undefined

undefined

undefined

undefined



Comparing with R4, this may indicate that the conconception, in regard to the two symbolic representations, is inverted. In other words, something not divided is unity of the given, where nothing divided is actally the undefined.

This may indicate that when any value is multipled by the undefined, the result is unity.

1CST7 [1, 2, 0, 0]

$$\frac{N_1 \cdot N_2 - N_2^2}{N_1^2} - R_0 = 0$$

$$\frac{N_1 - N_2}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2 + N_1) - N_2} - R_2 = 0$$

$$\frac{2 \cdot N_1 \cdot N_2 - N_2}{((2 \cdot N_1 \cdot N_2 - N_2) + N_1) - 1} - R_3 = 0$$

$$\frac{N_1^2}{(N_1^2 + N_1) - N_2} - R_4 = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2) - N_2}{((N_1 - 1) + N_1^2 \cdot N_2 + N_1 \cdot N_2) - N_2} - R_5 = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2) - N_2}{N_1^2} - R_6 = 0$$

$$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_2) + N_2} - R_7 = 0$$

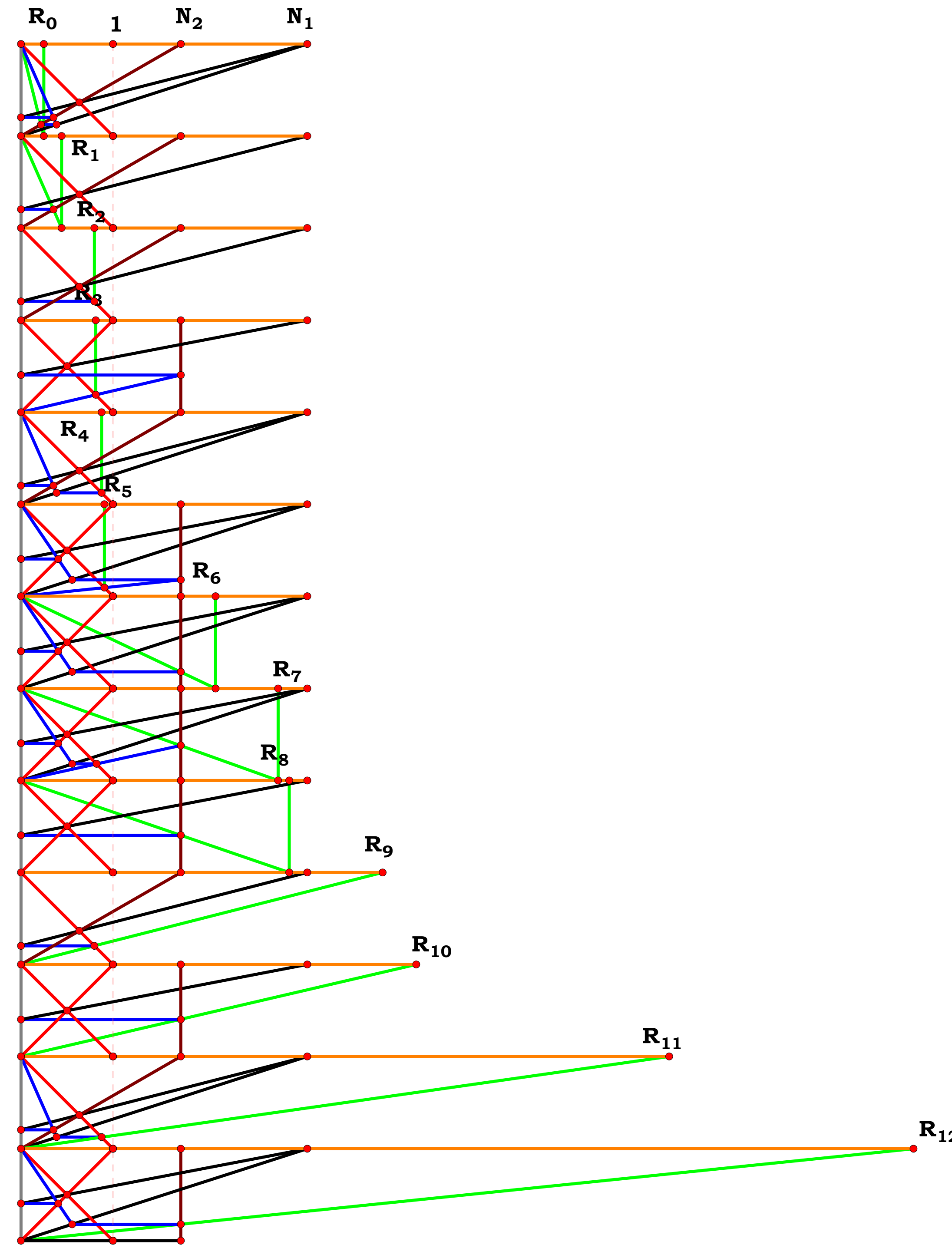
$$\frac{2 \cdot N_1 \cdot N_2 - N_2}{N_1} - R_8 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_9 = 0$$

$$\frac{2 \cdot N_1 \cdot N_2 - N_2}{N_1 - 1} - R_{10} = 0$$

$$\frac{N_1^2}{N_1 - N_2} - R_{11} = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2) - N_2}{N_1 - 1} - R_{12} = 0$$



1CST7 [1, 0, 3, 0]

$$\frac{N_1 \cdot N_3 - N_3}{N_1^2} - R_0 = 0$$

$$\frac{N_1 \cdot N_3 - N_3}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_3}{(N_1 + N_1 \cdot N_3) - N_3} - R_2 = 0$$

$$\frac{(N_1 - N_3) + N_1 \cdot N_3}{(2 \cdot N_1 - 2 \cdot N_3) + N_1 \cdot N_3} - R_3 = 0$$

$$\frac{N_1^2 \cdot N_3}{(N_1^2 + N_1 \cdot N_3) - N_3} - R_4 = 0$$

$$\frac{(N_1^2 + N_1) - N_3}{(N_1^2 + 2 \cdot N_1) - 2 \cdot N_3} - R_5 = 0$$

$$\frac{(N_1^2 + N_1) - N_3}{N_1^2} - R_6 = 0$$

$$\frac{N_1^2}{(N_1^2 - N_1) + N_3} - R_7 = 0$$

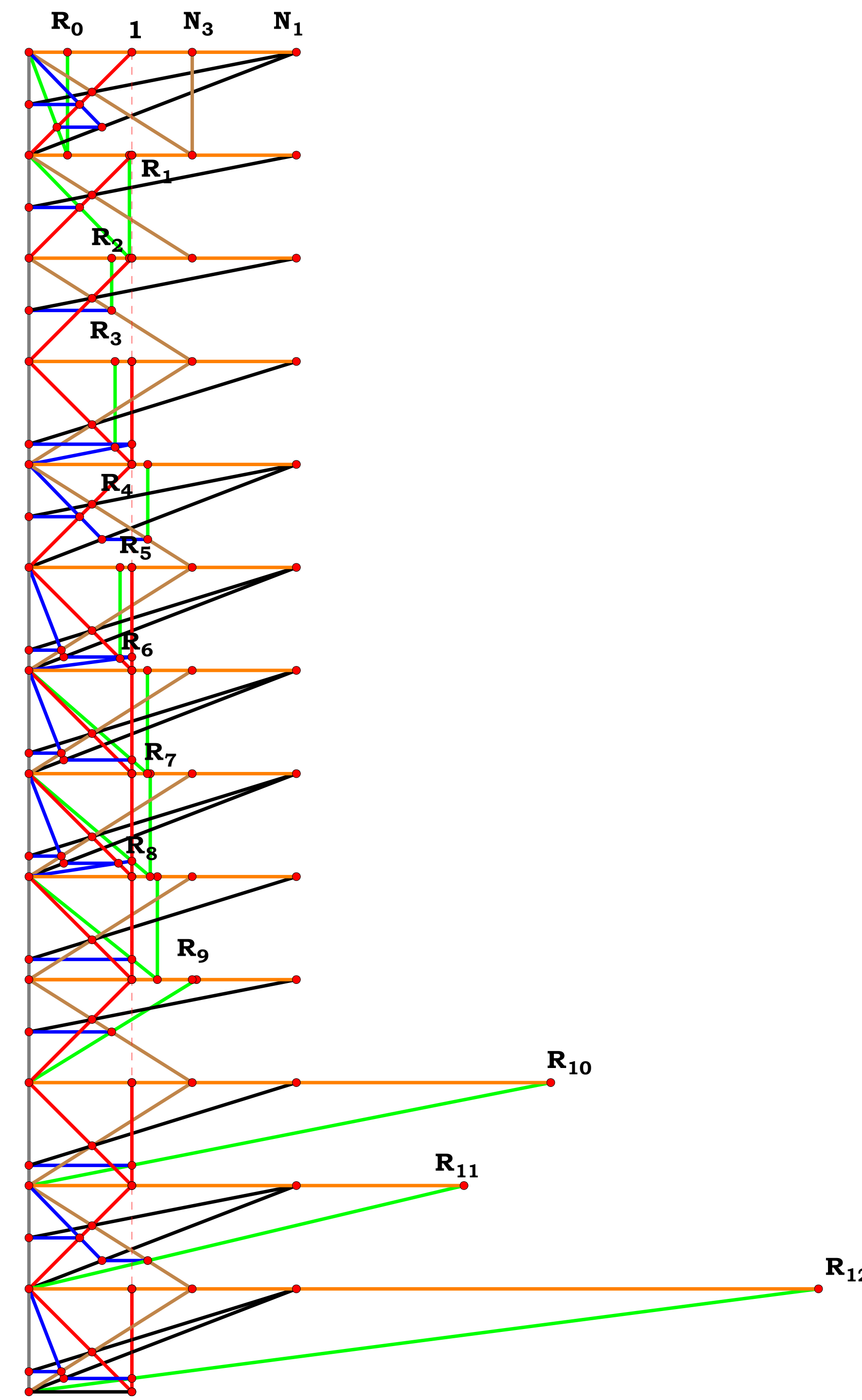
$$\frac{(N_1 + N_1 \cdot N_3) - N_3}{N_1 \cdot N_3} - R_8 = 0$$

$$\frac{N_1}{N_1 - 1} - R_9 = 0$$

$$\frac{(N_1 + N_1 \cdot N_3) - N_3}{N_1 - N_3} - R_{10} = 0$$

$$\frac{N_1^2}{N_1 - 1} - R_{11} = 0$$

$$\frac{(N_1^2 + N_1) - N_3}{N_1 - N_3} - R_{12} = 0$$



1CST7 [1, 0, 0, 4]

$$\frac{N_1-1}{N_1^2}-R_0 = 0$$

$$\frac{N_1-1}{N_1}-R_1 = 0$$

$$\frac{N_1}{(N_1+N_1)-1}-R_2 = 0$$

$$\frac{(N_1 \cdot N_4^2+N_1 \cdot N_4)-N_4^2}{(((N_1 \cdot N_4+N_1)-N_4)+N_1 \cdot N_4^2)-N_4^2}-R_3 = 0$$

$$\frac{N_1^2}{(N_1^2+N_1)-1}-R_4 = 0$$

$$\frac{(N_1^2 \cdot N_4+N_1 \cdot N_4^2)-N_4^2}{((N_1 \cdot N_4^2-N_4^2)+N_1^2+N_1 \cdot N_4)-N_4}-R_5 = 0$$

$$\frac{(N_1^2+N_1 \cdot N_4)-N_4}{N_1^2}-R_6 = 0$$

$$\frac{N_1^2}{(N_1^2-N_1)+1}-R_7 = 0$$

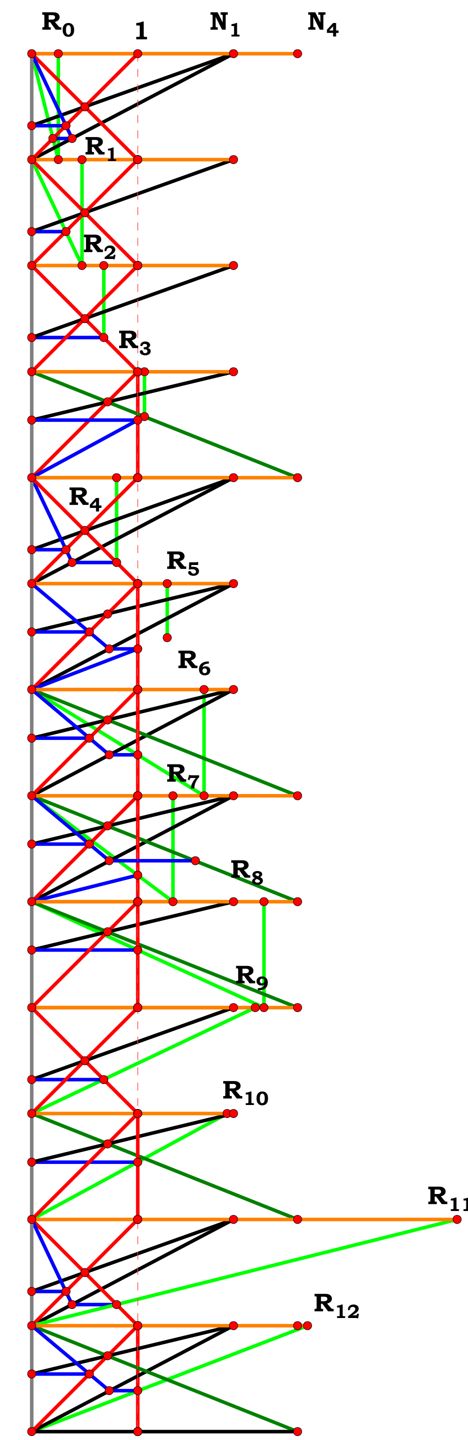
$$\frac{(N_1 \cdot N_4+N_1)-N_4}{N_1}-R_8 = 0$$

$$\frac{N_1}{N_1-1}-R_9 = 0$$

$$\frac{(N_1 \cdot N_4+N_1)-N_4}{N_1 \cdot N_4-N_4}-R_{10} = 0$$

$$\frac{N_1^2}{N_1-1}-R_{11} = 0$$

$$\frac{(N_1^2+N_1 \cdot N_4)-N_4}{N_1 \cdot N_4-N_4}-R_{12} = 0$$



1CST7 [0, 2, 3, 0]

$N_2 \cdot N_3 - N_2^2 \cdot N_3 - R_0 = 0$

$N_3 - N_2 \cdot N_3 - R_1 = 0$

$\dfrac{N_2 \cdot N_3}{(N_2+N_3) - N_2 \cdot N_3} - R_2 = 0$

$\dfrac{N_2}{(N_2+1) - N_3} - R_3 = 0$

$\dfrac{N_3}{(1+N_3) - N_2 \cdot N_3} - R_4 = 0$

$\dfrac{2 \cdot N_2 - N_2 \cdot N_3}{((1-N_3)+2 \cdot N_2) - N_2 \cdot N_3} - R_5 = 0$

$2 \cdot N_2 - N_2 \cdot N_3 - R_6 = 0$

$\dfrac{N_2}{(1-N_2)+N_2 \cdot N_3} - R_7 = 0$

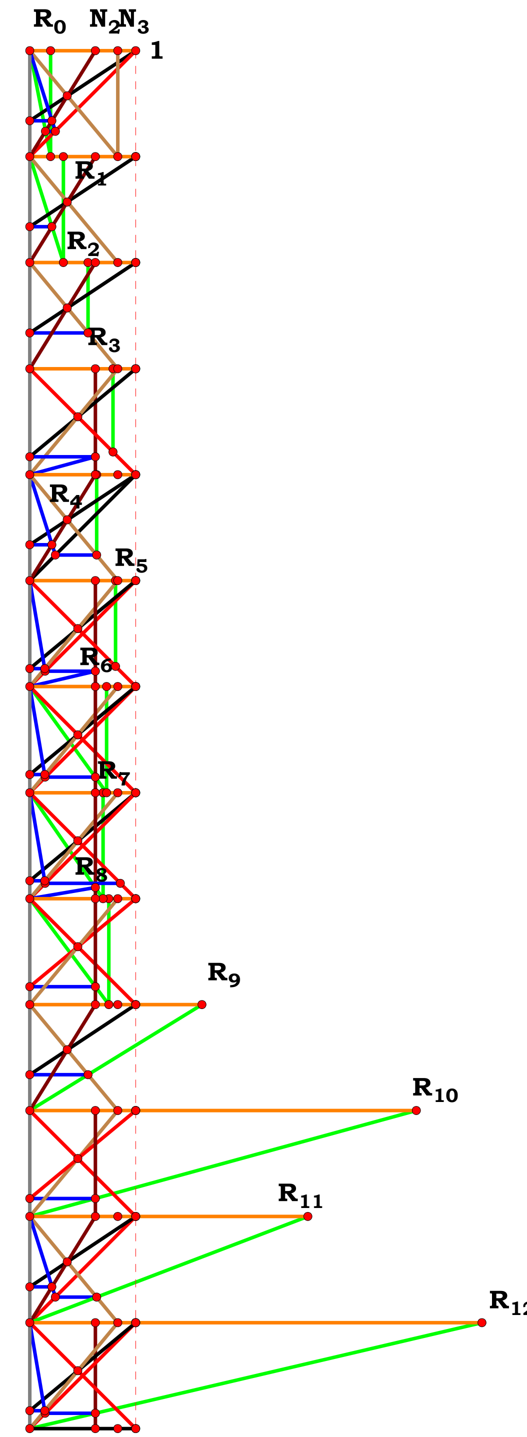
$\dfrac{N_2}{N_3} - R_8 = 0$

$\dfrac{N_2}{1-N_2} - R_9 = 0$

$\dfrac{N_2}{1-N_3} - R_{10} = 0$

$\dfrac{1}{1-N_2} - R_{11} = 0$

$\dfrac{2 \cdot N_2 - N_2 \cdot N_3}{1-N_3} - R_{12} = 0$



1CST7 [0, 2, 0, 4]

$N_2 - N_2^2 - R_0 = 0$

$1 - N_2 - R_1 = 0$

$N_2 - R_2 = 0$

$N_4 - R_3 = 0$

$\frac{1}{2 - N_2} - R_4 = 0$

$N_4 - R_5 = 0$

$N_2 - R_6 = 0$

$N_2 - R_7 = 0$

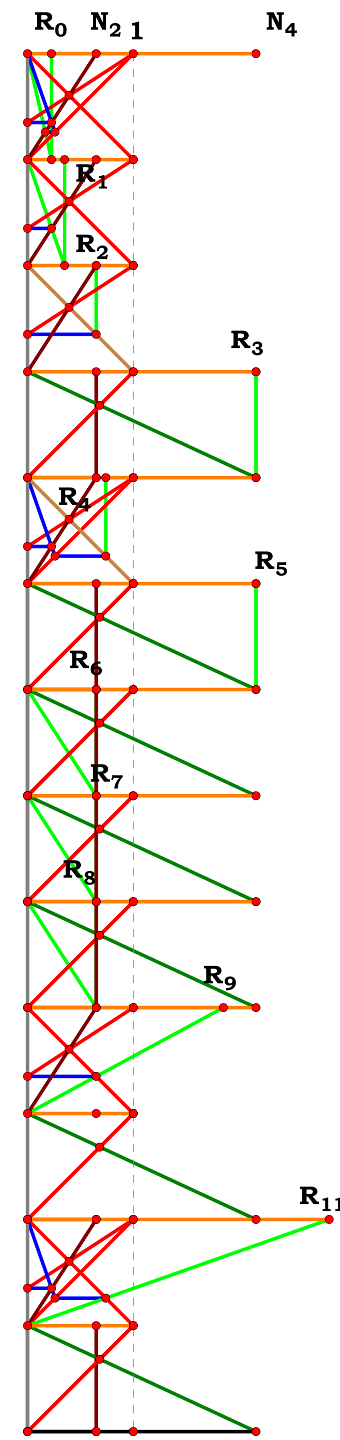
$N_2 - R_8 = 0$

$\frac{N_2}{1 - N_2} - R_9 = 0$

undefined

$\frac{1}{1 - N_2} - R_{11} = 0$

undefined



1CST7 [0, 0, 3, 4]

$$\frac{0}{1}-R_0 = 0$$

$$0-R_1 = 0$$

$$N_3-R_2 = 0$$

$$\frac{(N_4^2+N_3\cdot N_4)-N_3\cdot N_4^2}{(((N_4+N_3)-N_3\cdot N_4)+N_4^2)-N_3\cdot N_4^2}-R_3 = 0$$

$$N_3-R_4 = 0$$

$$\frac{(N_4+N_4^2)-N_3\cdot N_4^2}{((N_4^2-N_3\cdot N_4^2)+1+N_4)-N_3\cdot N_4}-R_5 = 0$$

$$(1+N_4)-N_3\cdot N_4-R_6 = 0$$

$$\frac{1}{N_3}-R_7 = 0$$

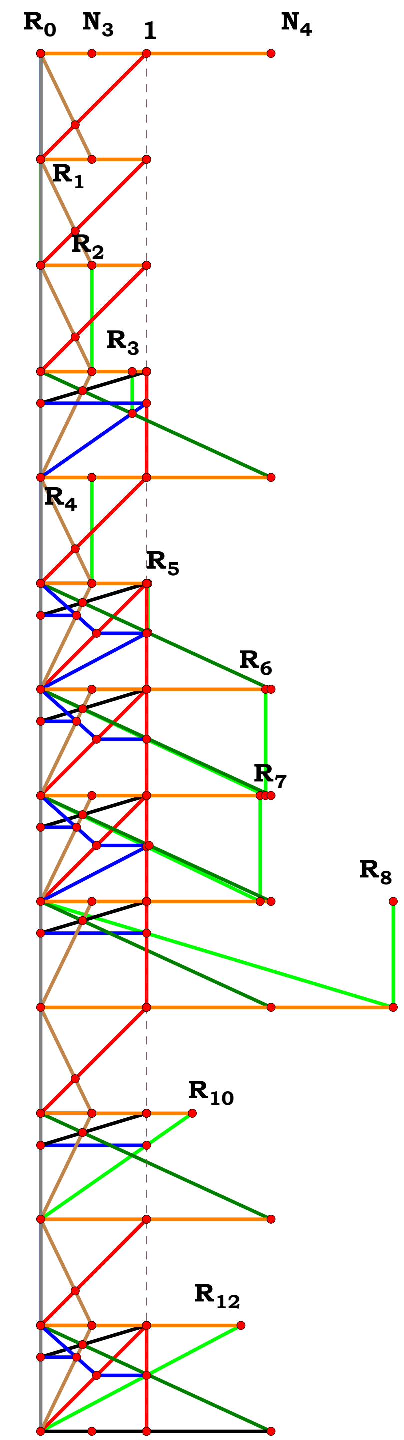
$$\frac{(N_4+N_3)-N_3\cdot N_4}{N_3}-R_8 = 0$$

undefined

$$\frac{(N_4+N_3)-N_3\cdot N_4}{N_4-N_3\cdot N_4}-R_{10} = 0$$

undefined

$$\frac{(1+N_4)-N_3\cdot N_4}{N_4-N_3\cdot N_4}-R_{12} = 0$$



1CST7 [1, 2, 3, 0]

$$\frac{N_1 \cdot N_2 \cdot N_3 - N_2{}^2 \cdot N_3}{N_1{}^2} - R_0 = 0$$

$$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_2 \cdot N_3}{(N_1 \cdot N_2 + N_1 \cdot N_3) - N_2 \cdot N_3} - R_2 = 0$$

$$\frac{(N_1 \cdot N_2 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3}{(((N_1 \cdot N_2 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3) + N_1) - N_3} - R_3 = 0$$

$$\frac{N_1{}^2 \cdot N_3}{(N_1{}^2 + N_1 \cdot N_3) - N_2 \cdot N_3} - R_4 = 0$$

$$\frac{(N_1{}^2 \cdot N_2 + N_1 \cdot N_2) - N_2 \cdot N_3}{((N_1 \cdot N_3) + N_1{}^2 \cdot N_2 + N_1 \cdot N_2) - N_2 \cdot N_3} - R_5 = 0$$

$$\frac{(N_1{}^2 \cdot N_2 + N_1 \cdot N_2) - N_2 \cdot N_3}{N_1{}^2} - R_6 = 0$$

$$\frac{N_1{}^2 \cdot N_2}{(N_1{}^2 - N_1 \cdot N_2) + N_2 \cdot N_3} - R_7 = 0$$

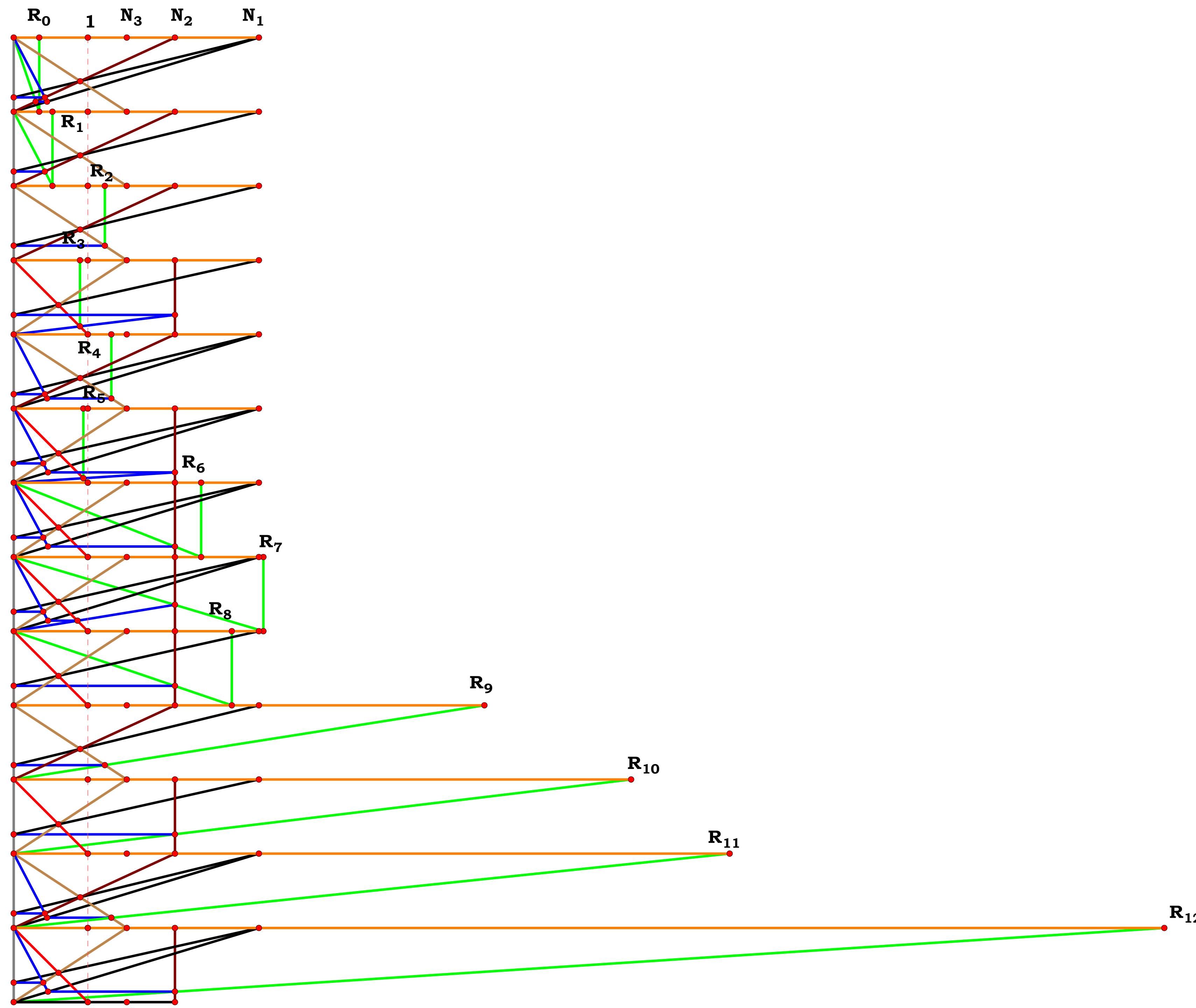
$$\frac{(N_1 \cdot N_2 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3}{N_1 \cdot N_3} - R_8 = 0$$

$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_9 = 0$$

$$\frac{(N_1 \cdot N_2 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3}{N_1 - N_3} - R_{10} = 0$$

$$\frac{N_1{}^2}{N_1 - N_2} - R_{11} = 0$$

$$\frac{(N_1{}^2 \cdot N_2 + N_1 \cdot N_2) - N_2 \cdot N_3}{N_1 - N_3} - R_{12} = 0$$



1CST7 [1, 0, 3, 4]

$$\frac{N_1 \cdot N_3 - N_3}{N_1^2} - R_0 = 0$$

$$\frac{N_1 \cdot N_3 - N_3}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_3}{N_1 + N_1 \cdot N_3 \cdot N_3} - R_2 = 0$$

$$\frac{(N_1 \cdot N_4^2 + N_1 \cdot N_3 \cdot N_4) - N_3 \cdot N_4^2}{(((N_1 \cdot N_4 + N_1 \cdot N_3) - N_3 \cdot N_4) + N_1 \cdot N_4^2) - N_3 \cdot N_4^2} - R_3 = 0$$

$$\frac{N_1^2 \cdot N_3}{(N_1^2 + N_1 \cdot N_3) - N_3} - R_4 = 0$$

$$\frac{(N_1^2 \cdot N_4 + N_1 \cdot N_4^2) - N_3 \cdot N_4^2}{((N_1 \cdot N_4^2 - N_3 \cdot N_4^2) + N_1^2 + N_1 \cdot N_4) - N_3 \cdot N_4} - R_5 = 0$$

$$\frac{(N_1^2 + N_1 \cdot N_4) - N_3 \cdot N_4}{N_1^2} - R_6 = 0$$

$$\frac{N_1^2}{(N_1^2 - N_1) + N_3} - R_7 = 0$$

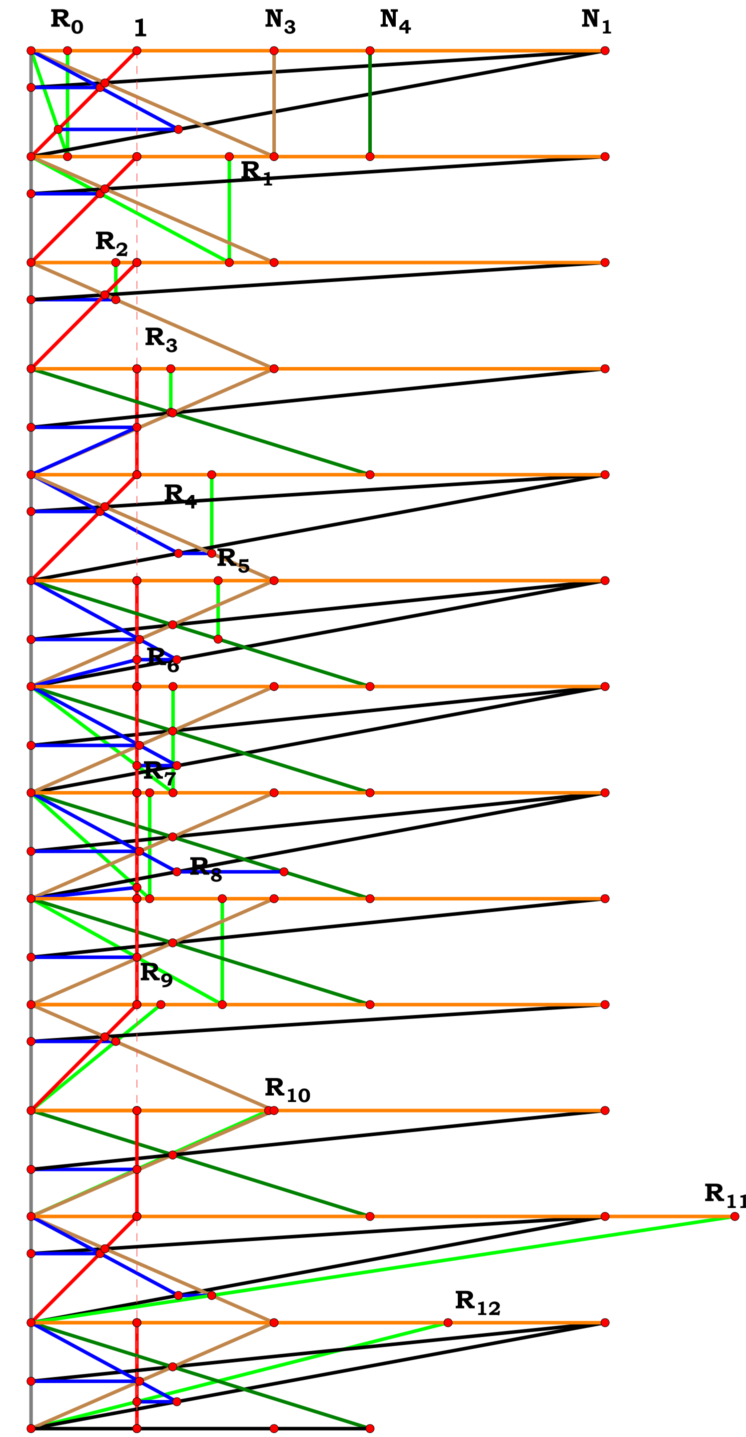
$$\frac{(N_1 \cdot N_4 + N_1 \cdot N_3) - N_3 \cdot N_4}{N_1 \cdot N_3} - R_8 = 0$$

$$\frac{N_1}{N_1 - 1} - R_9 = 0$$

$$\frac{(N_1 \cdot N_4 + N_1 \cdot N_3) - N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} - R_{10} = 0$$

$$\frac{N_1^2}{N_1 - 1} - R_{11} = 0$$

$$\frac{(N_1^2 + N_1 \cdot N_4) - N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} - R_{12} = 0$$



1CST7 [0, 2, 3, 4]

$$N_2\cdot N_3-N_2^2\cdot N_3-R_0 = 0$$

$$N_3-N_2\cdot N_3-R_1 = 0$$

$$\frac{N_2\cdot N_3}{(N_2+N_3)-N_2\cdot N_3}-R_2 = 0$$

$$\frac{(N_2\cdot N_4^2+N_2\cdot N_3\cdot N_4)-N_2\cdot N_3\cdot N_4^2}{(((N_2\cdot N_4+N_2\cdot N_3)-N_2\cdot N_3\cdot N_4)+N_4^2)-N_3\cdot N_4^2}-R_3 = 0$$

$$\frac{N_3}{(1+N_3)-N_2\cdot N_3}-R_4 = 0$$

$$\frac{(N_2\cdot N_4+N_2\cdot N_4^2)-N_2\cdot N_3\cdot N_4^2}{((N_4^2-N_3\cdot N_4^2)+N_2+N_2\cdot N_4)-N_2\cdot N_3\cdot N_4}-R_5 = 0$$

$$(N_2+N_2\cdot N_4)-N_2\cdot N_3\cdot N_4-R_6 = 0$$

$$\frac{N_2}{(1-N_2)+N_2\cdot N_3}-R_7 = 0$$

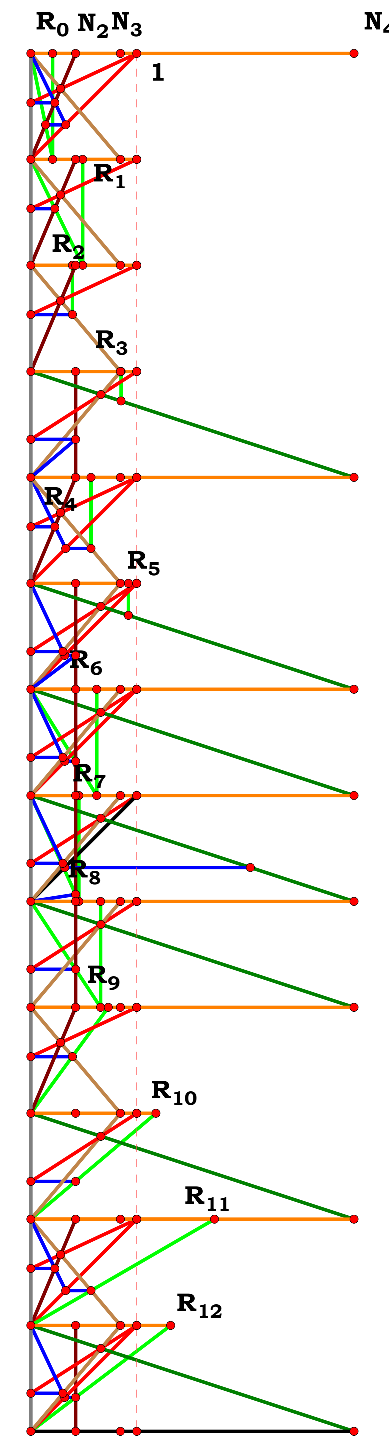
$$\frac{(N_2\cdot N_4+N_2\cdot N_3)-N_2\cdot N_3\cdot N_4}{N_3}-R_8 = 0$$

$$\frac{N_2}{1-N_2}-R_9 = 0$$

$$\frac{(N_2\cdot N_4+N_2\cdot N_3)-N_2\cdot N_3\cdot N_4}{N_4-N_3\cdot N_4}-R_{10} = 0$$

$$\frac{1}{1-N_2}-R_{11} = 0$$

$$\frac{(N_2+N_2\cdot N_4)-N_2\cdot N_3\cdot N_4}{N_4-N_3\cdot N_4}-R_{12} = 0$$



1CST7 [1, 2, 0, 4]

$$\frac{N_1 \cdot N_2 - N_2^2}{N_1^2} - R_0 = 0$$

$$\frac{N_1 - N_2}{N_1} - R_1 = 0$$

$$\frac{N_1 \cdot N_2}{(N_1 \cdot N_2 + N_1) - N_2} - R_2 = 0$$

$$\frac{(N_1 \cdot N_2 \cdot N_4^2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_4^2}{(((N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2) - N_2 \cdot N_4) + N_1 \cdot N_4^2) - N_4^2} - R_3 = 0$$

$$\frac{N_1^2}{(N_1^2 + N_1) - N_2} - R_4 = 0$$

$$\frac{(N_1^2 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4^2) - N_2 \cdot N_4^2}{((N_1 \cdot N_4^2 - N_4^2) + N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_4} - R_5 = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_4}{N_1^2} - R_6 = 0$$

$$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_2) + N_2} - R_7 = 0$$

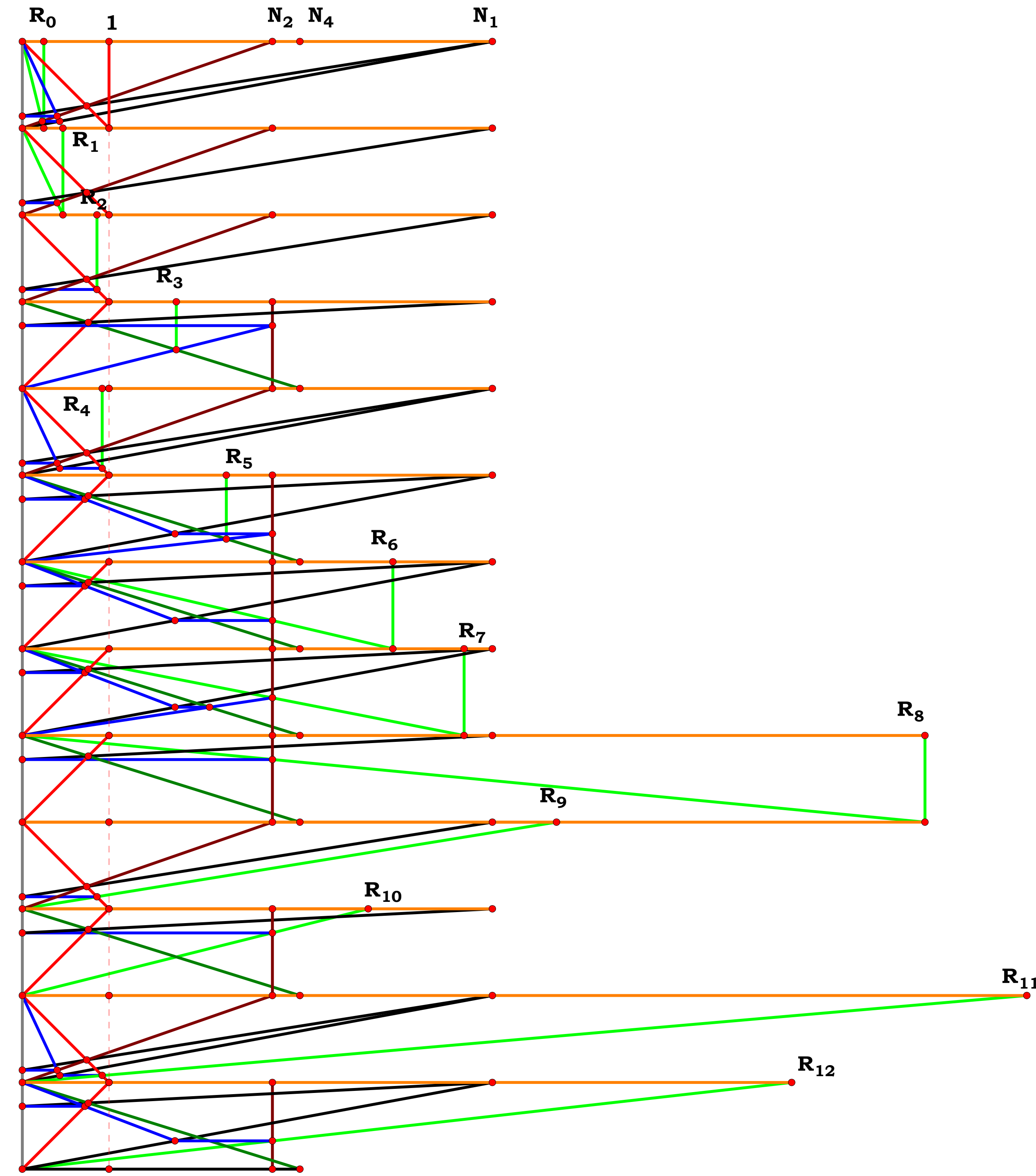
$$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_3} - R_8 = 0$$

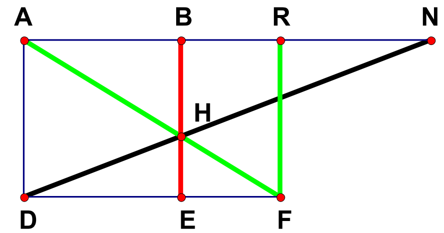
$$\frac{N_1 \cdot N_2}{N_1 - N_2} - R_9 = 0$$

$$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2) - N_2 \cdot N_4}{N_1 \cdot N_4 - N_4} - R_{10} = 0$$

$$\frac{N_1^2}{N_1 - N_2} - R_{11} = 0$$

$$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_4}{N_1 \cdot N_4 - N_4} - R_{12} = 0$$





1CST1R0

Unit.

$AB := 1$

Given.

$AN := -3$

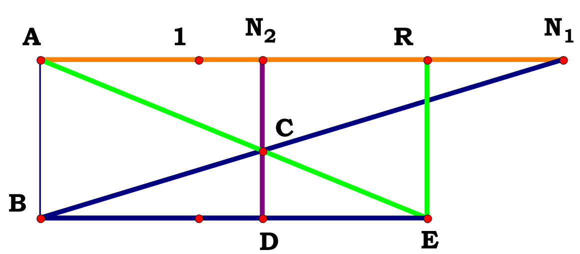
Descriptions.

$AD := AB \quad DE := AB \quad EH := \dfrac{AD^2}{AN} \quad EF := \dfrac{DE \cdot EH}{AD - EH}$

$DF := DE + EF \quad AR := DF$

Definitions.

$EH - \dfrac{1}{AN} = 0 \quad EF - \dfrac{1}{AN - 1} = 0 \quad DF - \dfrac{AN}{AN - 1} = 0 \quad \dfrac{AN}{AN - 1} - AR = 0$



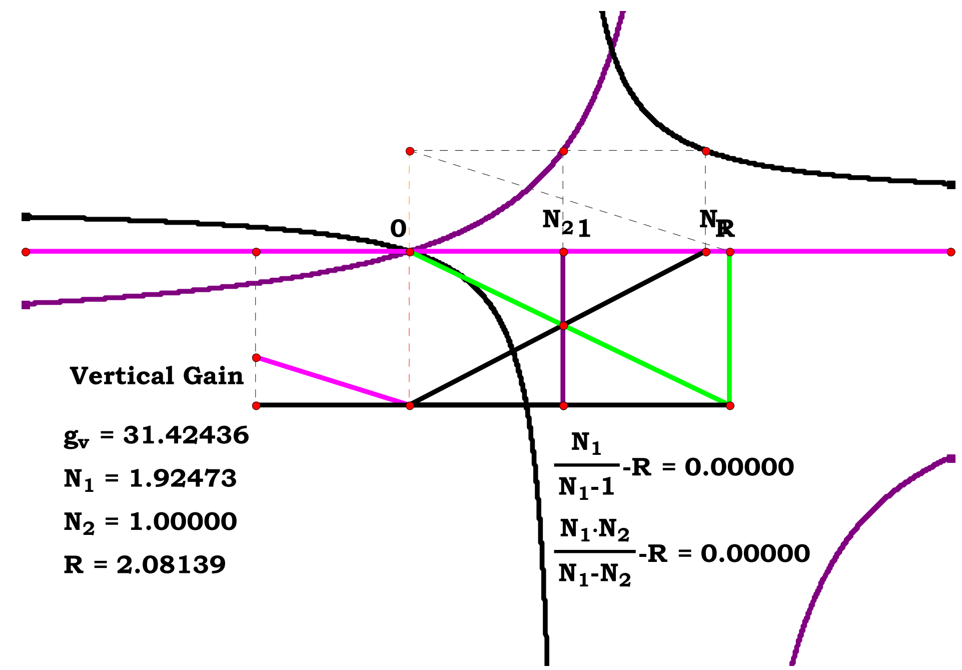
$N_1 := 3$

$N_2 := 2$

$ab := 1 \quad cd := \dfrac{ab \cdot N_2}{N_1} \quad de := \dfrac{N_2 \cdot cd}{ab - cd} \quad be := N_2 + de \quad ar := be$

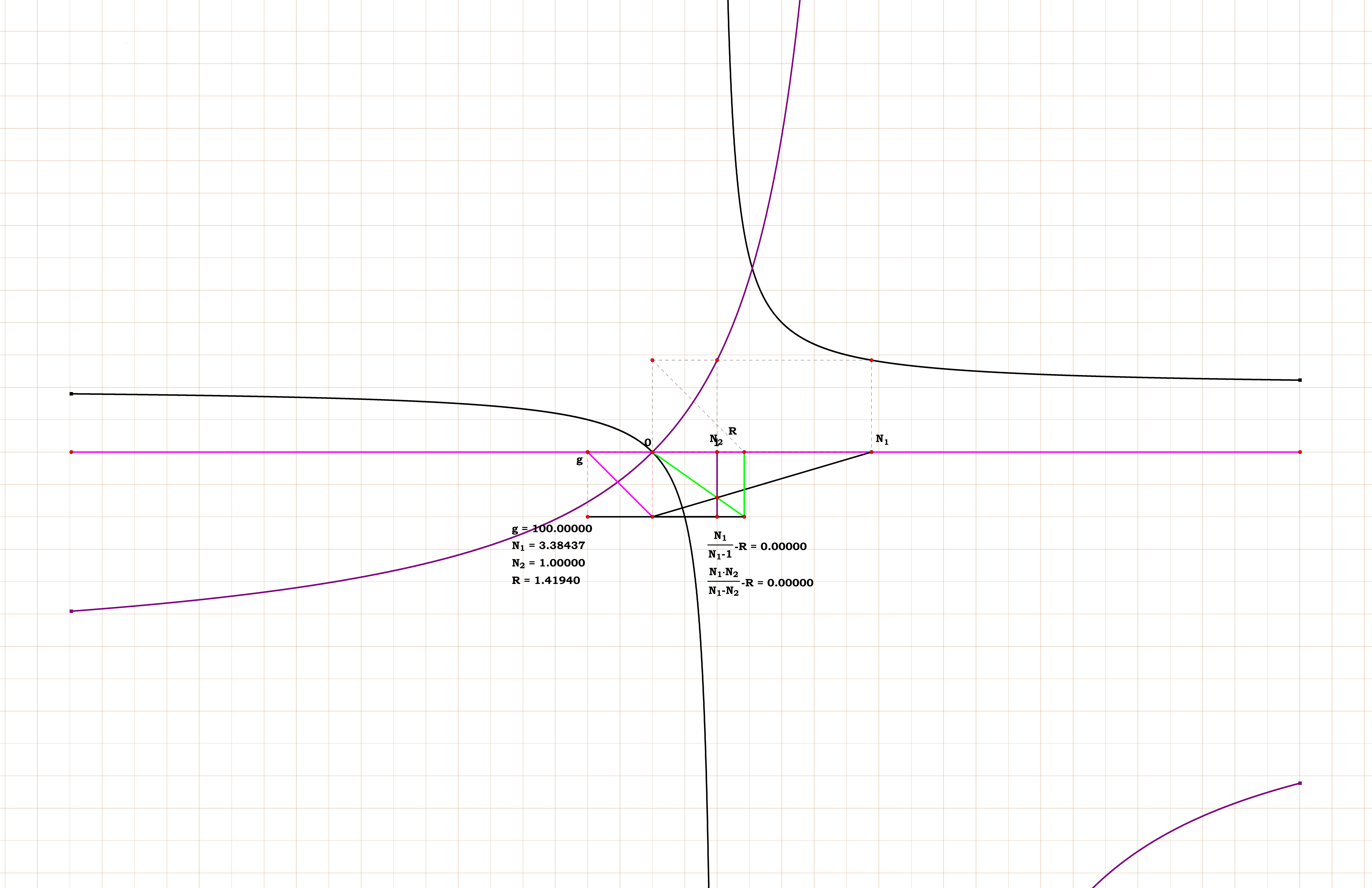
Definitions.

$cd - \dfrac{N_2}{N_1} = 0 \qquad de - \dfrac{N_2^{\,2}}{N_1 - N_2} = 0 \qquad ar - \dfrac{N_1 \cdot N_2}{N_1 - N_2} = 0$

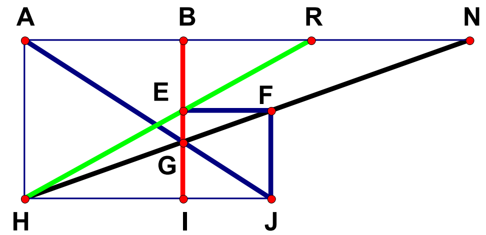


Two Transforms.

0, 0.	0
1, 0.	$\frac{N_1}{N_1 - 1}$
0, 2.	$-\frac{N_2}{N_2 - 1}$
1, 2.	$\frac{N_1 \cdot N_2}{N_1 - N_2}$



0
g
N_2
R
N_1
g = 100.00000
N_1 = 3.38437
N_2 = 1.00000
R = 1.41940
$\frac{N_1}{N_1-1}$-R = 0.00000
$\frac{N_1 \cdot N_2}{N_1-N_2}$-R = 0.00000



1 CST1R1

Unit.

$AB := 1$

Given.

$AN := 3$

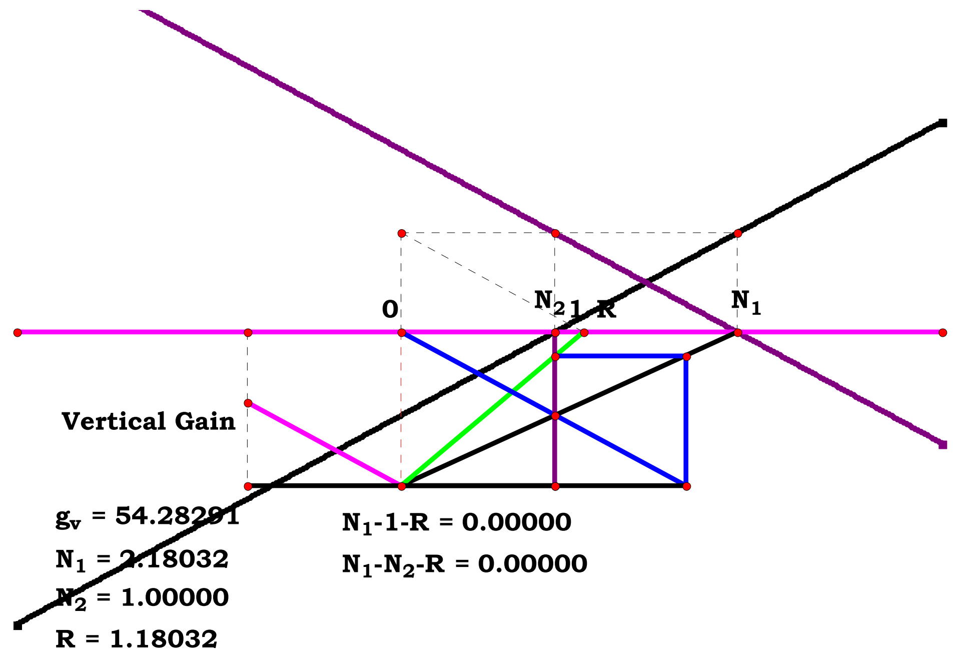
Descriptions.

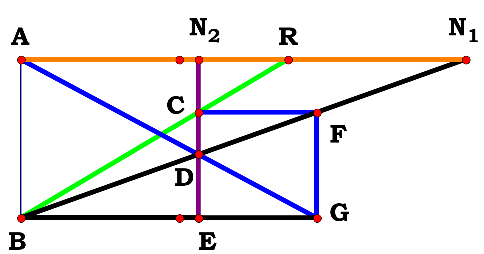
$AH := AB \quad HI := AB \quad HJ := \dfrac{AN}{AN - 1}$

$FJ := \dfrac{AH \cdot HJ}{AN} \quad EI := FJ \quad AR := \dfrac{HI^2}{EI}$

Definitions.

$(AN - 1) - AR = 0$





$N_1 := 3$

$N_2 := 2$

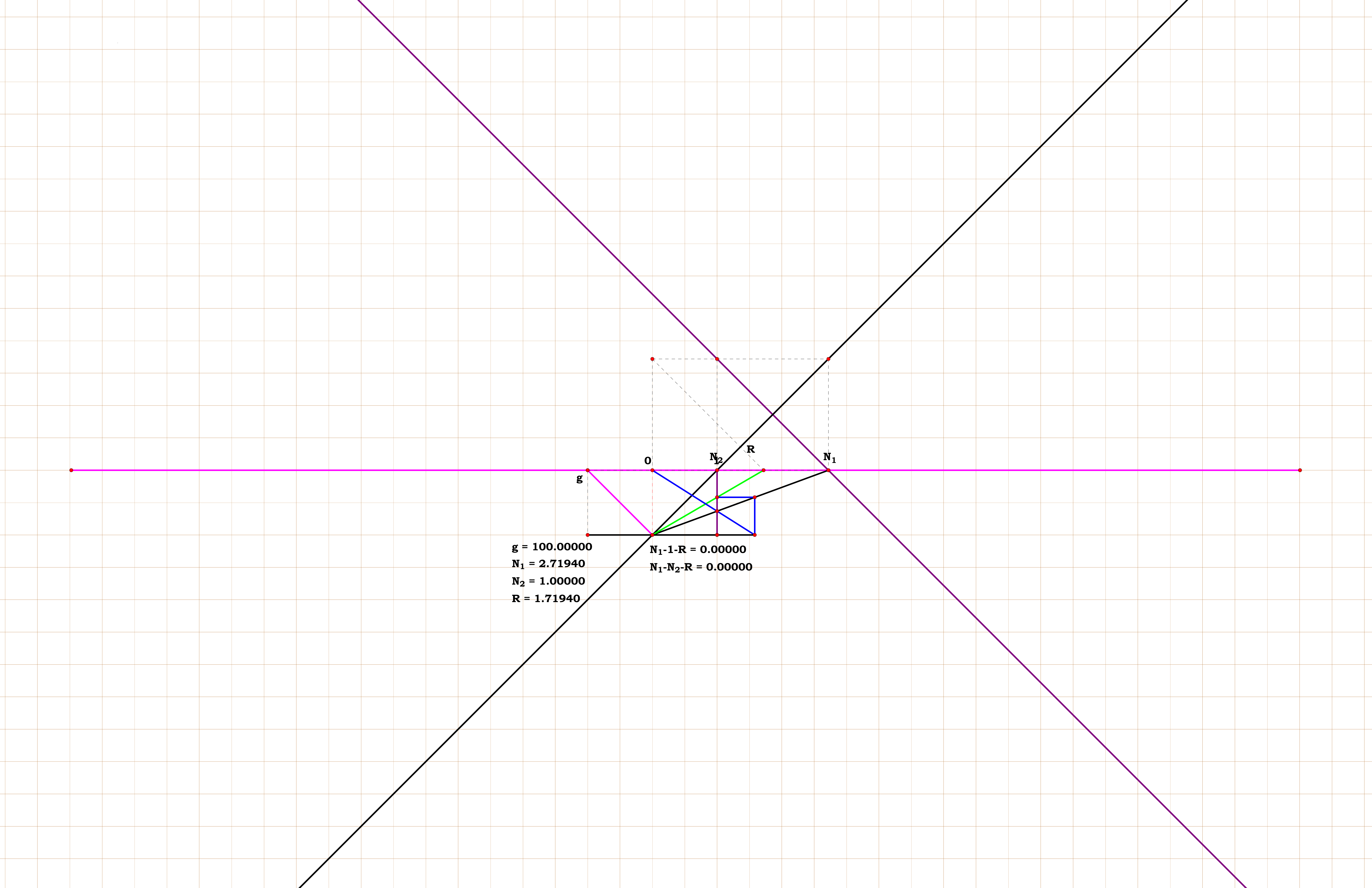
$ab := 1 \quad be := N_2 \quad bg := \dfrac{N_1 \cdot N_2}{N_1 - N_2} \quad fg := \dfrac{ab \cdot bg}{N_1} \quad ce := fg \quad ar := \dfrac{ab \cdot N_2}{ce}$

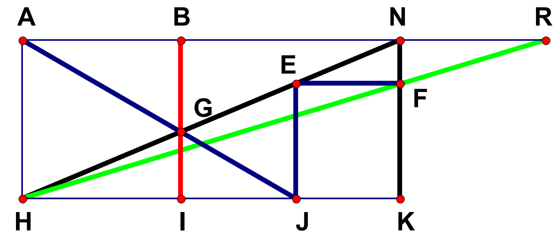
Definitions.

$fg - \dfrac{N_2}{N_1 - N_2} = 0 \qquad ar - \left(N_1 - N_2\right) = 0$

Two Transforms.

0, 0.	0
1, 0.	$N_1 - 1$
0, 2.	$1 - N_2$
1, 2.	$N_1 - N_2$


0
N_2
1
R
N_1
g
g = 100.00000
N_1 = 2.71940
N_2 = 1.00000
R = 1.71940
N_1-1-R = 0.00000
N_1-N_2-R = 0.00000



1CST1R2

Unit.

$AB := 1$

Given.

$AN := 3$

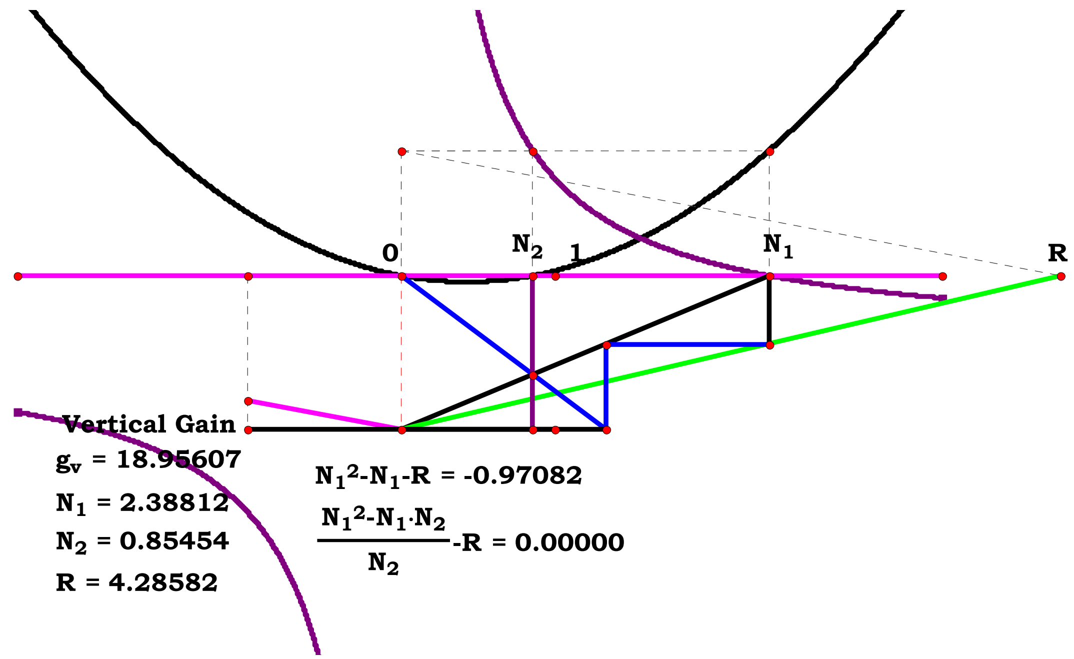
Descriptions.

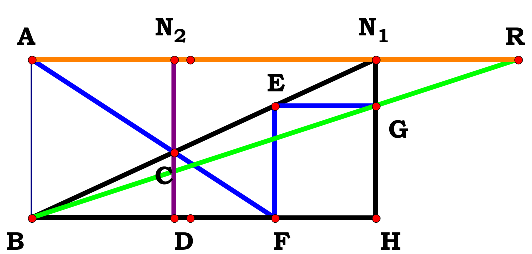
$AH := AB \quad HI := AB \quad HJ := \dfrac{AN}{AN-1}$

$EJ := \dfrac{AH \cdot HJ}{AN} \quad AR := \dfrac{AN \cdot AB}{EJ}$

Definitions.

$EJ - \dfrac{1}{AN-1} = 0 \quad AN^2 - AN - AR = 0$





$N_1 := 3$

$N_2 := 2$

$ab := 1 \quad bf := \dfrac{N_1 \cdot N_2}{N_1 - N_2} \quad ef := \dfrac{N_2}{N_1 - N_2} \quad ar := \dfrac{N_1}{N_2} \cdot \left(N_1 - N_2\right)$

Definitions.

$ar - \dfrac{N_1^{\,2} - N_1 \cdot N_2}{N_2} = 0$

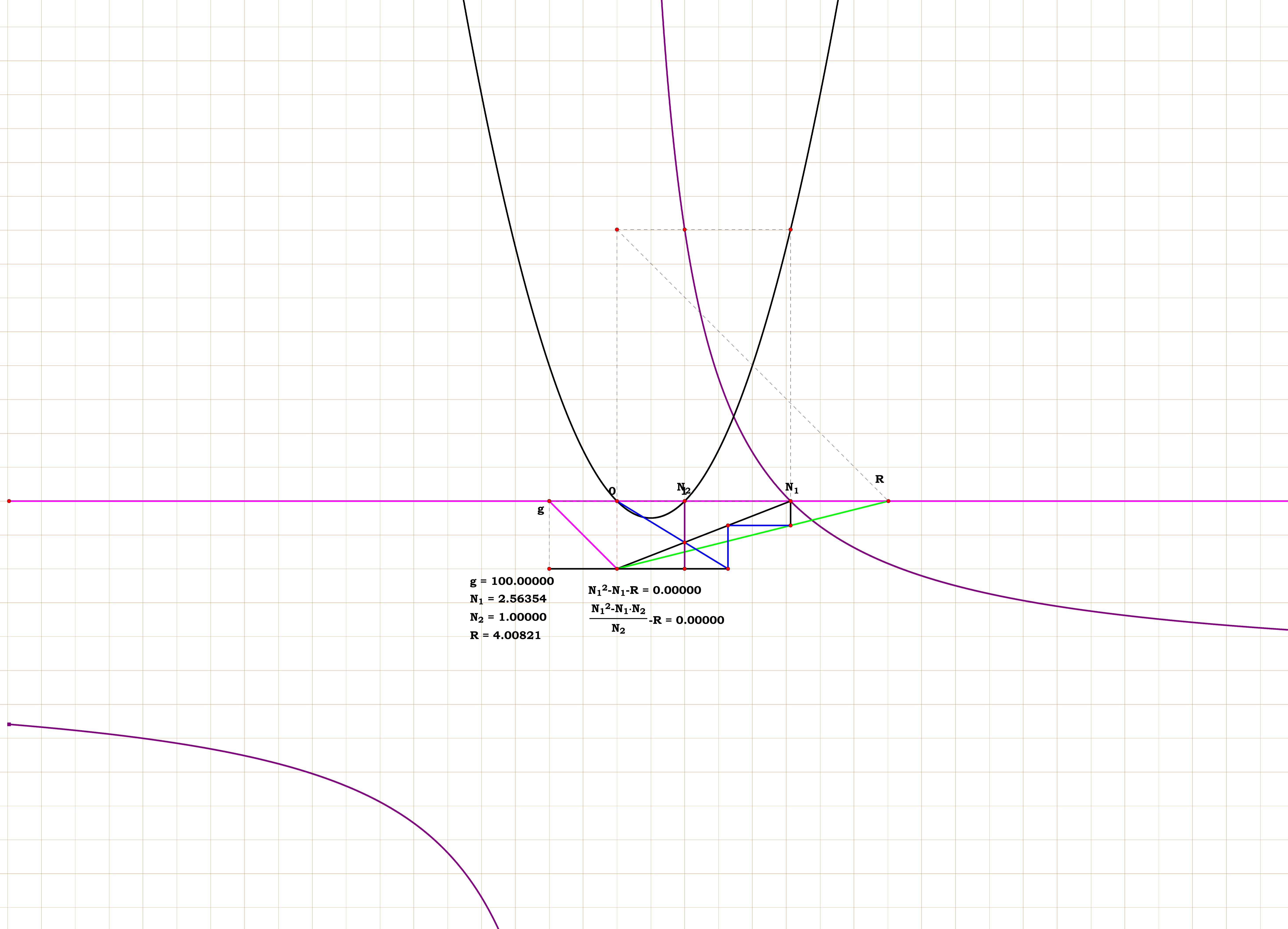
Two Transforms.

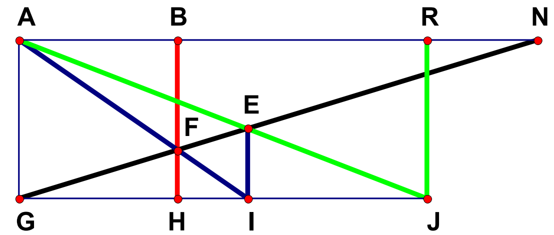
0, 0. 0

1, 0. $N_1 \cdot (N_1 - 1)$

0, 2. $-\frac{N_2 - 1}{N_2}$

1, 2. $\frac{{N_1}^2 - N_1 \cdot N_2}{N_2}$


O
N 1
N 2
N 1
R
g
g = 100.00000
N_1 = 2.56354
N_2 = 1.00000
R = 4.00821
$N_1^2 - N_1 - R = 0.00000$
$\frac{N_1^2 - N_1 \cdot N_2}{N_2} - R = 0.00000$



1CST1R3

Unit.

$AB := 1$

Given.

$AN := 5$

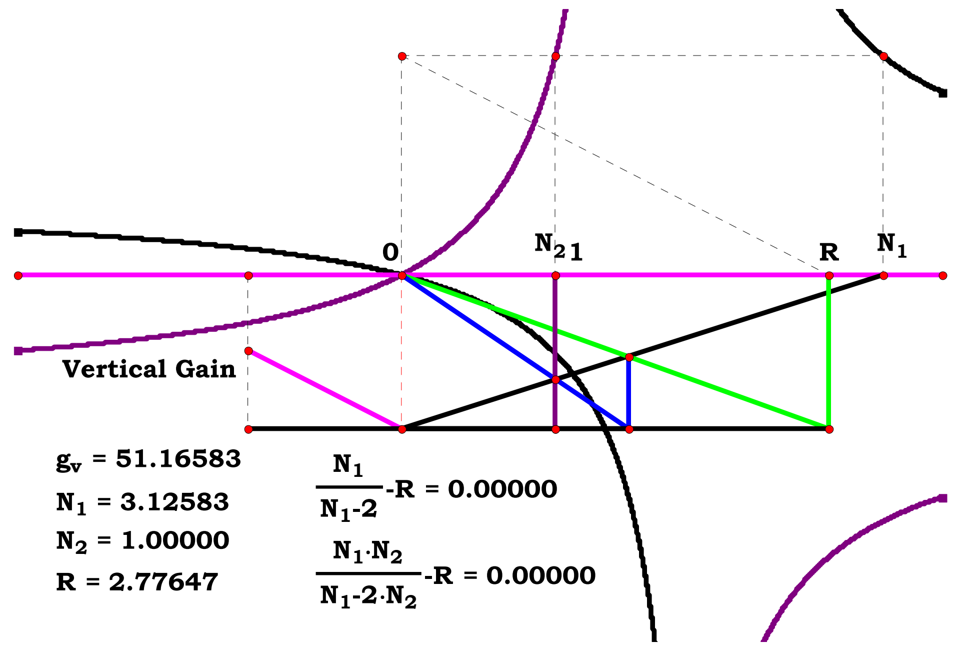
Descriptions.

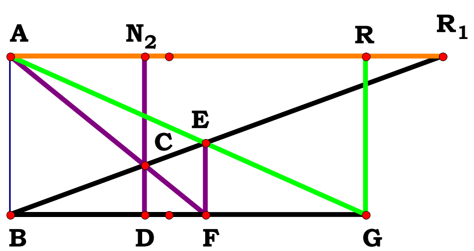
$AG := AB \quad GH := AB \quad EI := \frac{1}{AN-1}$

$GI := \frac{AN}{AN-1} \quad GJ := \frac{GI \cdot AG}{AG-EI} \quad AR := GJ$

Definitions.

$AR - \frac{AN}{AN-2} = 0$





$N_1 := 4$

$N_2 := 3$

$ab := 1 \quad bd := N_2 \quad ef := \frac{N_2}{N_1 - N_2} \quad bf := \frac{N_1 \cdot N_2}{N_1 - N_2} \quad bg := \frac{bf \cdot ab}{ab - ef} \quad ar := bg$

Definitions.

$ar - \frac{N_1 \cdot N_2}{N_1 - 2N_2} = 0$

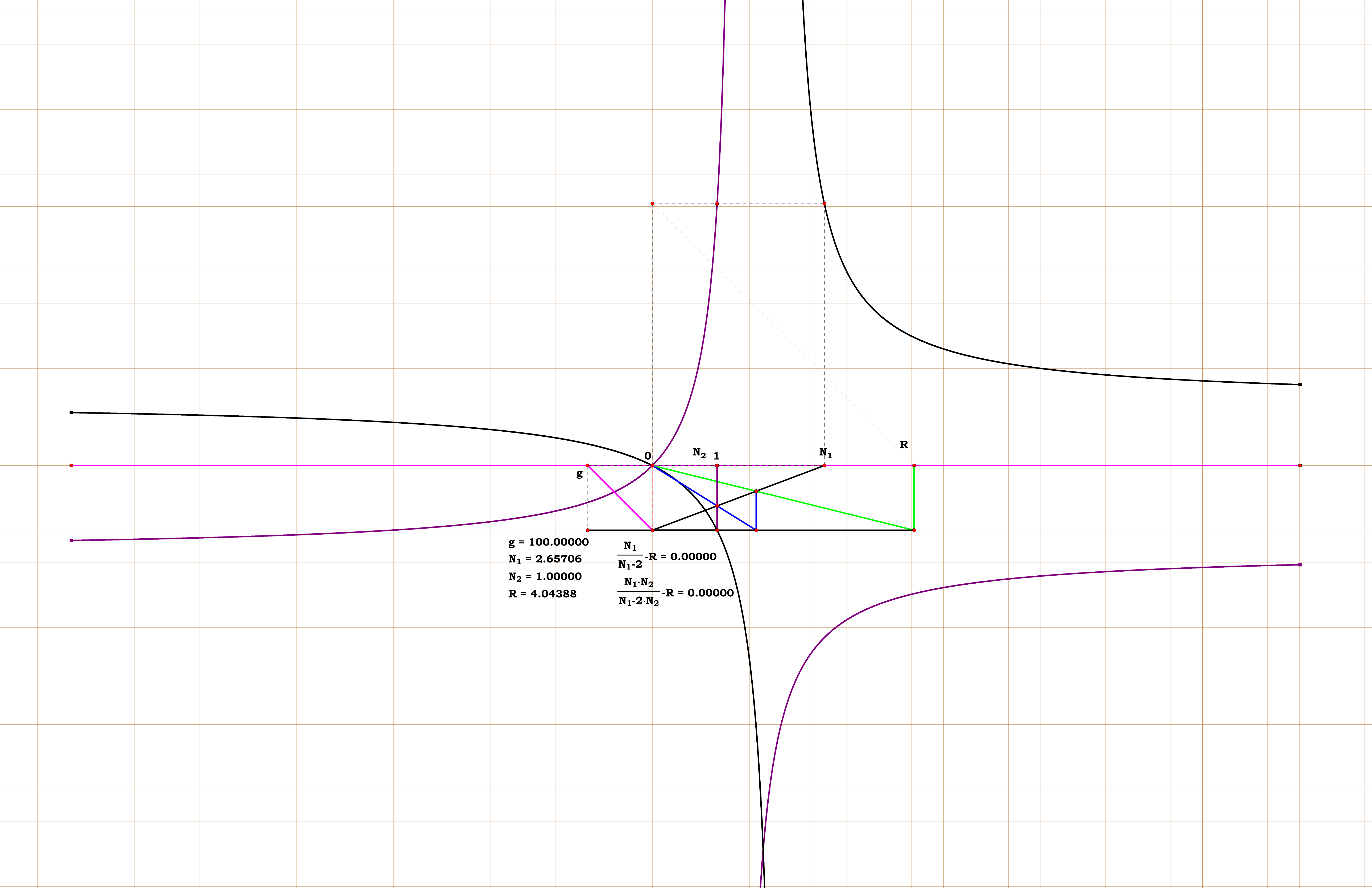
Two Transforms.

0, 0. -1

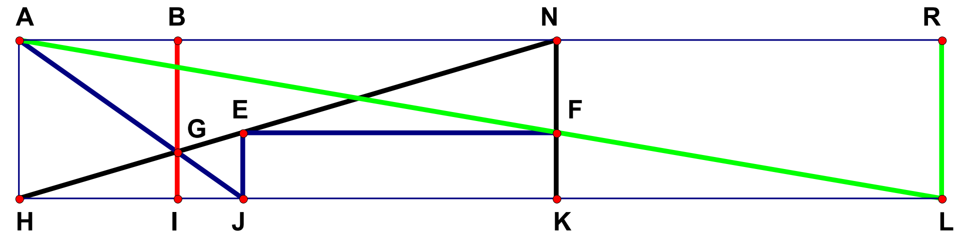
1, 0. $\dfrac{N_1}{N_1 - 2}$

0, 2. $-\dfrac{N_2}{2 \cdot N_2 - 1}$

1, 2. $\dfrac{N_1 \cdot N_2}{N_1 - 2N_2}$



0
N2
1
N1
R
g
g = 100.00000
N1 = 2.65706
N2 = 1.00000
R = 4.04388
N1 / (N1-2) - R = 0.00000
N1·N2 / (N1-2·N2) - R = 0.00000



1CST1R4

Unit. $AB := 1$

Given. $AN := 5$

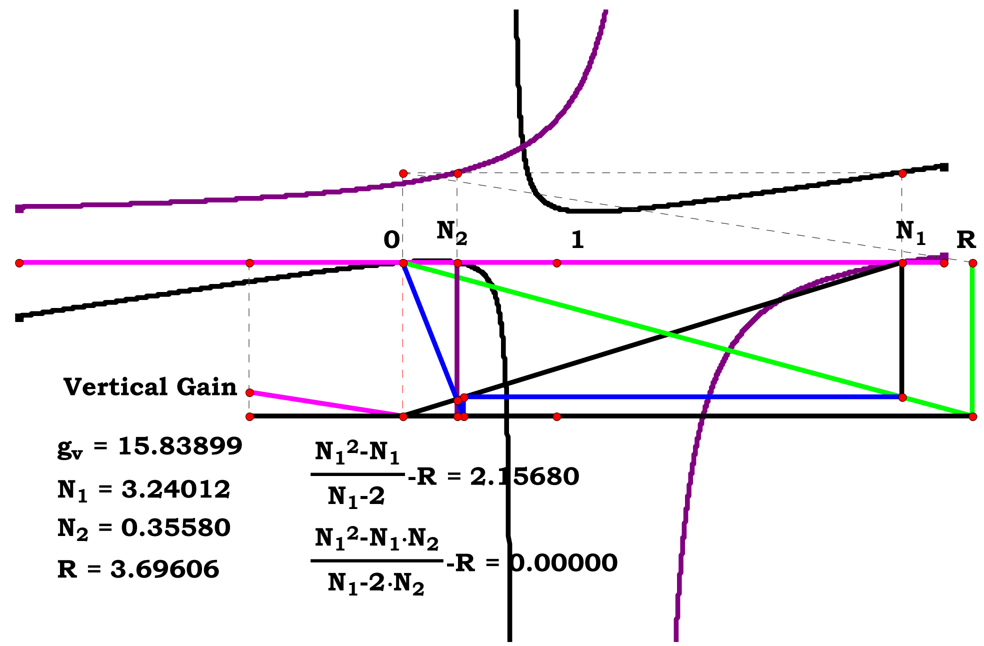
Descriptions.

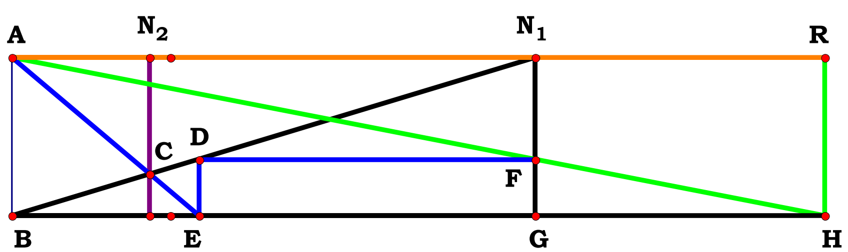
$AH := AB \quad HI := AB$

$EJ := \dfrac{1}{AN - 1} \quad HL := \dfrac{AN \cdot AB}{AB - EJ} \quad AR := HL$

Definitions.

$$AR - \frac{AN^2 - AN}{AN - 2} = 0$$





$N_1 := 5$

$N_2 := 2$

$ab := 1$

$de := \dfrac{N_2}{N_1 - N_2} \quad bh := \dfrac{N_1 \cdot ab}{ab - de} \quad ar := bh$

Definitions.

$$ar - \frac{N_1^{\,2} - N_1 \cdot N_2}{N_1 - 2 \cdot N_2} = 0$$

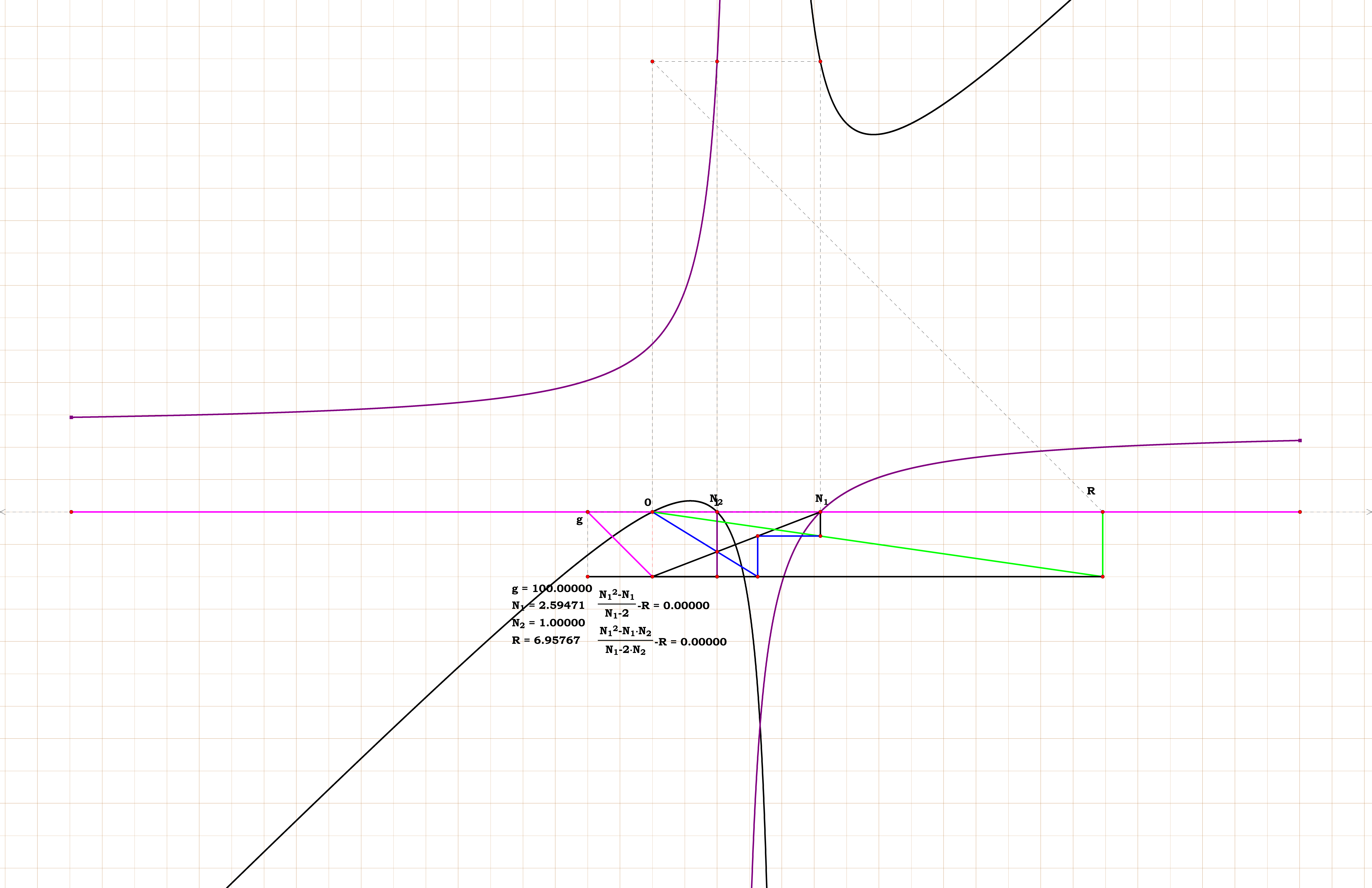
Two Transforms.

0, 0. $\quad 0$

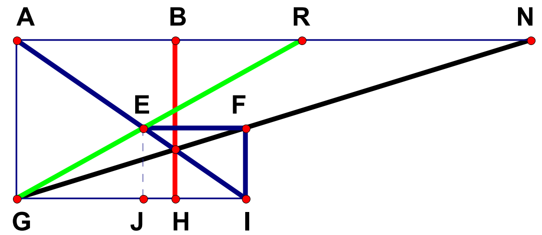
1, 0. $\quad \dfrac{N_1 \cdot (N_1 - 1)}{N_1 - 2}$

0, 2. $\quad \dfrac{N_2 - 1}{2 \cdot N_2 - 1}$

1, 2. $\quad \dfrac{N_1^{\,2} - N_1 \cdot N_2}{N_1 - 2 \cdot N_2}$



0
N_2
N_1
R
g
g = 100.00000
N_1 = 2.59471
N_2 = 1.00000
R = 6.95767
$\frac{N_1^2-N_1}{N_1-2}-R = 0.00000$
$\frac{N_1^2-N_1 \cdot N_2}{N_1-2 \cdot N_2}-R = 0.00000$



1CST1R5

Unit.

$AB := 1$

Given.

$AN := 3$

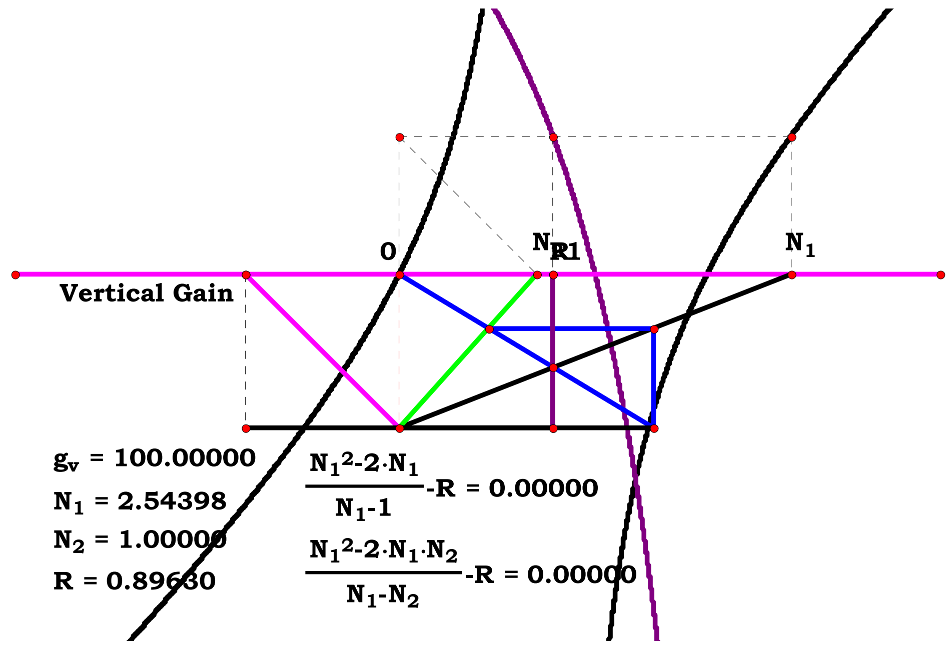
Descriptions.

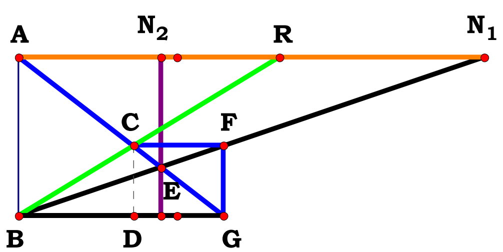
$AG := AB \quad GI := \dfrac{AN}{AN-1} \quad FI := \dfrac{1}{AN-1} \quad IJ := \dfrac{GI \cdot FI}{AG}$

$GJ := GI - IJ \quad EJ := FI \quad AR := \dfrac{GJ \cdot AB}{EJ}$

Definitions.

$$AR - \frac{AN^2 - 2AN}{AN-1} = 0$$





$N_1 := 9$

$N_2 := 2$

$ab := 1$

$bg := \dfrac{N_1 \cdot N_2}{N_1 - N_2} \quad fg := \dfrac{N_2}{N_1 - N_2} \quad dg := \dfrac{bg \cdot fg}{ab} \quad bd := bg - dg \quad cd := fg \quad ar := \dfrac{bd \cdot ab}{cd}$

Definitions.

$$dg - \frac{N_1 \cdot N_2^2}{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2} = 0 \quad bd - \frac{N_1^2 \cdot N_2 - 2 \cdot N_1 \cdot N_2^2}{N_1^2 - 2 \cdot N_1 \cdot N_2 + N_2^2} = 0 \quad ar - \frac{N_1^2 - 2 \cdot N_1 \cdot N_2}{N_1 - N_2} = 0$$

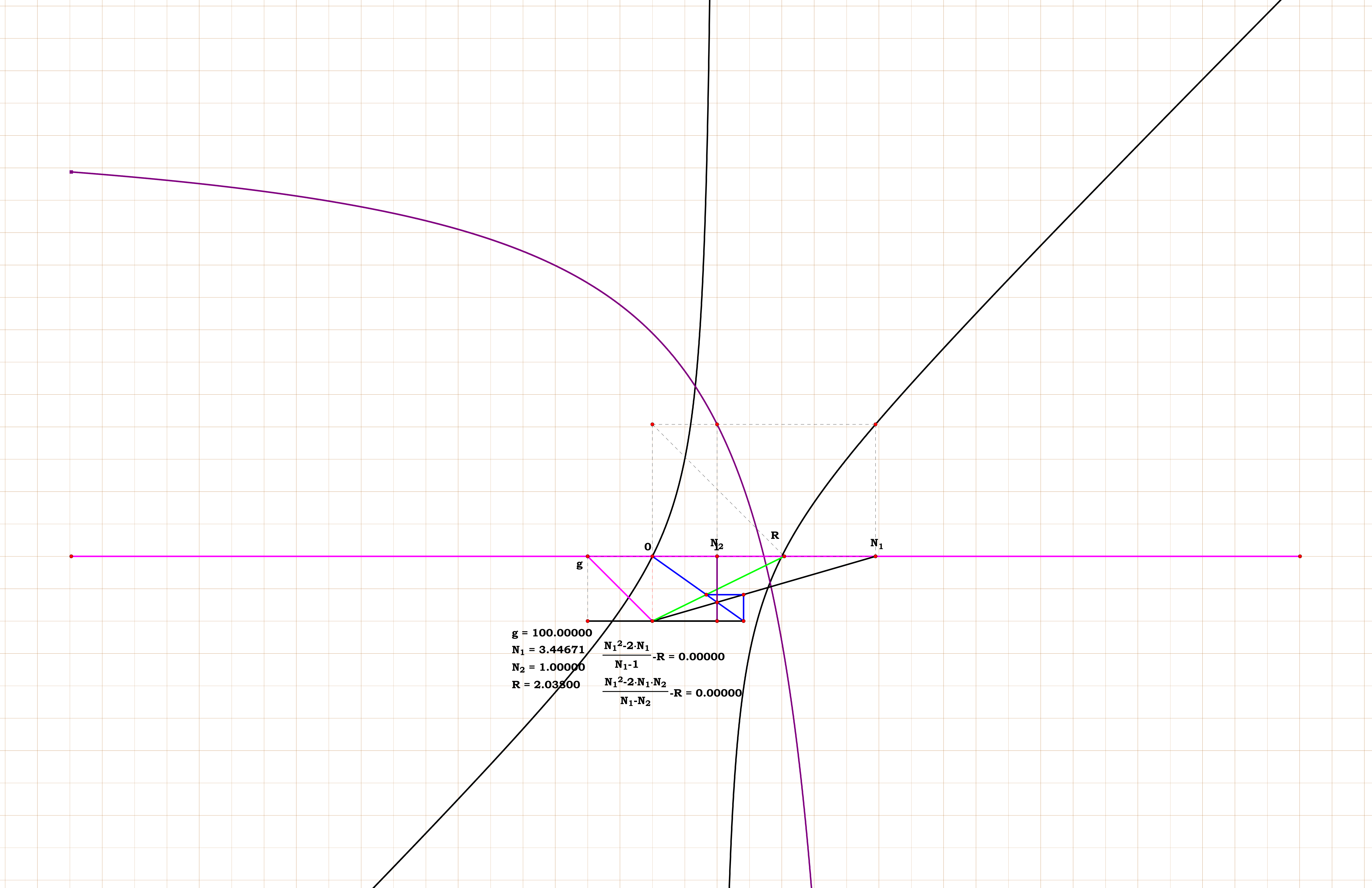
Two Transforms.

0, 0. 0

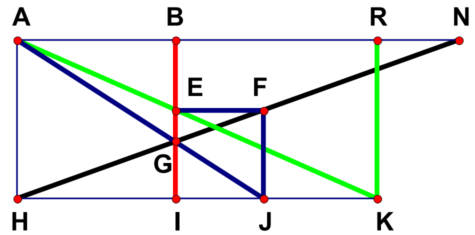
1, 0. $\dfrac{N_1 \cdot (N_1 - 2)}{N_1 - 1}$

0, 2. $\dfrac{2 \cdot N_2 - 1}{N_2 - 1}$

1, 2. $\dfrac{{N_1}^2 - 2 \cdot N_1 \cdot N_2}{N_1 - N_2}$



0
N2
1
R
N1
g
g = 100.00000
N1 = 3.44671
N2 = 1.00000
R = 2.03800
(N1^2-2·N1)/(N1-1)-R = 0.00000
(N1^2-2·N1·N2)/(N1-N2)-R = 0.00000



1CST1R6

Unit.

$AB := 1$

Given.

$AN := 4$

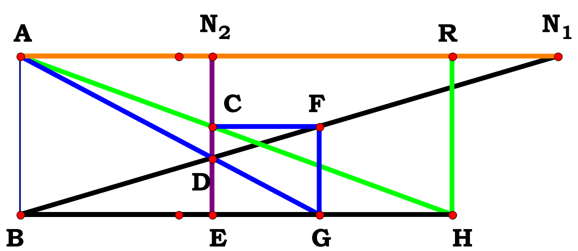
Descriptions.

$AH := AB \quad HI := AB \quad FJ := \dfrac{1}{AN-1} \quad EI := FJ$

$HK := \dfrac{HI^2}{HI-EI} \quad AR := HK$

Definitions.

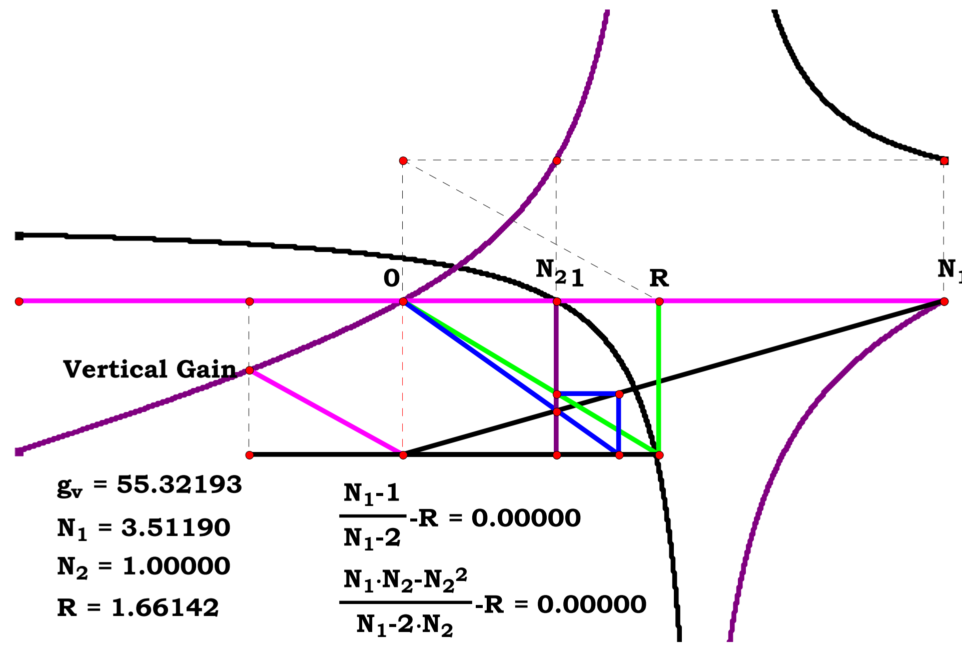
$$AR - \frac{AN-1}{AN-2} = 0$$



$N_1 := 3$

$N_2 := 2$

$ab := 1$



$be := N_2 \quad fg := \dfrac{N_2}{N_1-N_2} \quad ce := fg \quad bh := \dfrac{ab \cdot be}{ab-ce} \quad ar := bh$

Descriptions.

$$ar - \frac{N_2 \cdot N_1 - N_2^{\,2}}{N_1 - 2 \cdot N_2} = 0$$

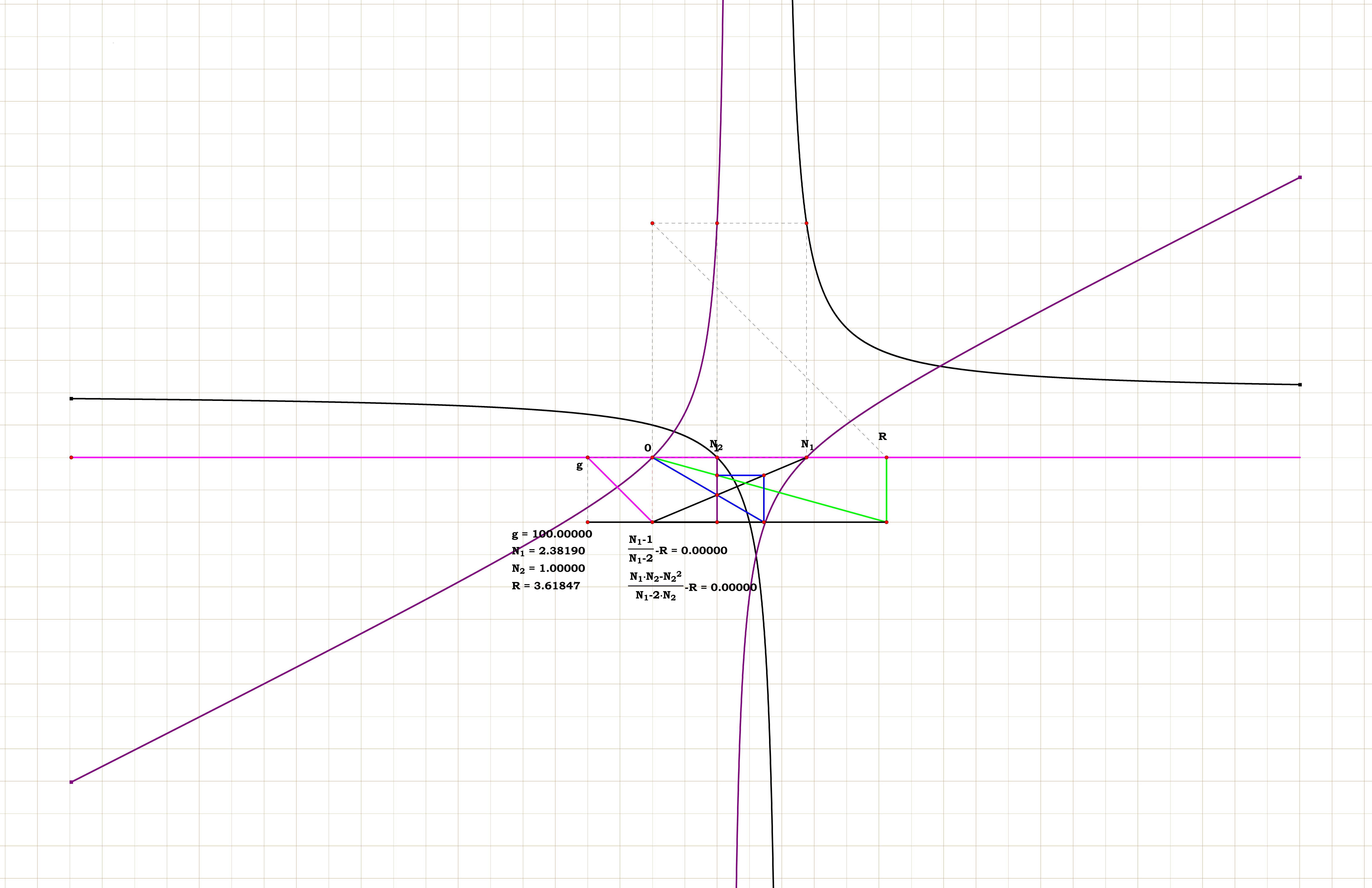
Two Transforms.

0, 0. $\quad 0$

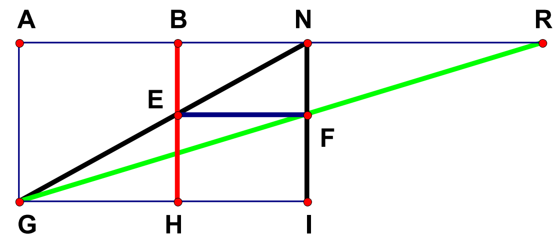
1, 0. $\quad \dfrac{N_1 - 1}{N_1 - 2}$

0, 2. $\quad \dfrac{N_2 \cdot (N_2 - 1)}{2 \cdot N_2 - 1}$

1, 2. $\quad \dfrac{N_2 \cdot N_1 - N_2^{\,2}}{N_1 - 2 \cdot N_2}$



0
N_2
N_1
R
g
$g = 100.00000$
$N_1 = 2.38190$
$N_2 = 1.00000$
$R = 3.61847$
$\frac{N_1-1}{N_1-2}-R = 0.00000$
$\frac{N_1 \cdot N_2 - N_2^2}{N_1 - 2 \cdot N_2}-R = 0.00000$



1CST1R7

Unit.

$AB := 1$

Given.

$AN := 3$

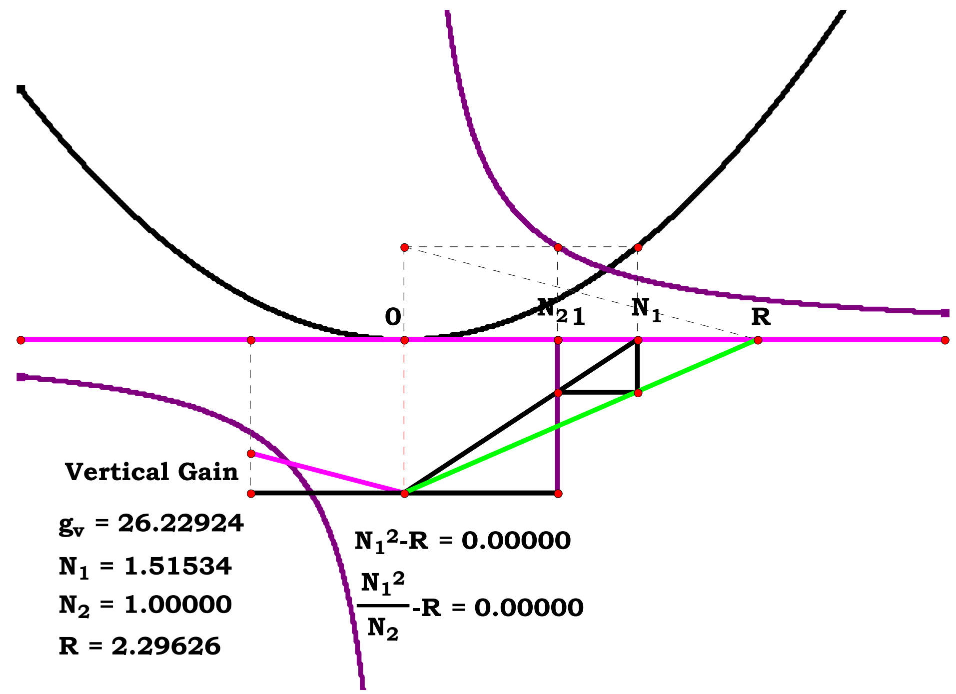
Descriptions.

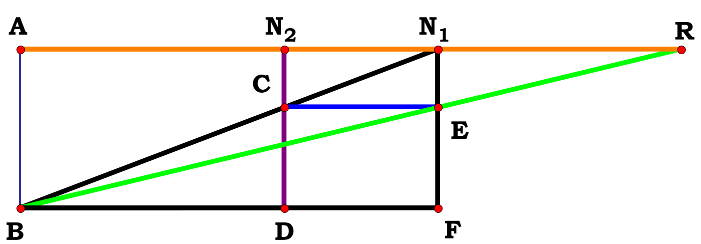
$AG := AB \quad BN := AN - AB \quad BE := \dfrac{AG \cdot BN}{AN}$

$NF := BE \quad FI := AG - NF \quad AR := \dfrac{AN \cdot AG}{FI}$

Definitions.

$BE - \dfrac{AN - 1}{AN} = 0 \quad AR - AN^2 = 0$





$N_1 := 3$

$N_2 := 2$

$ab := 1$

$df := N_1 - N_2 \quad cn := \dfrac{ab \cdot df}{N_1} \quad en := cn \quad ef := ab - cn \quad ar := \dfrac{N_1}{ef}$

Definitions.

$cn - \dfrac{N_1 - N_2}{N_1} = 0 \quad ef - \dfrac{N_2}{N_1} = 0 \quad ar - \dfrac{{N_1}^2}{N_2} = 0$

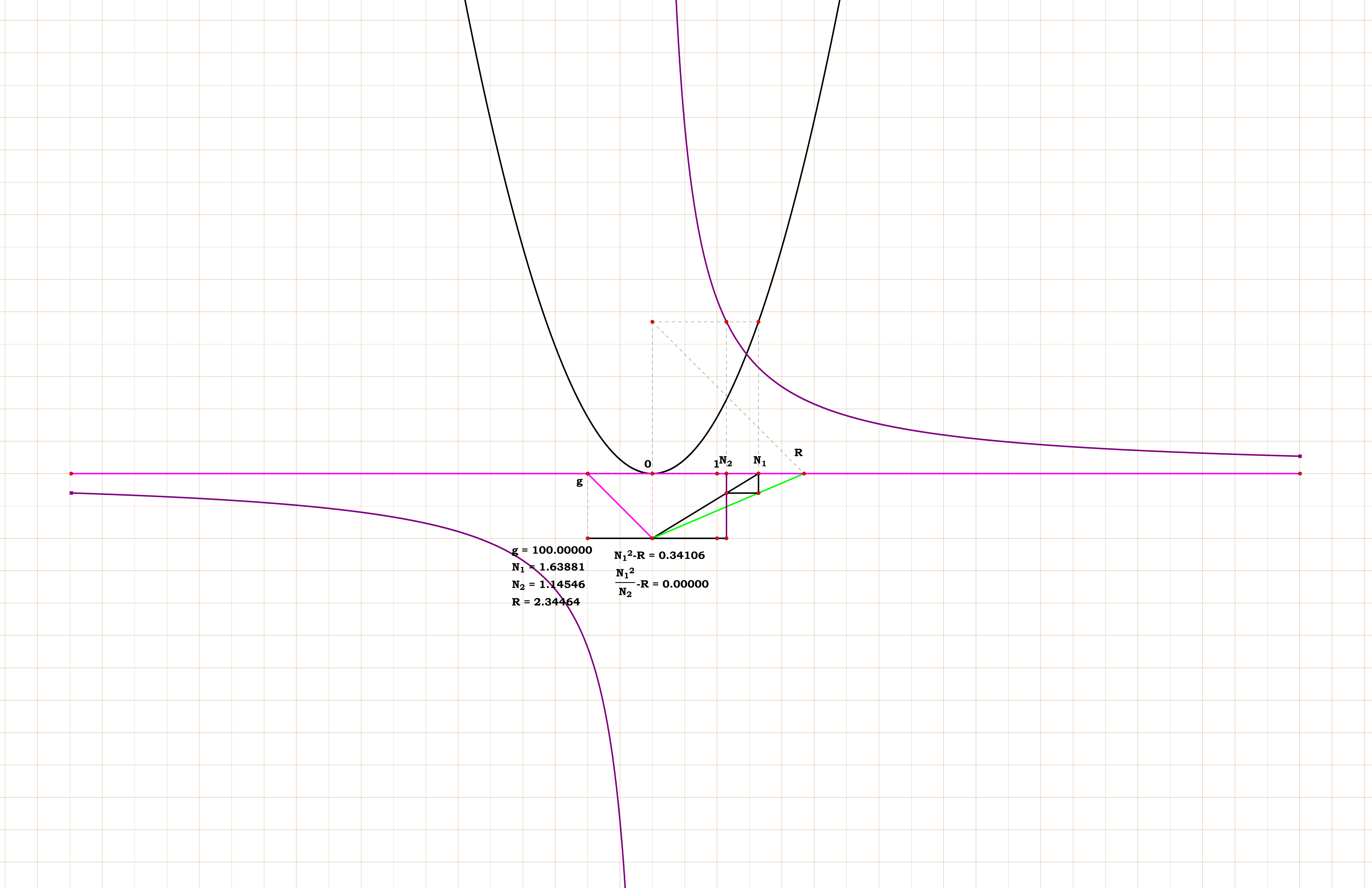
Two Transforms.

0, 0. 1

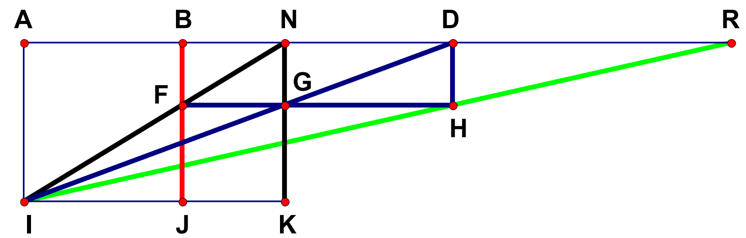
1, 0. $N_1^{\,2}$

0, 2. $\frac{1}{N_2}$

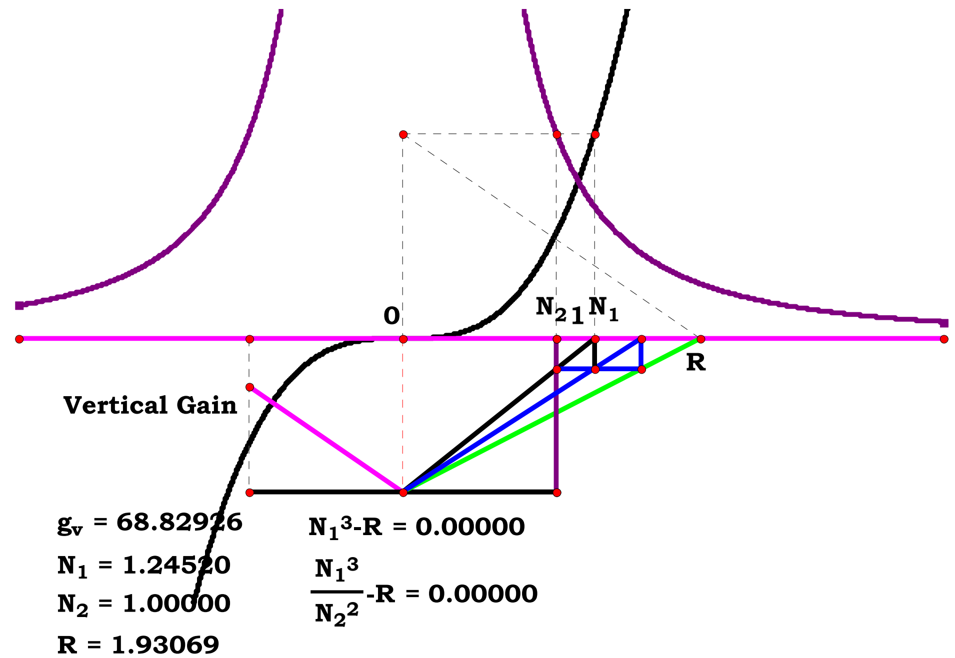
1, 2. $\frac{N_1^{\,2}}{N_2}$



0
1
N2
N1
R
g
g = 100.00000
N1 = 1.63881
N2 = 1.14546
R = 2.34464
N1²-R = 0.34106
N1²/N2 -R = 0.00000



1CST1R8



Unit. $AB := 1$

Given. $AN := 3$

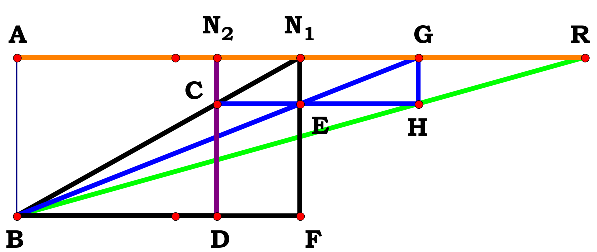
Descriptions.

$AI := AB \quad AD := AN^2 \quad BF := \dfrac{AN - 1}{AN}$

$DH := BF \quad AR := \dfrac{AD \cdot AI}{AI - DH}$

Definitions.

$AR - AN^3 = 0$



$N_1 := 3$

$N_2 := 2$

$ab := 1$

$ag := \dfrac{N_1^{\,2}}{N_2} \quad cn := \dfrac{N_1 - N_2}{N_1} \quad ar := \dfrac{ag \cdot ab}{ab - cn}$

Definitions.

$$ar - \frac{N_1^{\,3}}{N_2^{\,2}} = 0$$

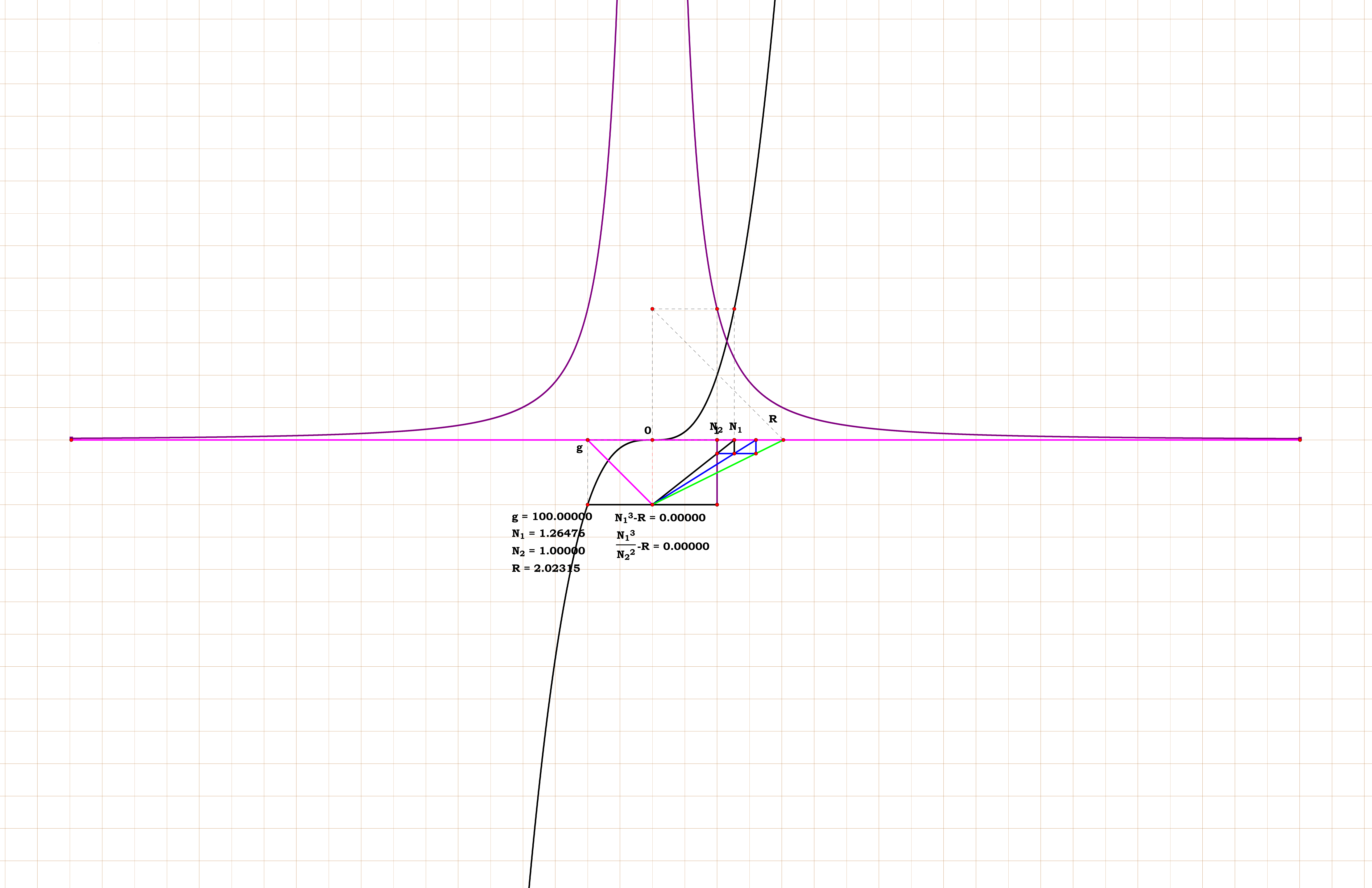
Two Transforms.

0, 0. 1

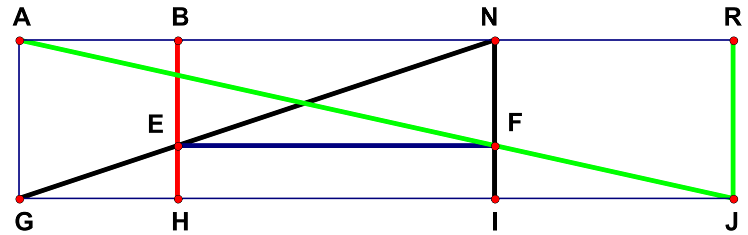
1, 0. $N_1{}^3$

0, 2. $\dfrac{1}{N_2{}^2}$

1, 2. $\dfrac{N_1{}^3}{N_2{}^2}$



0
N_2
1
N_1
R
g
g = 100.00000
N_1 = 1.26475
N_2 = 1.00000
R = 2.02315
N_1^3-R = 0.00000
$\frac{N_1^3}{N_2^2}$-R = 0.00000



1CST1R9

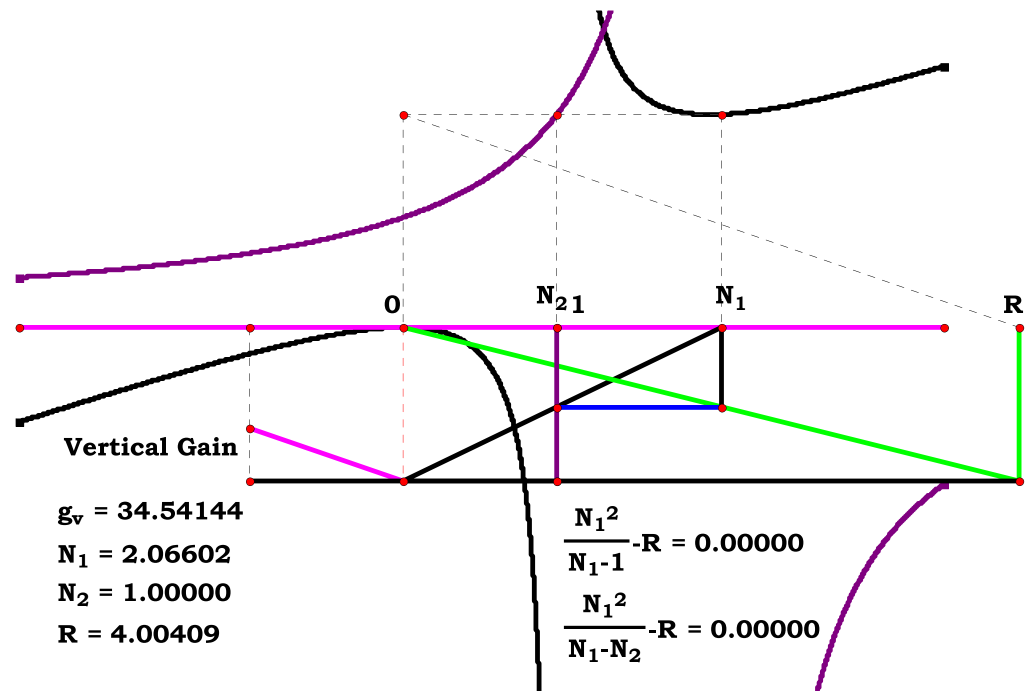
Unit. $AB := 1$ **Given.** $AN := 3$

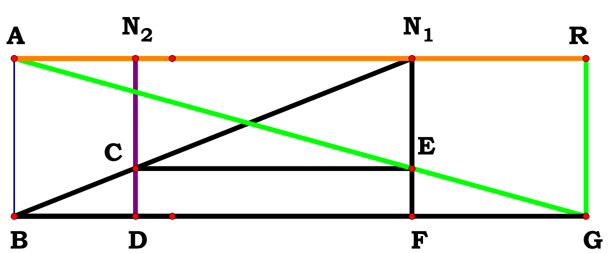
Descriptions.

$$BE := \frac{AN-1}{AN} \quad NF := BE \quad GJ := \frac{AN \cdot AB}{NF} \quad AR := GJ$$

Definitions.

$$AR - \frac{AN^2}{AN-1} = 0$$





$$N_1 := 3$$

$$N_2 := 2$$

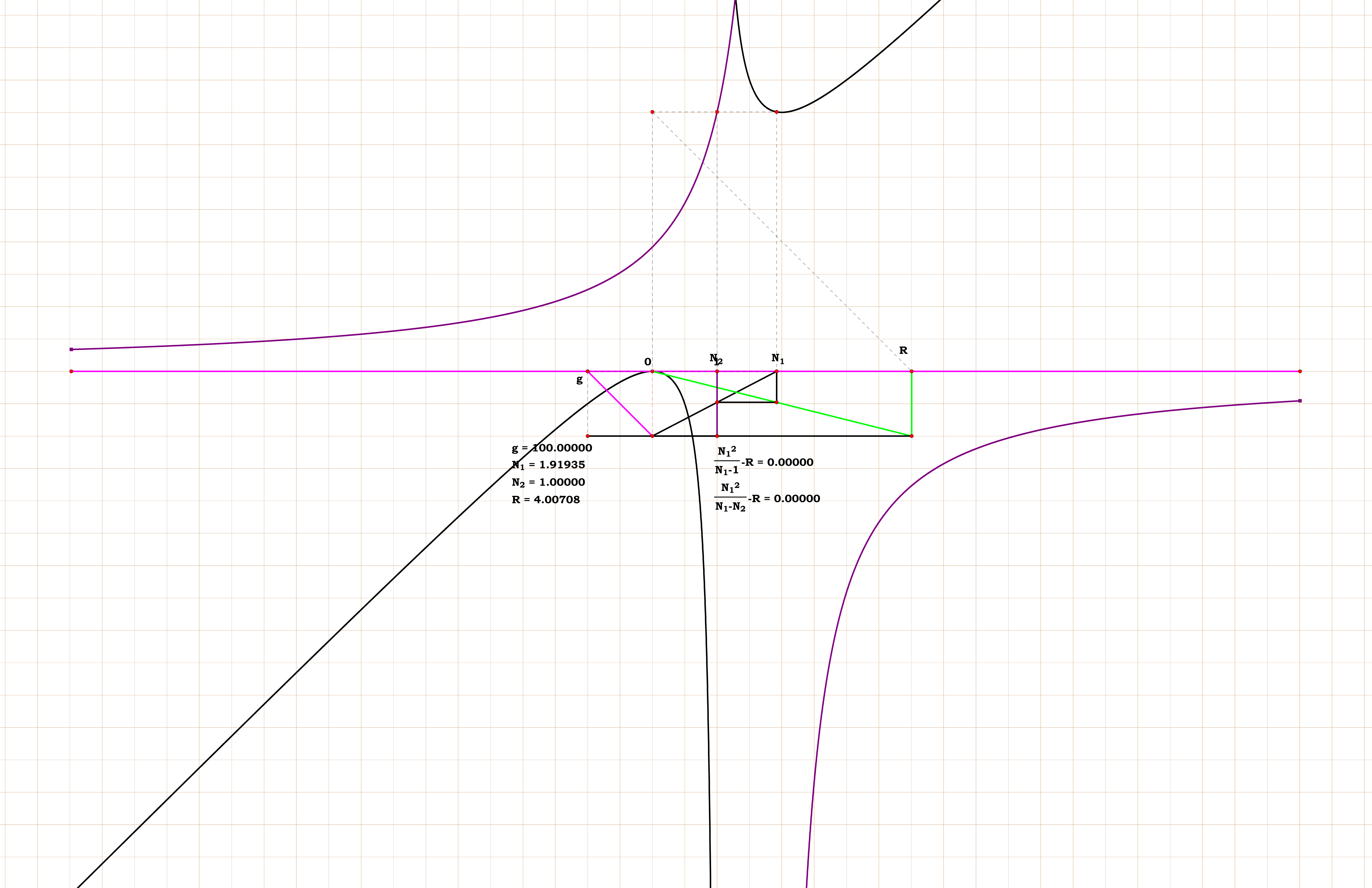
$$ab := 1 \quad cn := \frac{N_1 - N_2}{N_1} \quad bg := \frac{N_1 \cdot ab}{cn} \quad ar := bg$$

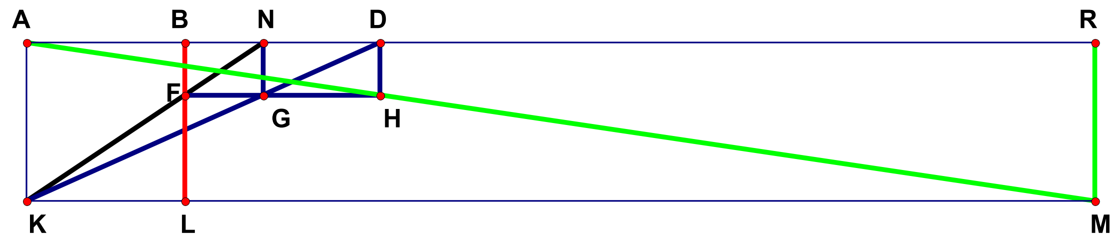
Definitions.

$$ar - \frac{N_1^{\,2}}{N_1 - N_2} = 0$$

Two Transforms.

0, 0.	0
1, 0.	$\frac{N_1^{\,2}}{N_1 - 1}$
0, 2.	$-\frac{1}{N_2 - 1}$
1, 2.	$\frac{N_1^{\,2}}{N_1 - N_2}$


O
N_2
N_1
R
g
g = 100.00000
N_1 = 1.91935
N_2 = 1.00000
R = 4.00708
$\frac{N_1^2}{N_1-1}$-R = 0.00000
$\frac{N_1^2}{N_1-N_2}$-R = 0.00000



1CST1R10

Unit. $AB := 1$ **Given.** $AN := 3$

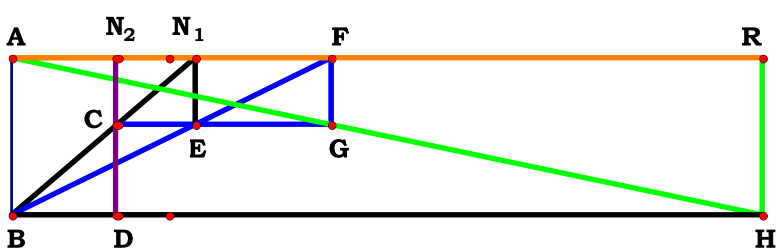
Descriptions.

$$AD := AN^2 \qquad BF := \frac{AN - 1}{AN}$$

$$DH := BF \qquad AR := \frac{AD \cdot AB}{DH}$$

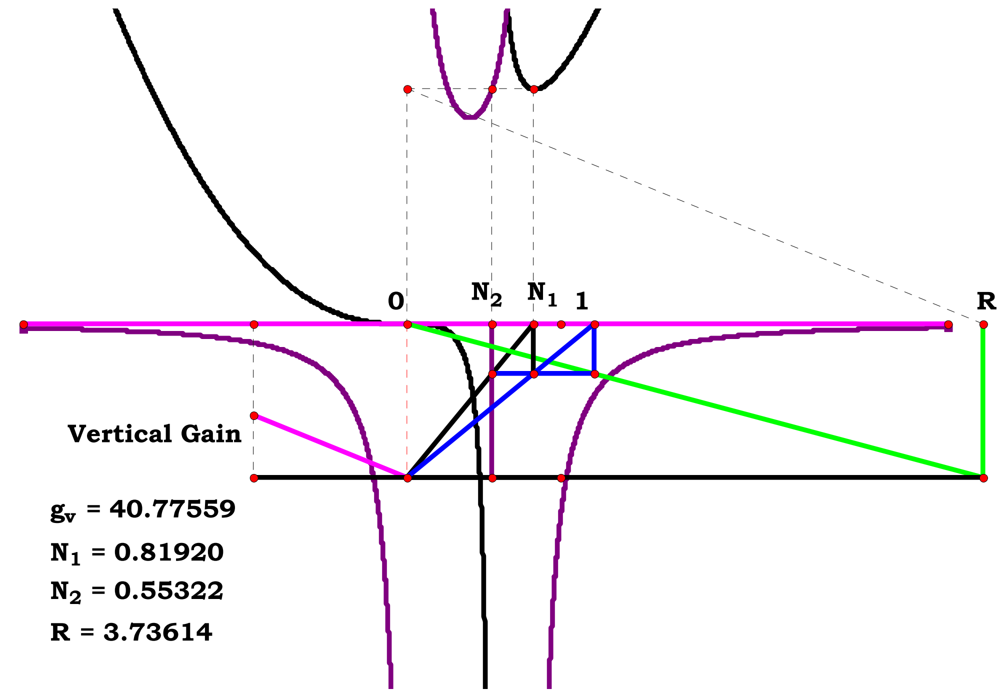
Definitions.

$$AR - \frac{AN^3}{AN - 1} = 0$$



$$N_1 := 3$$

$$N_2 := 2$$



$$af := \frac{N_1^{\,2}}{N_2} \qquad cn := \frac{N_1 - N_2}{N_1} \qquad fg := cn \quad ar := \frac{af}{cn}$$

Definitions.

$$ar - \frac{N_1^{\,3}}{N_2 \cdot N_1 - N_2^{\,2}} = 0$$

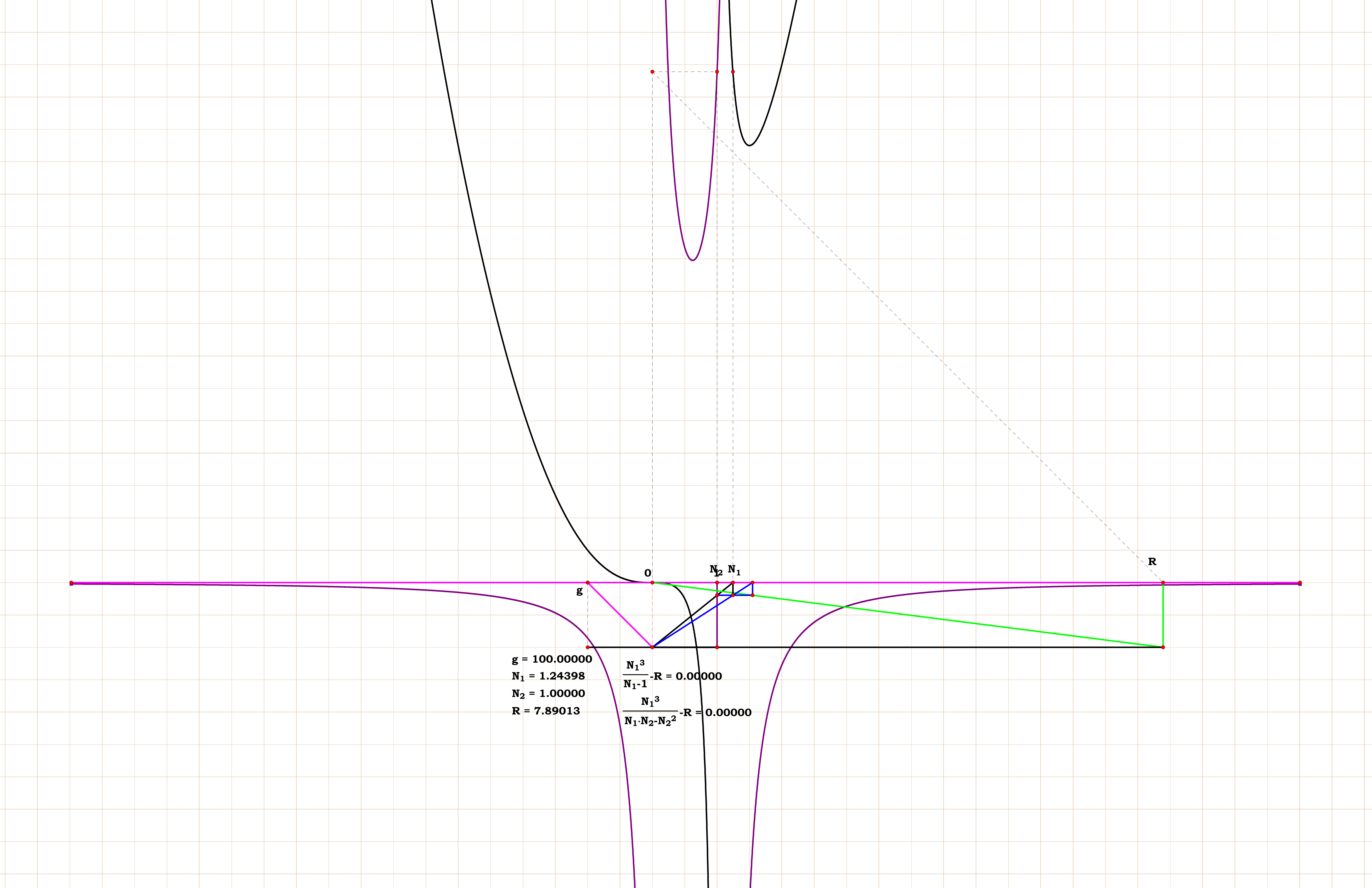
Two Transforms.

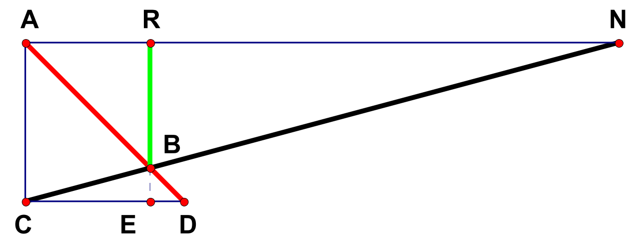
0, 0. 0

1, 0. $\frac{N_1{}^3}{N_1 - 1}$

0, 2. $-\frac{1}{N_2 \cdot (N_2 - 1)}$

1, 2. $\frac{N_1{}^3}{N_2 \cdot N_1 - N_2{}^2}$


O
N_2
N_1
R
g
g = 100.00000
N_1 = 1.24398
N_2 = 1.00000
R = 7.89013
$\frac{N_1^3}{N_1-1}-R = 0.00000$
$\frac{N_1^3}{N_1 \cdot N_2-N_2^2}-R = 0.00000$



1CST2R0

Unit. $AC := 1$ **Given.** $AN := 4$

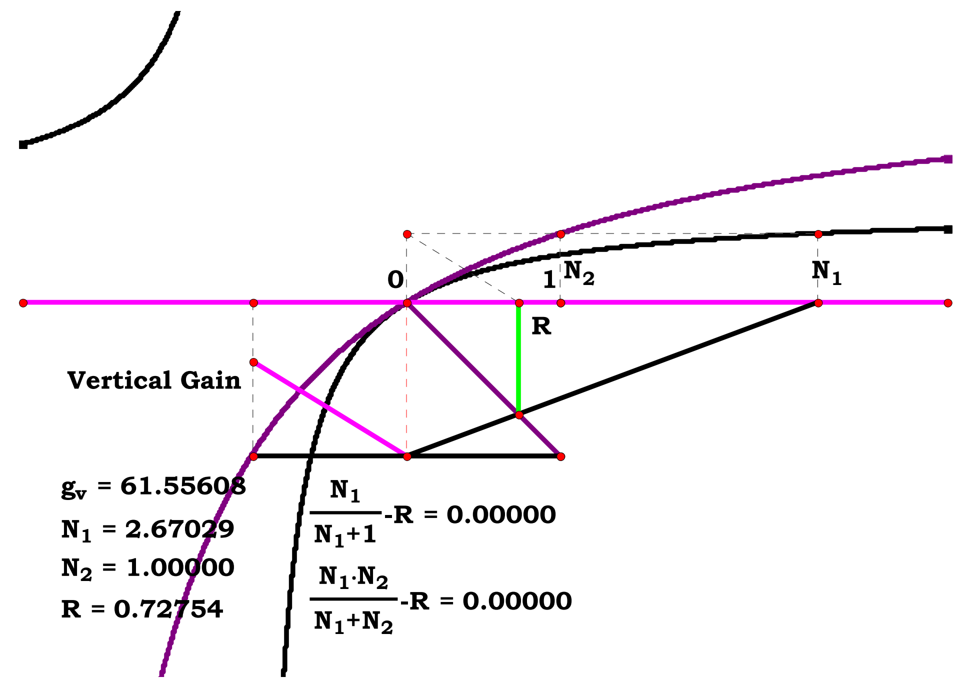
Descriptions.

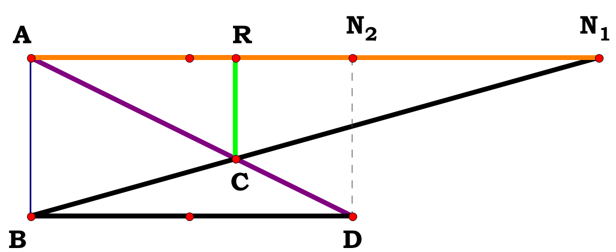
$$CD := AC \qquad AR := \frac{CD \cdot AN}{CD + AN}$$

$$DE := AC - AR$$

Definitions.

$$DE - \frac{1}{AN + 1} = 0 \qquad AR - \frac{AN}{AN + 1} = 0$$





$$N_1 := 3$$

$$N_2 := 2$$

Definitions.

$$ar := \frac{N_1 \cdot N_2}{N_1 + N_2}$$

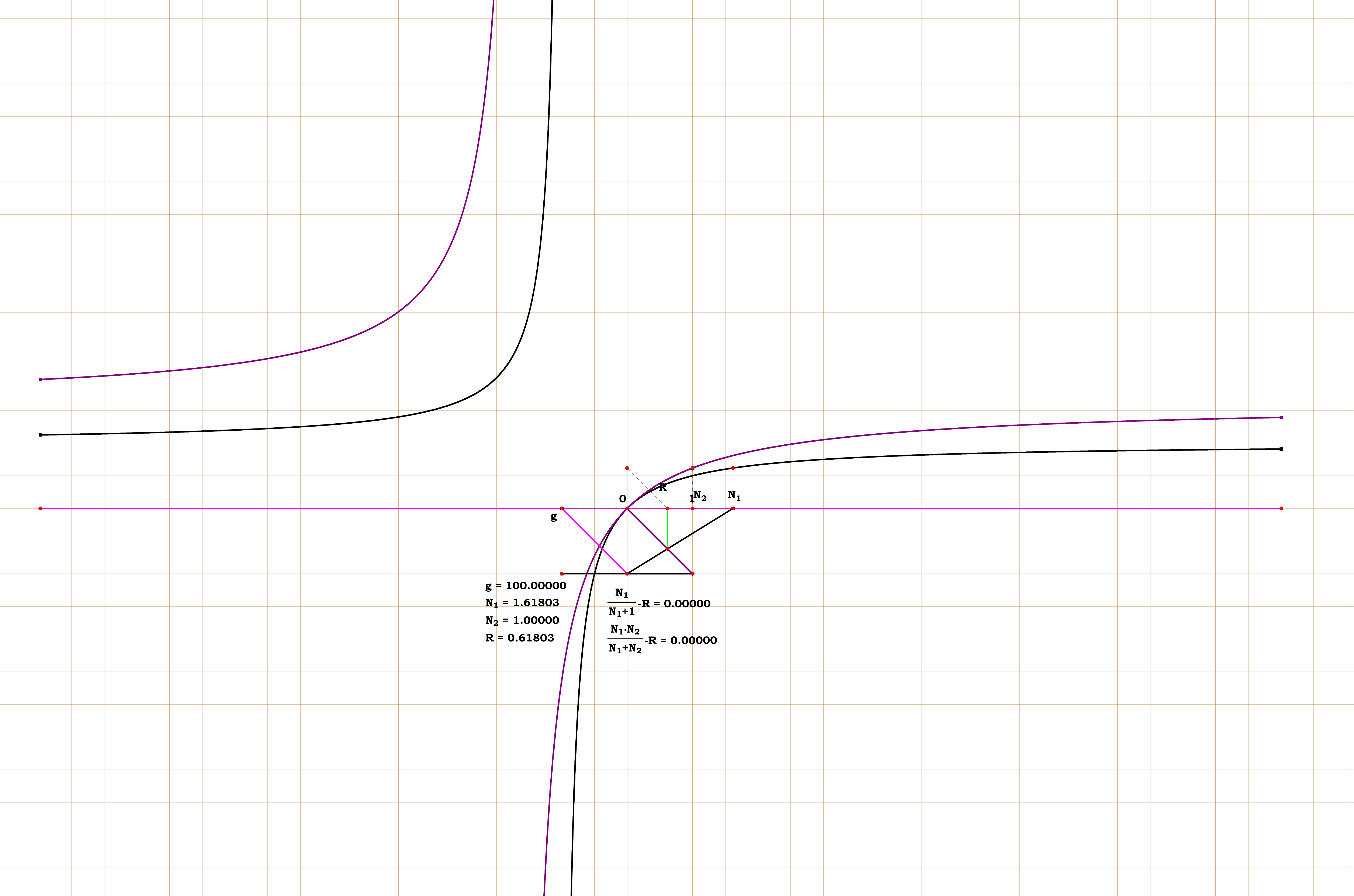
Two Transforms.

0, 0. $\frac{1}{2}$

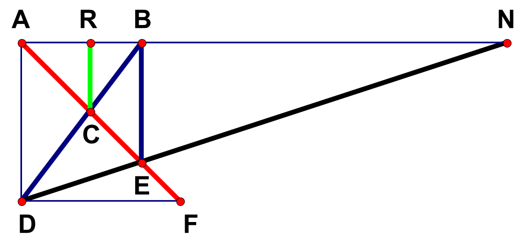
1, 0. $\frac{N_1}{N_1 + 1}$

0, 2. $\frac{N_2}{1 + N_2}$

1, 2. $\frac{N_1 \cdot N_2}{N_1 + N_2}$



0
R
1
N_2
N_1
g
g = 100.00000
N_1 = 1.61803
N_2 = 1.00000
R = 0.61803
$\frac{N_1}{N_1+1}$-R = 0.00000
$\frac{N_1 \cdot N_2}{N_1+N_2}$-R = 0.00000



1CST2R1

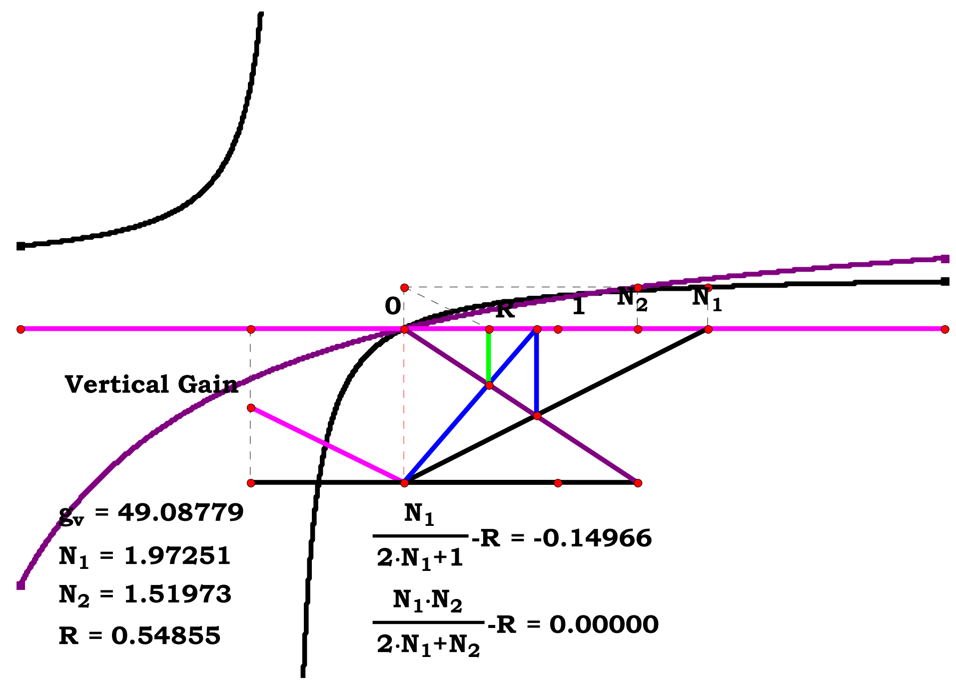
Unit. $AD := 1$ **Given.** $AN := 3$

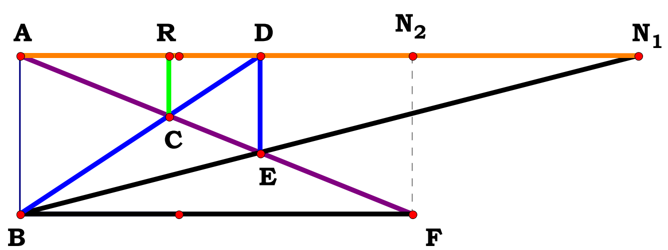
Descriptions.

$DF := AD \quad AB := \frac{AN}{AN+1} \quad AR := \frac{DF \cdot AB}{DF + AB}$

Definitions.

$$AR - \frac{AN}{2AN+1} = 0$$





$N_1 := 3$

$N_2 := 2$

$ab := 1$

$ad := \frac{N_1 \cdot N_2}{N_1 + N_2} \quad ar := \frac{N_2 \cdot ad}{N_2 + ad}$

Descriptions.

$$ar - \frac{N_1 \cdot N_2}{2 \cdot N_1 + N_2} = 0$$

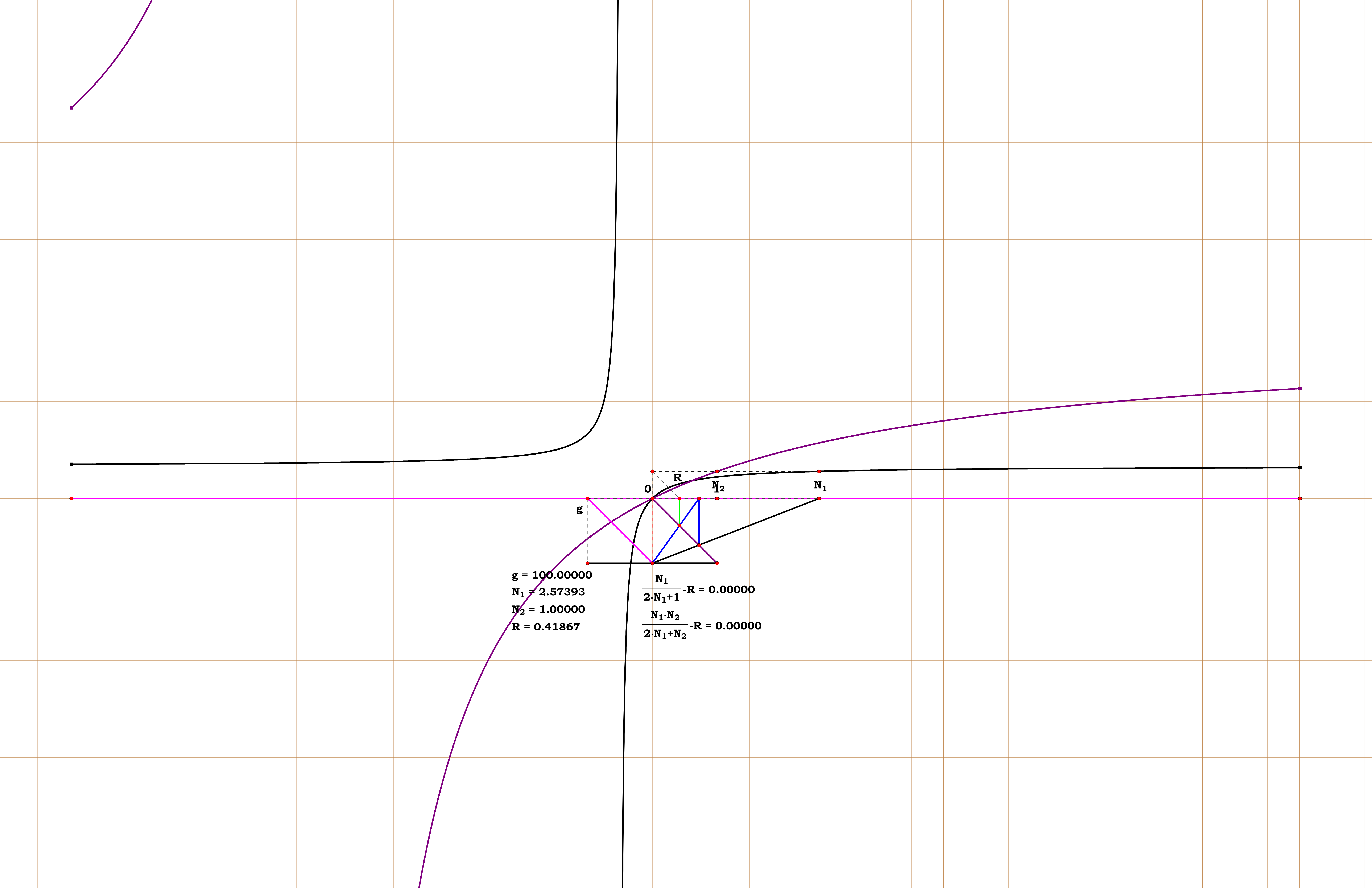
Two Transforms.

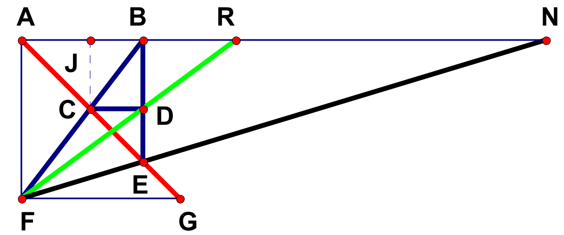
0, 0. $\frac{1}{3}$

1, 0. $\frac{N_1}{2 \cdot N_1 + 1}$

0, 2. $\frac{N_2}{2 + N_2}$

1, 2. $\frac{N_1 \cdot N_2}{2 \cdot N_1 + N_2}$


R
0
N2
N1
g
g = 100.00000
N1 = 2.57393
N2 = 1.00000
R = 0.41867
$\frac{N_1}{2 \cdot N_1 + 1} - R = 0.00000$
$\frac{N_1 \cdot N_2}{2 \cdot N_1 + N_2} - R = 0.00000$



1CST2R2

Unit. $AF := 1$ **Given.** $AN := 4$

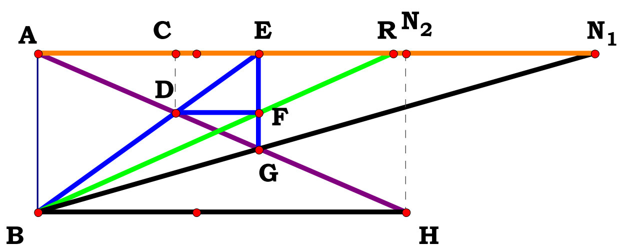
Descriptions.

$$AJ := \frac{AN}{2AN+1} \qquad JC := AJ$$

$$BD := JC \qquad AB := \frac{AN}{AN+1} \qquad AR := \frac{AB \cdot AF}{AF - BD}$$

Definitions.

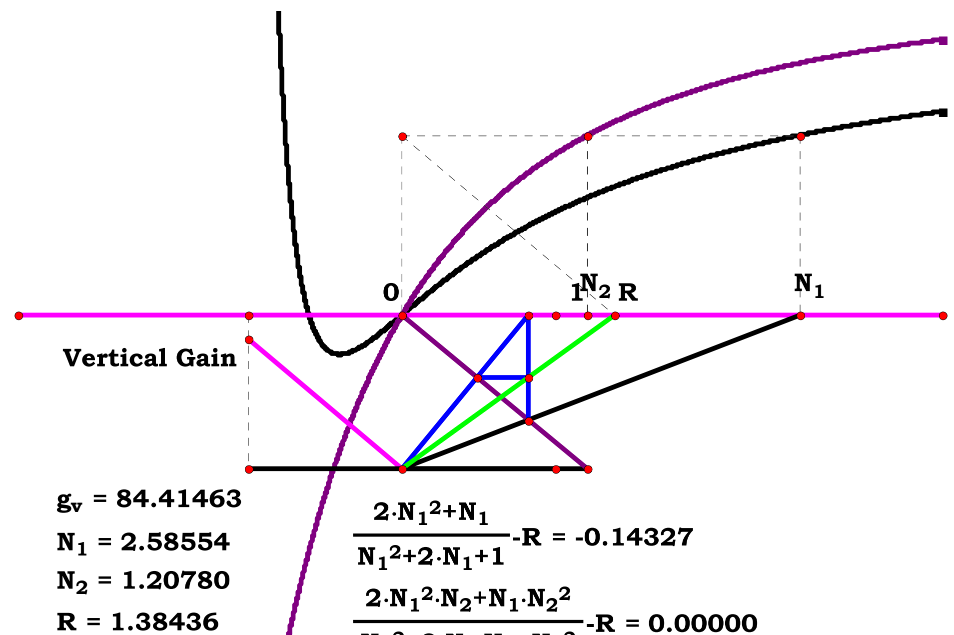
$$AR - \frac{2 \cdot AN^2 + AN}{AN^2 + 2 \cdot AN + 1} = 0$$



$N_1 := 3$

$N_2 := 2$

$ab := 1$



$$ae := \frac{N_1 \cdot N_2}{N_1 + N_2} \qquad ac := \frac{N_2 \cdot ae}{ae + N_2} \qquad cd := \frac{ab \cdot ac}{N_2} \qquad ar := \frac{ae \cdot ab}{ab - cd}$$

Definitions.

$$ac - \frac{N_1 \cdot N_2}{2 \cdot N_1 + N_2} = 0 \qquad cd - \frac{N_1}{2N_1 + N_2} = 0 \qquad ar - \frac{2 \cdot N_1^{\,2} \cdot N_2 + N_1 \cdot N_2^{\,2}}{N_1^{\,2} + 2 \cdot N_1 \cdot N_2 + N_2^{\,2}} = 0$$

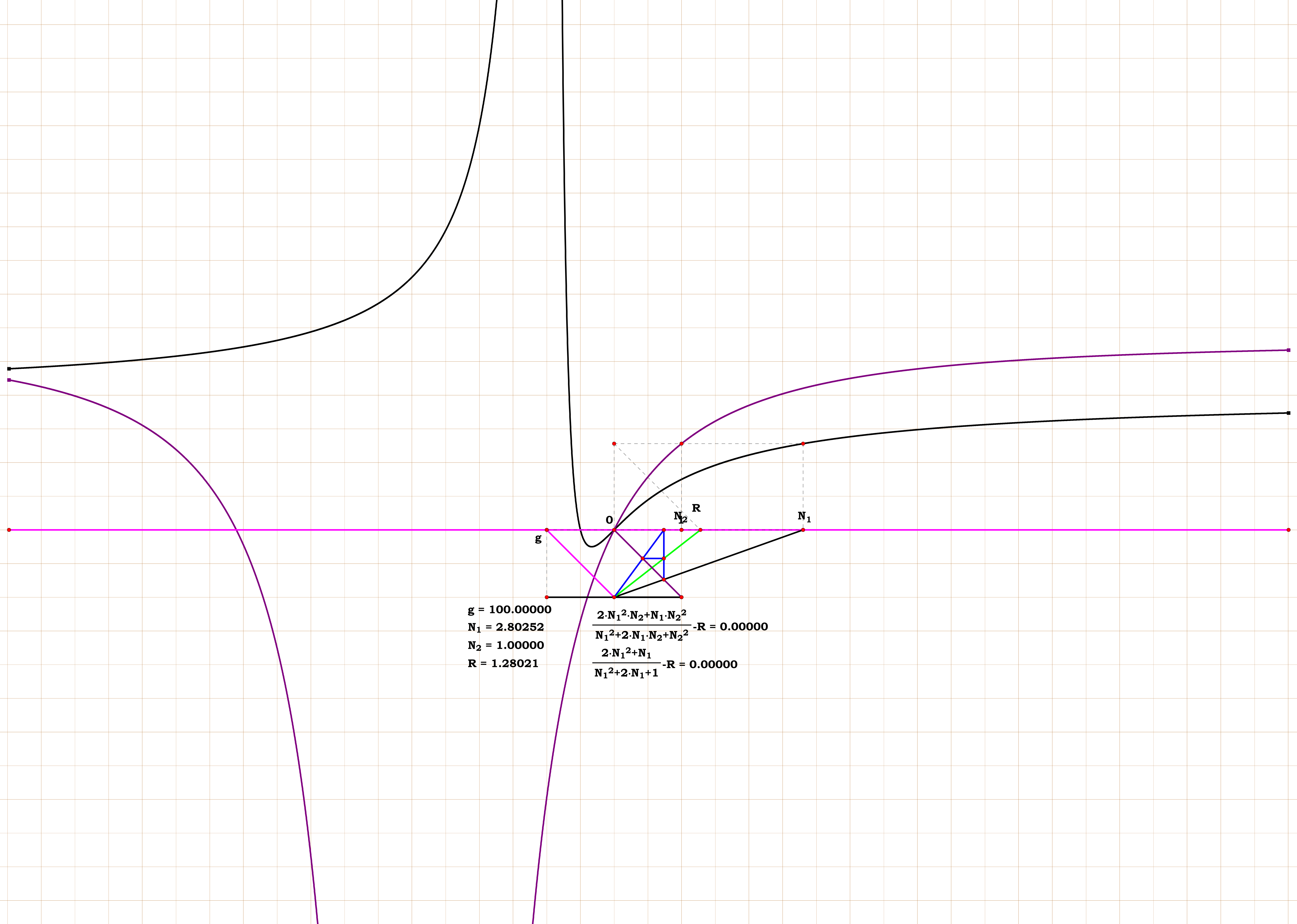
Two Transforms.

0, 0. $$\frac{3}{4}$$

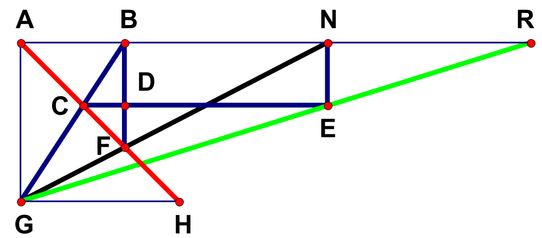
1, 0. $$\frac{2 \cdot N_1^{\,2} + N_1}{N_1^{\,2} + 2 \cdot N_1 + 1}$$

0, 2. $$\frac{2 \cdot N_2 + N_2^{\,2}}{1 + 2 \cdot N_2 + N_2^{\,2}}$$

1, 2. $$\frac{2 \cdot N_1^{\,2} \cdot N_2 + N_1 \cdot N_2^{\,2}}{N_1^{\,2} + 2 \cdot N_1 \cdot N_2 + N_2^{\,2}}$$



0
N_2
R
N_1
g
g = 100.00000
N_1 = 2.80252
N_2 = 1.00000
R = 1.28021
$\frac{2 \cdot N_1^2 \cdot N_2 + N_1 \cdot N_2^2}{N_1^2 + 2 \cdot N_1 \cdot N_2 + N_2^2} - R = 0.00000$
$\frac{2 \cdot N_1^2 + N_1}{N_1^2 + 2 \cdot N_1 + 1} - R = 0.00000$



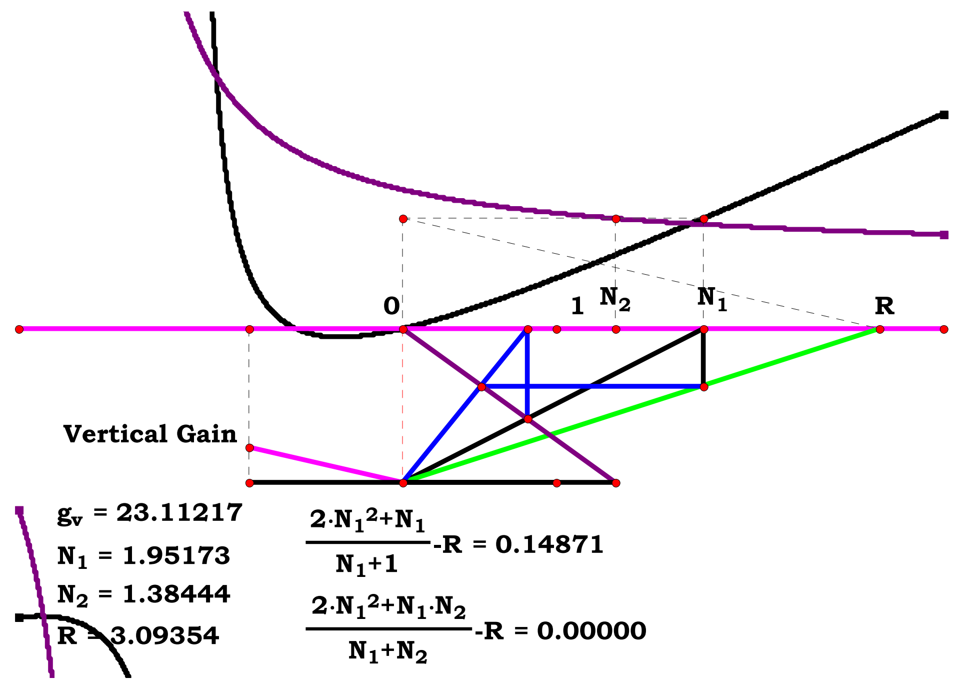
1CST2R3

Unit.

$AG := 1$

Given.

$AN := 2$

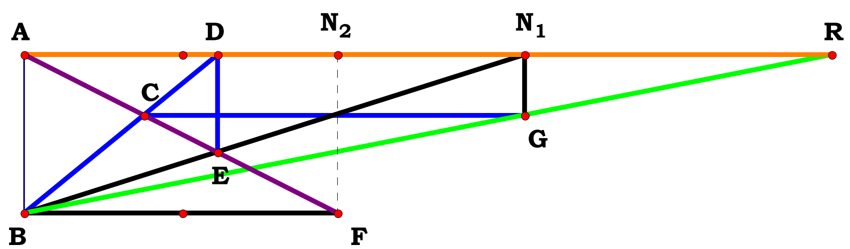


Descriptions.

$$GH := AG \quad NE := \frac{AN}{2AN+1} \quad AR := \frac{AN \cdot AG}{AG - NE}$$

Definitions.

$$AR - \frac{2 \cdot AN^2 + AN}{AN + 1} = 0$$



$$N_1 := 3 \quad N_2 := 2 \quad ab := 1$$

$$gn := \frac{N_1}{2N_1 + N_2} \quad ar := \frac{N_1 \cdot ab}{ab - gn}$$

Definitions.

$$ar - \frac{2 \cdot N_1^{\,2} + N_1 \cdot N_2}{N_1 + N_2} = 0$$

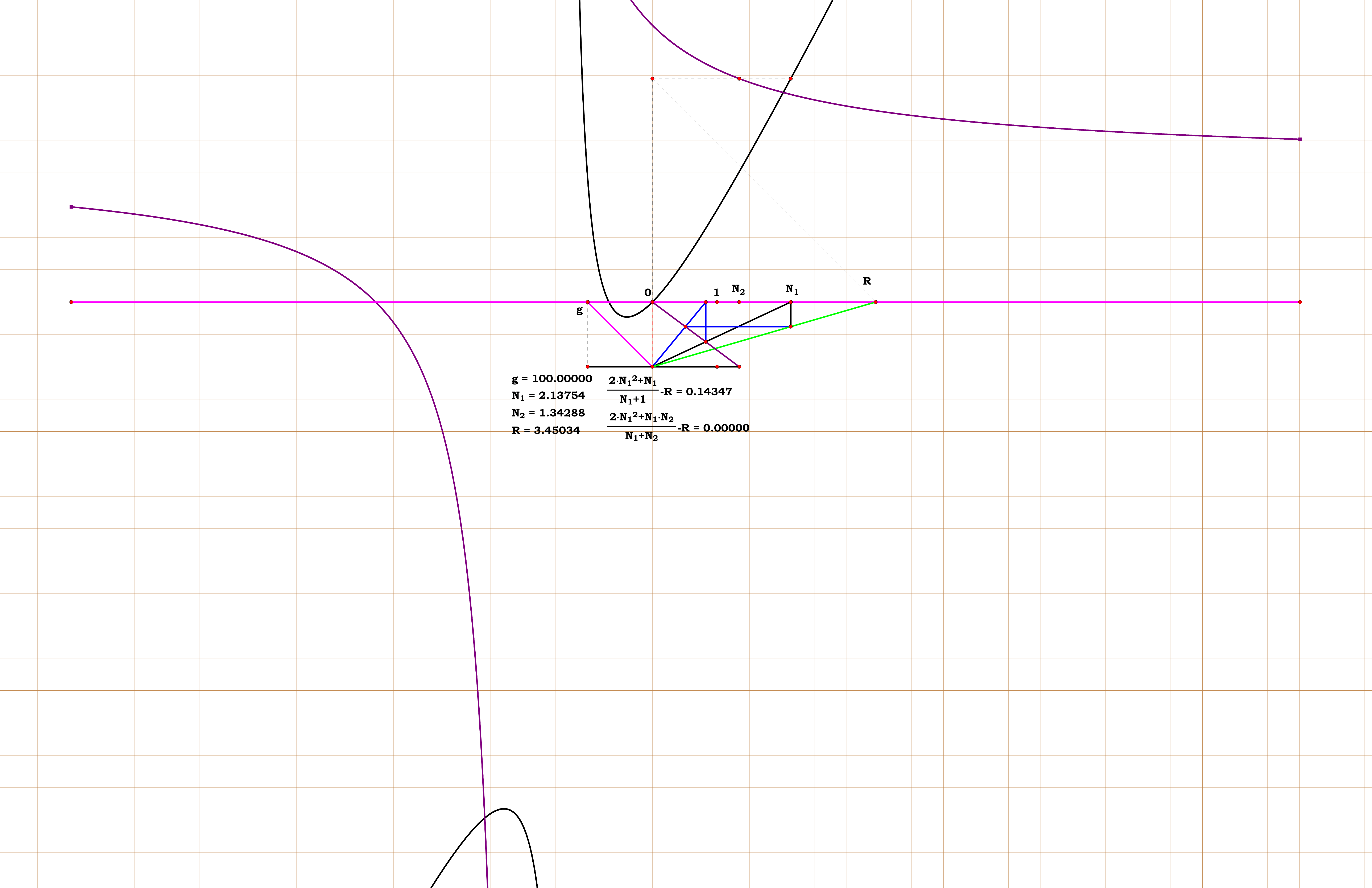
Two Transforms.

0, 0. $$\frac{3}{2}$$

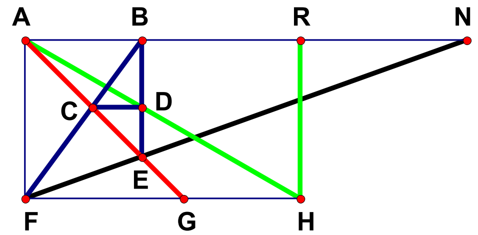
1, 0. $$\frac{2 \cdot N_1^{\,2} + N_1}{N_1 + 1}$$

0, 2. $$\frac{2 + N_2}{1 + N_2}$$

1, 2. $$\frac{2 \cdot N_1^{\,2} + N_1 \cdot N_2}{N_1 + N_2}$$



0
1
N2
N1
R
g
g = 100.00000
N1 = 2.13754
N2 = 1.34288
R = 3.45034
(2·N1²+N1)/(N1+1) -R = 0.14347
(2·N1²+N1·N2)/(N1+N2) -R = 0.00000



1CST2R4

Unit. $AF := 1$ **Given.** $AN := 3$

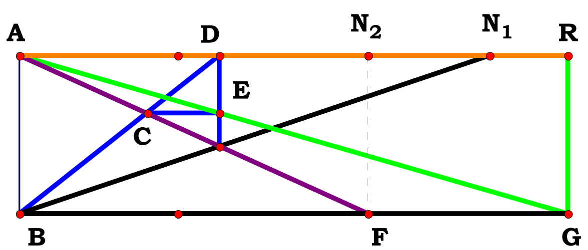
Descriptions.

$$FG := AF \qquad AB := \frac{AN}{AN+1} \qquad BD := \frac{AN}{2AN+1}$$

$$FH := \frac{AB \cdot AF}{BD} \qquad AR := FH$$

Definitions.

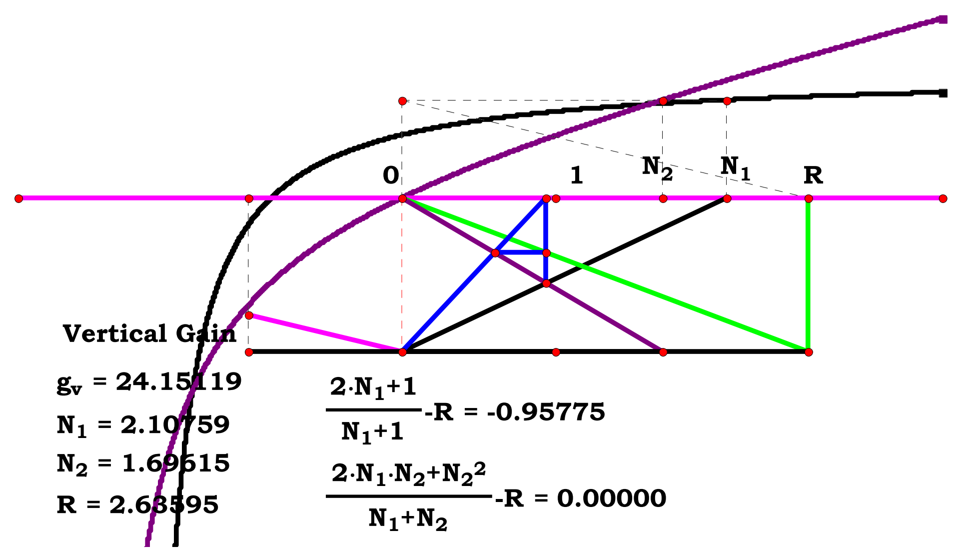
$$AR - \frac{2 \cdot AN + 1}{AN + 1} = 0$$



$N_1 := 3$

$N_2 := 2$

$ab := 1$



$$ad := \frac{N_1 \cdot N_2}{N_1 + N_2} \qquad de := \frac{N_1}{2N_1 + N_2} \qquad ar := \frac{ad}{de}$$

Definitions.

$$ar - \frac{2 \cdot N_1 \cdot N_2 + N_2^{\,2}}{N_1 + N_2} = 0$$

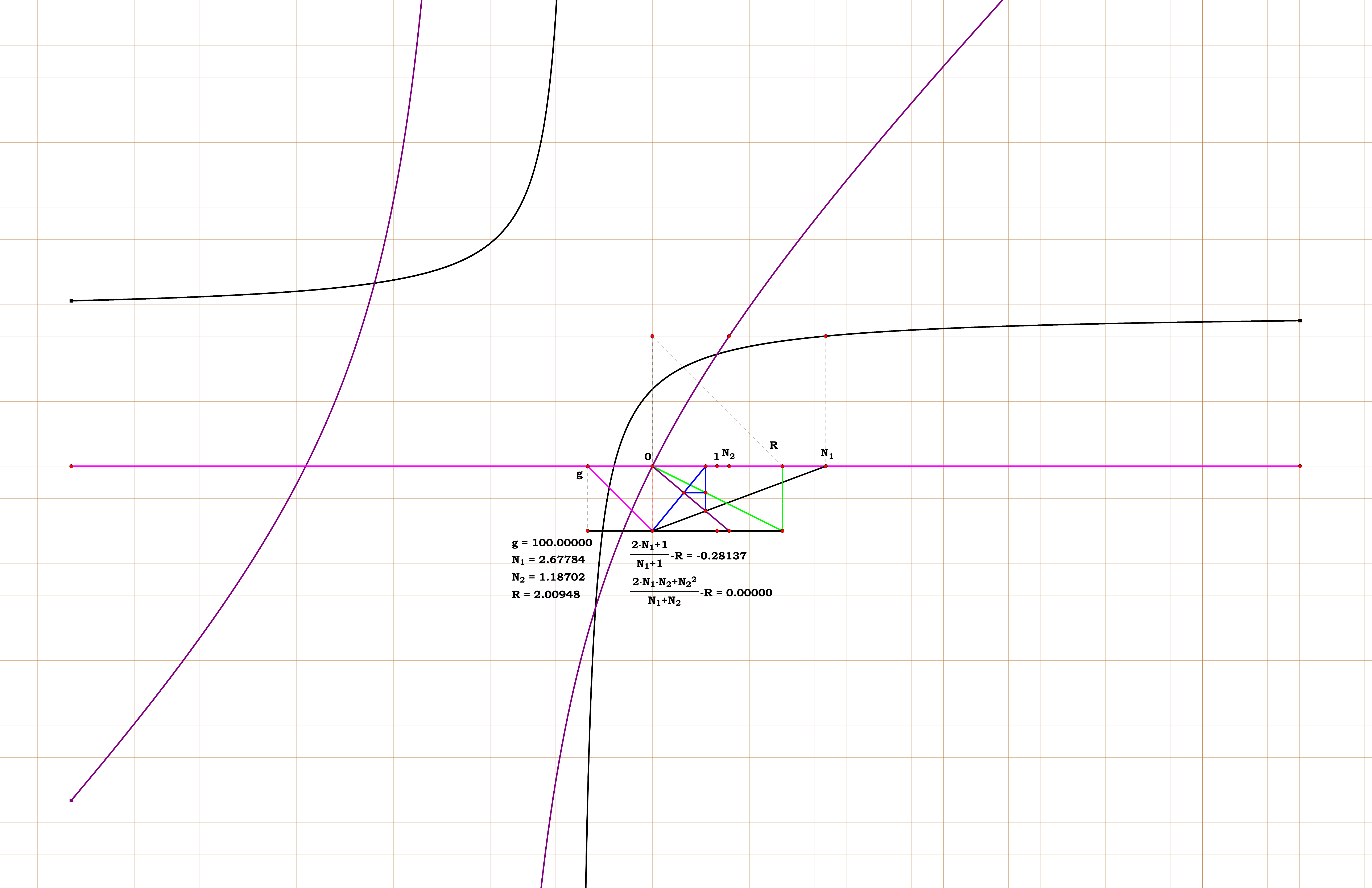
Two Transforms.

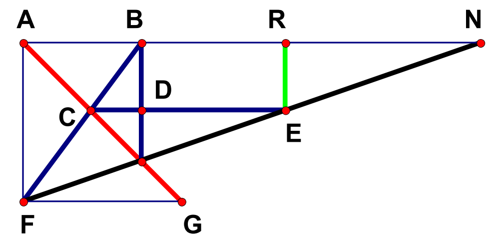
0, 0. $\frac{3}{2}$

1, 0. $\frac{2 \cdot N_1 + 1}{N_1 + 1}$

0, 2. $\frac{2 \cdot N_2 + N_2^{\,2}}{1 + N_2}$

1, 2. $\frac{2 \cdot N_1 \cdot N_2 + N_2^{\,2}}{N_1 + N_2}$


0
1
N_2
R
N_1
g
g = 100.00000
N_1 = 2.67784
N_2 = 1.18702
R = 2.00948
$\frac{2 \cdot N_1 + 1}{N_1 + 1} - R = -0.28137$
$\frac{2 \cdot N_1 \cdot N_2 + N_2^2}{N_1 + N_2} - R = 0.00000$



1CST2R5

Unit. $AF := 1$ **Given.** $AN := 3$

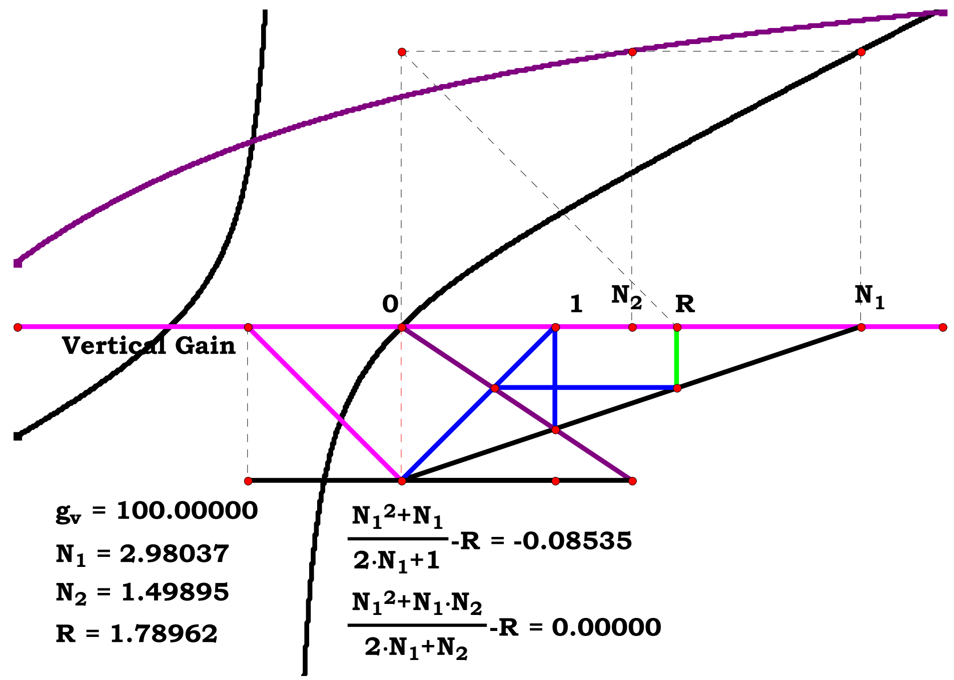
Descriptions.

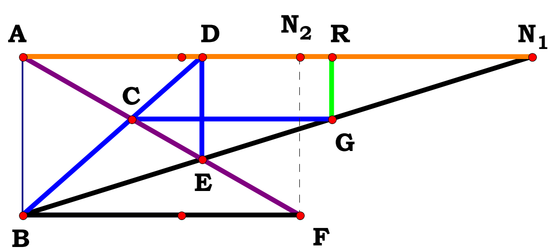
$FG := AF \quad BD := \dfrac{AN}{2AN+1} \quad RE := BD$

$RN := \dfrac{AN \cdot RE}{AF} \quad AR := AN - RN$

Definitions.

$$AR - \frac{AN^2 + AN}{2 \cdot AN + 1} = 0$$





$N_1 := 3 \quad N_2 := 2 \quad ab := 1$

$gr := \dfrac{N_1}{2N_1 + N_2} \quad nr := \dfrac{N_1 \cdot gr}{ab} \quad ar := N_1 - nr$

Definitions.

$$ar - \frac{N_1{}^2 + N_1 \cdot N_2}{2 \cdot N_1 + N_2} = 0$$

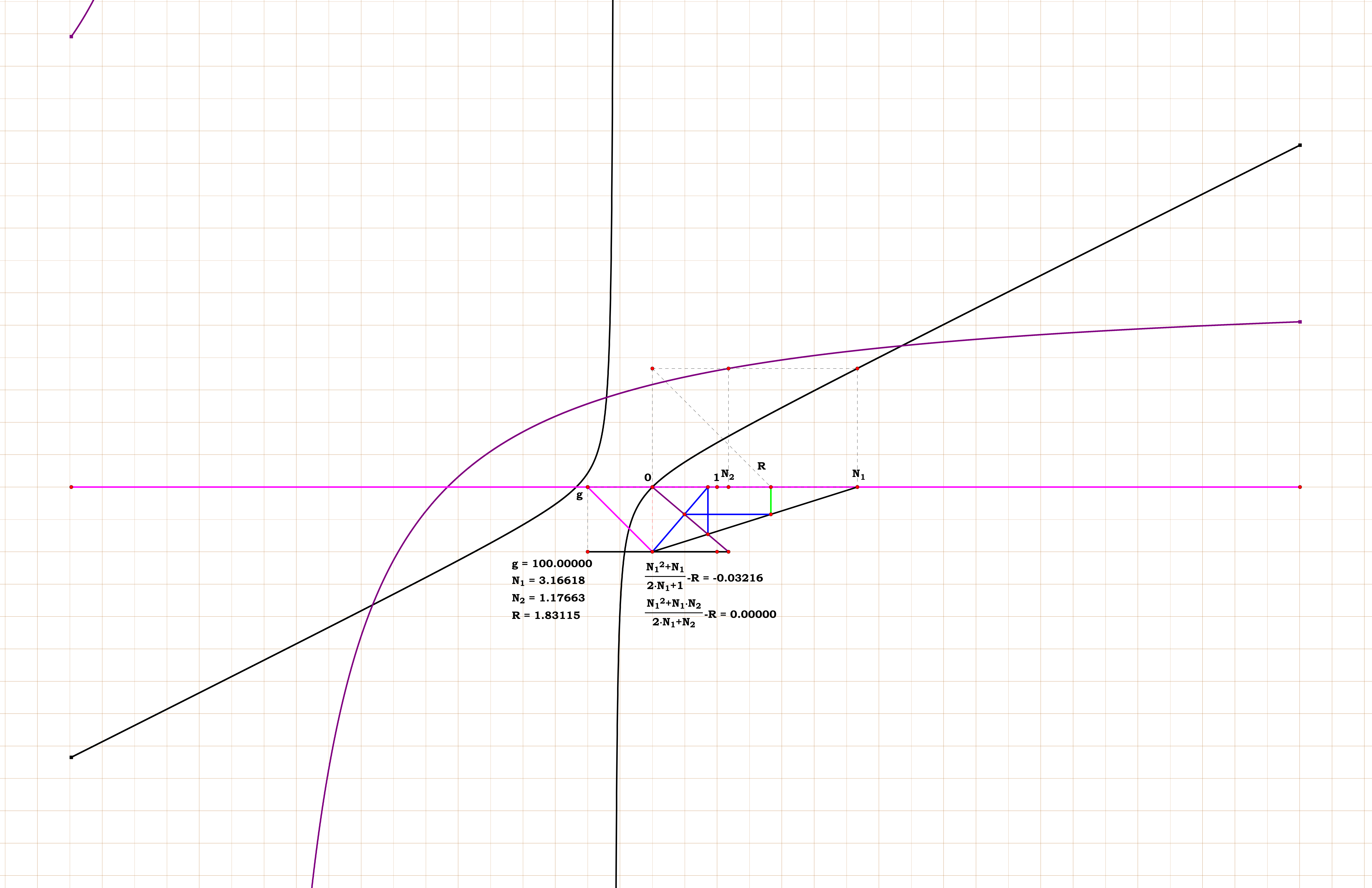
Two Transforms.

0, 0. $\frac{2}{3}$

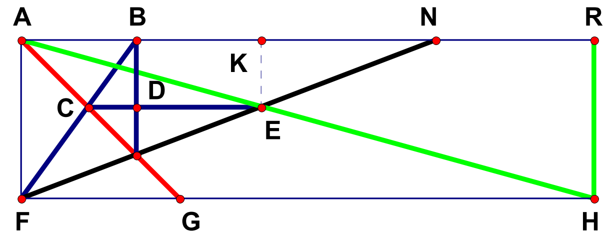
1, 0. $\frac{N_1{}^2 + N_1}{2 \cdot N_1 + 1}$

0, 2. $\frac{1 + N_2}{2 + N_2}$

1, 2. $\frac{N_1{}^2 + N_1 \cdot N_2}{2 \cdot N_1 + N_2}$



0
1
N_2
R
N_1
g
g = 100.00000
N_1 = 3.16618
N_2 = 1.17663
R = 1.83115
$\frac{N_1^2+N_1}{2 \cdot N_1+1}$-R = -0.03216
$\frac{N_1^2+N_1 \cdot N_2}{2 \cdot N_1+N_2}$-R = 0.00000



1CST2R6

Unit. $AF := 1$ **Given.** $AN := 3$

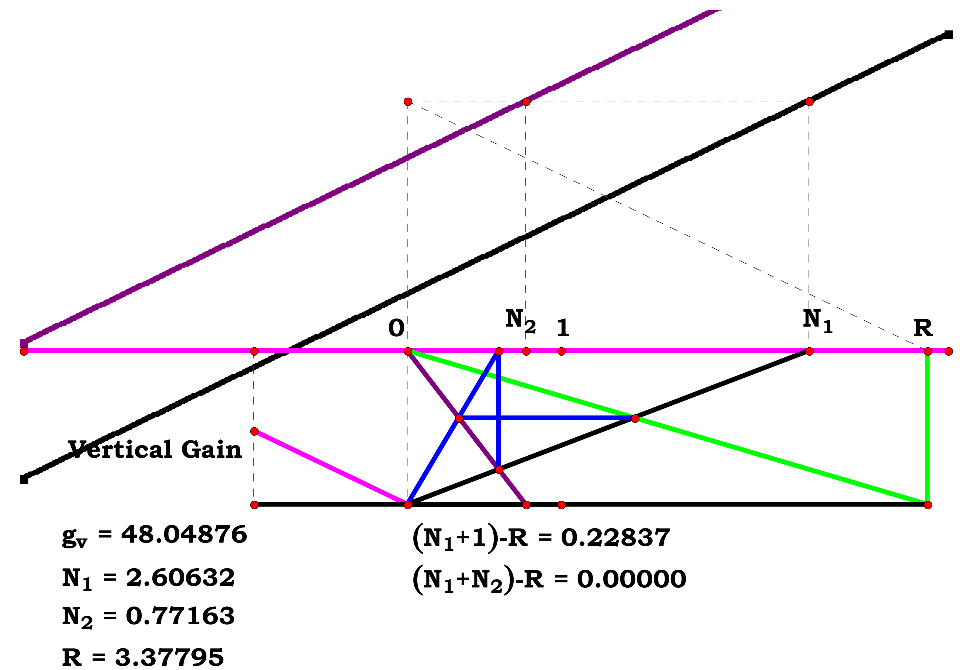
Descriptions.

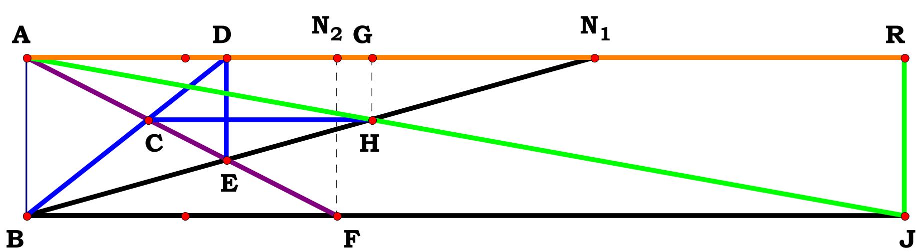
$$FG := AF \qquad AK := \frac{AN^2 + AN}{2 \cdot AN + 1} \qquad KE := \frac{AN}{2AN + 1}$$

$$FH := \frac{AK \cdot AF}{KE} \qquad AR := FH$$

Definitions.

$$AR - (AN + 1) = 0$$





$$N_1 := 3$$

$$N_2 := 2$$

$$ab := 1$$

$$gh := \frac{N_1}{2N_1 + N_2} \qquad ag := \frac{N_1{}^2 + N_1 \cdot N_2}{2 \cdot N_1 + N_2} \qquad ar := \frac{ag \cdot ab}{gh}$$

Definitions.

$$ar - \left(N_1 + N_2\right) = 0$$

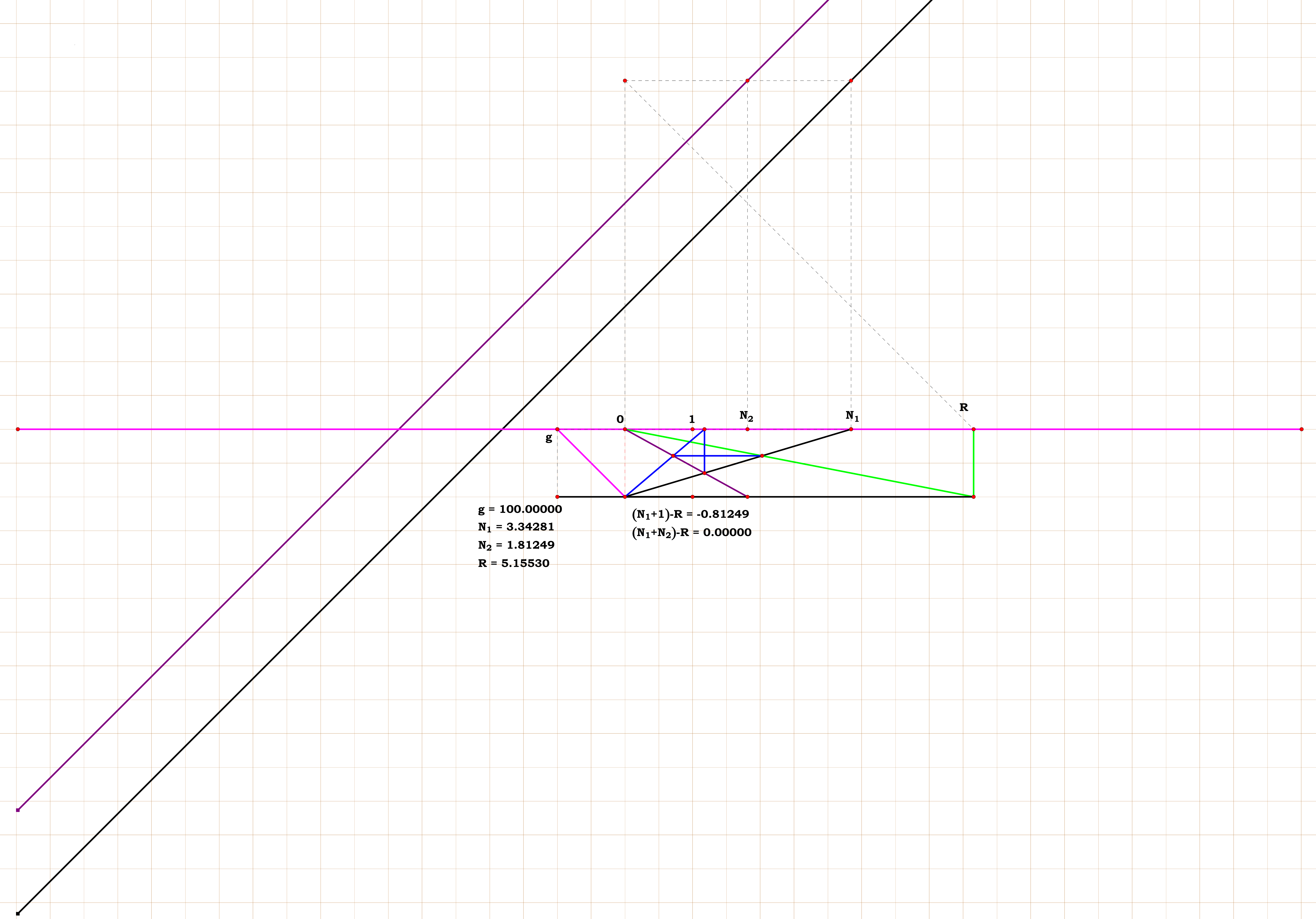
Two Transforms.

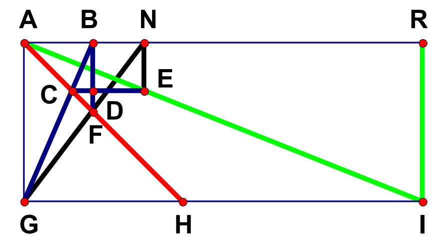
0, 0. $\mathbf{2}$

1, 0. $\mathbf{N_1 + 1}$

0, 2. $\mathbf{1 + N_2}$

1, 2. $\mathbf{(N_1 + N_2)}$


0
1
N_2
N_1
R
g
g = 100.00000
N_1 = 3.34281
N_2 = 1.81249
R = 5.15530
(N_1+1)-R = -0.81249
(N_1+N_2)-R = 0.00000



1CST2R7

Unit. $AG := 1$ **Given.** $AN := 5$

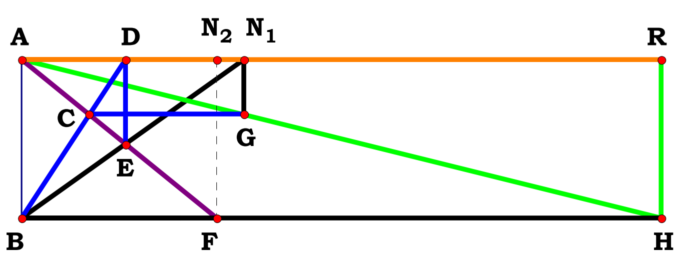
Descriptions.

$GH := AG \quad NE := \dfrac{AN}{2AN + 1}$

$GI := \dfrac{AN \cdot AG}{NE} \quad AR := GI$

Definitions.

$AR - (2AN + 1) = 0$



$N_1 := 3$

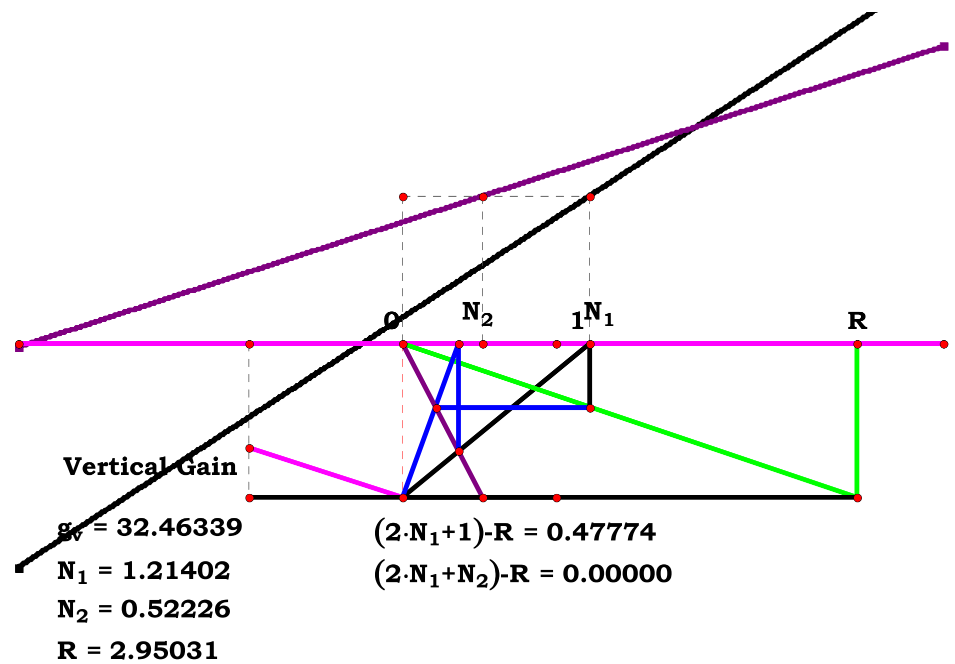
$N_2 := 2$

$ab := 1$

$gn := \dfrac{N_1}{2N_1 + N_2} \quad ar := \dfrac{N_1 \cdot ab}{gn}$

Definitions.

$ar - \left(2 \cdot N_1 + N_2\right) = 0$



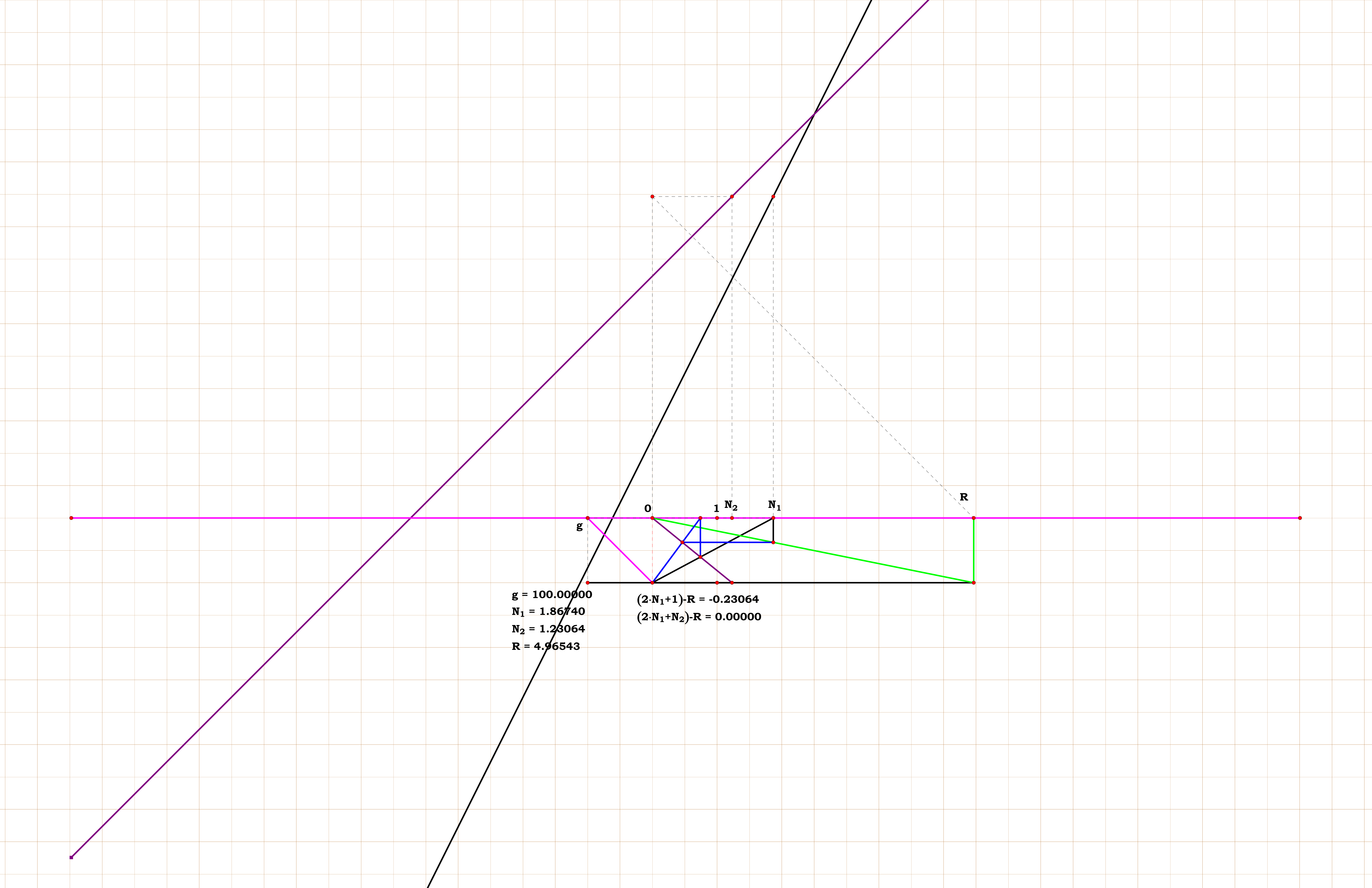
Two Transforms.

0, 0. $\mathbf{3}$

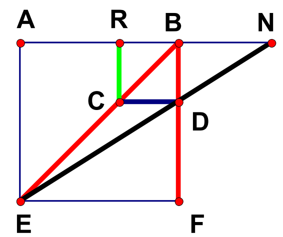
1, 0. $\mathbf{2 \cdot N_1 + 1}$

0, 2. $\mathbf{2 + N_2}$

1, 2. $\mathbf{\left(2 \cdot N_1 + N_2\right)}$



R
0
1
N2
N1
g
g = 100.00000
N1 = 1.86740
N2 = 1.23064
R = 4.96543
(2·N1+1)-R = -0.23064
(2·N1+N2)-R = 0.00000



1CST3R0

Unit.

$AB := 1$

Given.

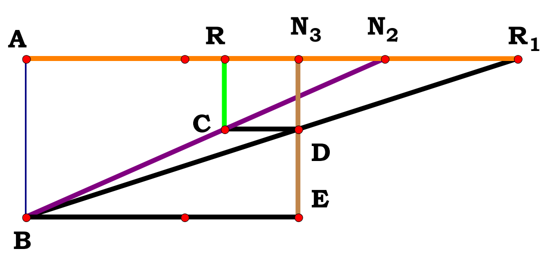
$AN := 3$

Descriptions.

$$BD := \frac{AN - 1}{AN} \quad AR := AB - BD$$

Definitions.

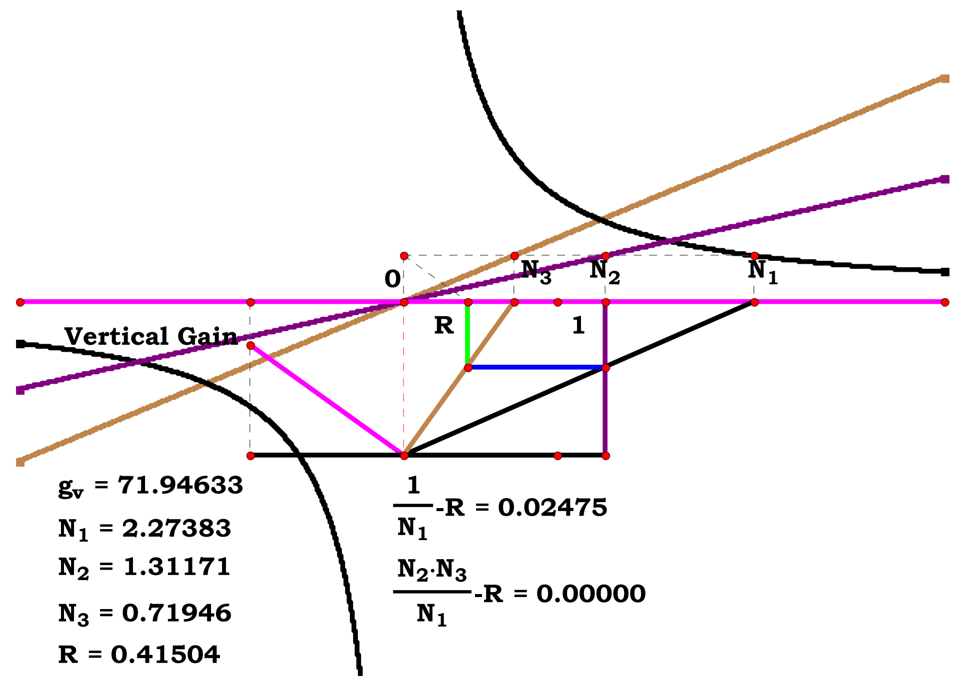
$$AR - \frac{1}{AN} = 0 \qquad AR - AN^{-1} = 0$$



$N_1 := 4$

$N_2 := 3$

$N_3 := 2$



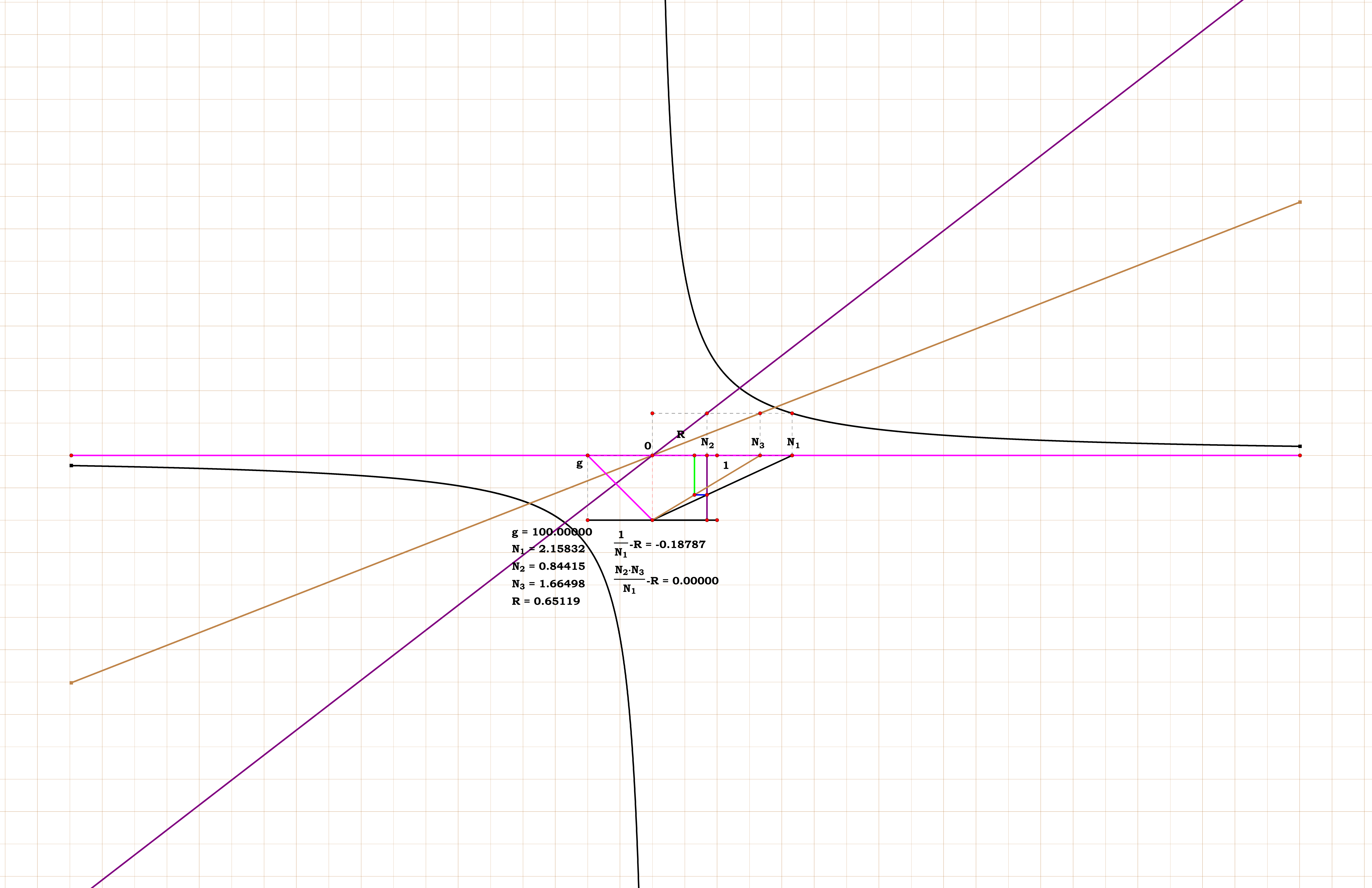
$$ab := 1 \qquad de := \frac{N_3}{N_1} \qquad cr := ab - de \qquad rn2 := \frac{N_2 \cdot cr}{ab} \qquad ar := N_2 - rn2$$

Definitions.

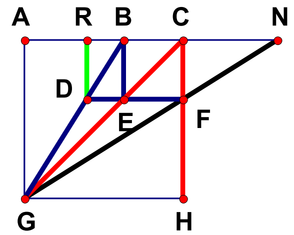
$$cr - \frac{N_1 - N_3}{N_1} = 0 \qquad ar - N_2 \cdot \frac{N_3}{N_1} = 0$$

Three Transforms.

0, 0, 0.	1	1, 2, 0.	$\frac{N_2}{N_1}$
1, 0, 0.	$\frac{1}{N_1}$	1, 0, 3.	$\frac{N_3}{N_1}$
0, 2, 0.	N_2	0, 2, 3.	$N_2 \cdot N_3$
0, 0, 3.	N_3	1, 2, 3.	$N_2 \cdot \frac{N_3}{N_1}$



R
0
N2
N3
N1
g
1
g = 100.00000
N1 = 2.15832
N2 = 0.84415
N3 = 1.66498
R = 0.65119
1/N1 -R = -0.18787
N2·N3/N1 -R = 0.00000



1CST3R1

Unit.

$AC := 1$

Given.

$AN := 3$

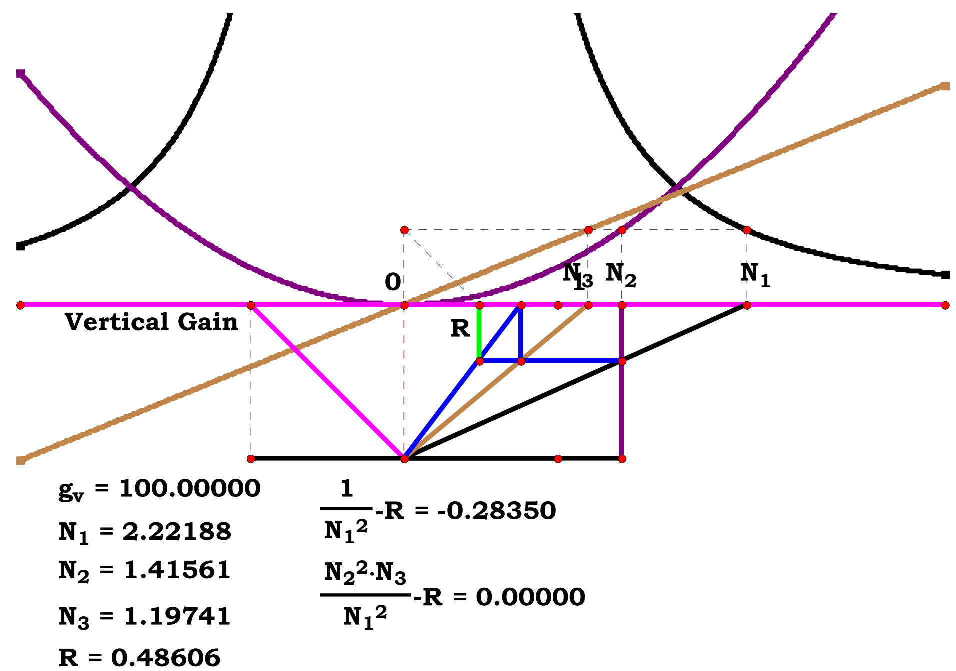
Descriptions.

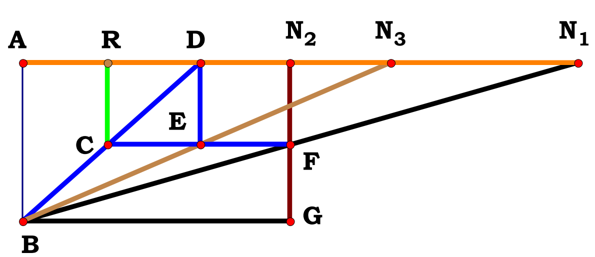
$$AB := \frac{1}{AN} \quad BC := \frac{AN-1}{AN} \quad BR := \frac{AB \cdot BC}{AC}$$

$$AR := AB - BR$$

Definitions.

$$AR - \frac{1}{AN^2} = 0 \quad AR - AN^{-2} = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$ab := 1 \quad ad := N_2 \cdot \frac{N_3}{N_1} \quad cr := \frac{N_1 - N_2}{N_1} \quad dr := \frac{ad \cdot cr}{ab} \quad ar := ad - dr$$

Definitions.

$$dr - \frac{N_2 \cdot N_3 \cdot N_1 - N_2^{\,2} \cdot N_3}{N_1^{\,2}} = 0 \qquad ar - \frac{N_2^{\,2} \cdot N_3}{N_1^{\,2}} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\frac{1}{N_1^{\,2}}$

0, 2, 0. $N_2^{\,2}$

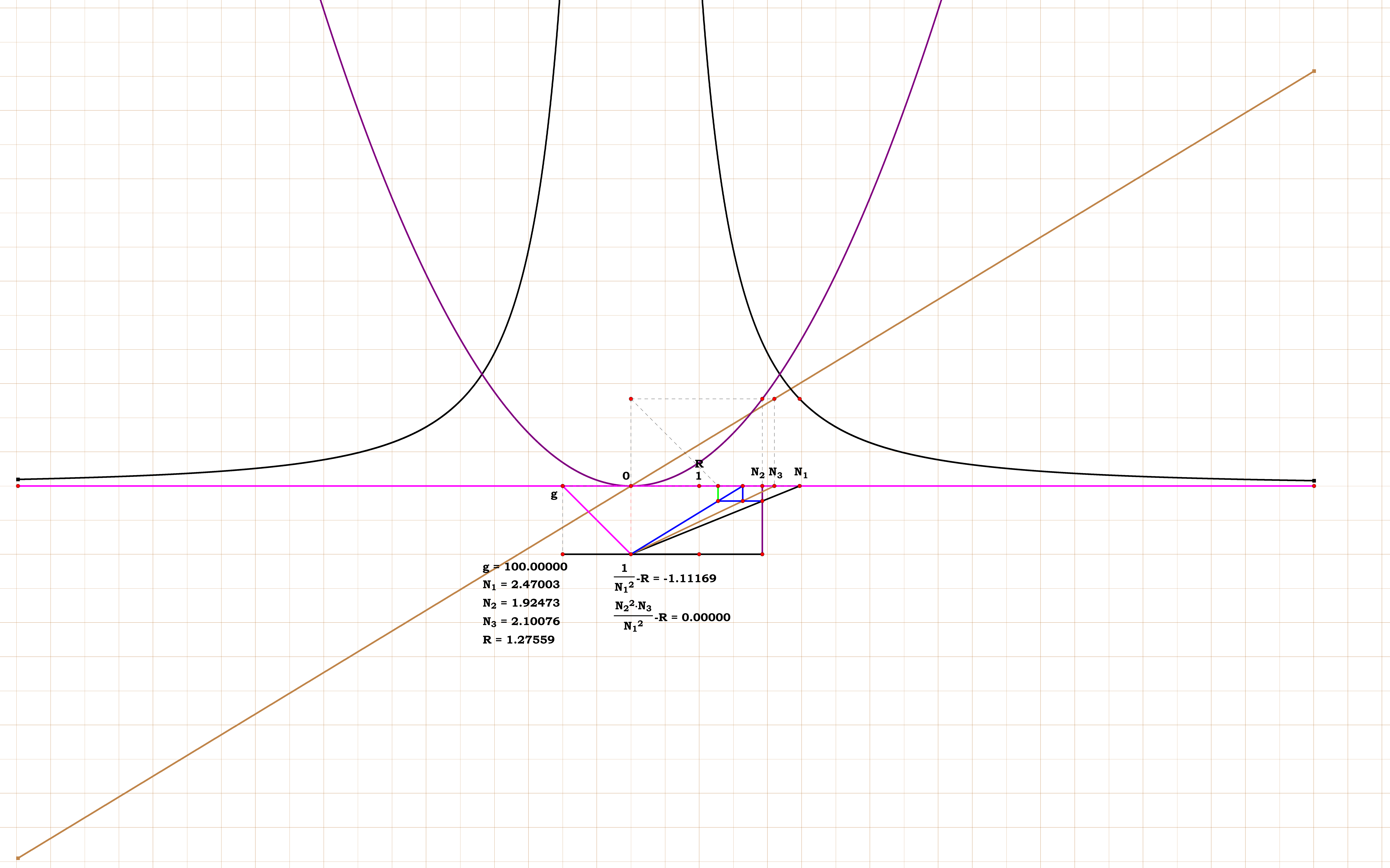
0, 0, 3. N_3

1, 2, 0. $\frac{N_2^{\,2}}{N_1^{\,2}}$

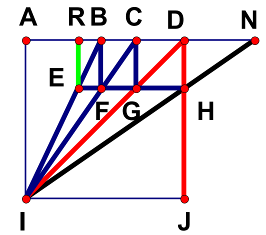
1, 0, 3. $\frac{N_3}{N_1^{\,2}}$

0, 2, 3. $N_2^{\,2} \cdot N_3$

1, 2, 3. $\frac{N_2^{\,2} \cdot N_3}{N_1^{\,2}}$



0
R
1
N2 N3 N1
g
g = 100.00000
N1 = 2.47003
N2 = 1.92473
N3 = 2.10076
R = 1.27559
1/N1² - R = -1.11169
N2²·N3/N1² - R = 0.00000



1CST3R2

Unit.

$AD := 1$

Given.

$AN := 2$

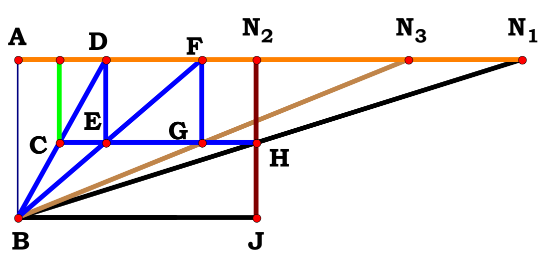
Descriptions.

$$DH := \frac{AN - 1}{AN} \qquad AB := \frac{1}{AN^2}$$

$$BR := \frac{AB \cdot DH}{AD} \qquad AR := AB - BR$$

Definitions.

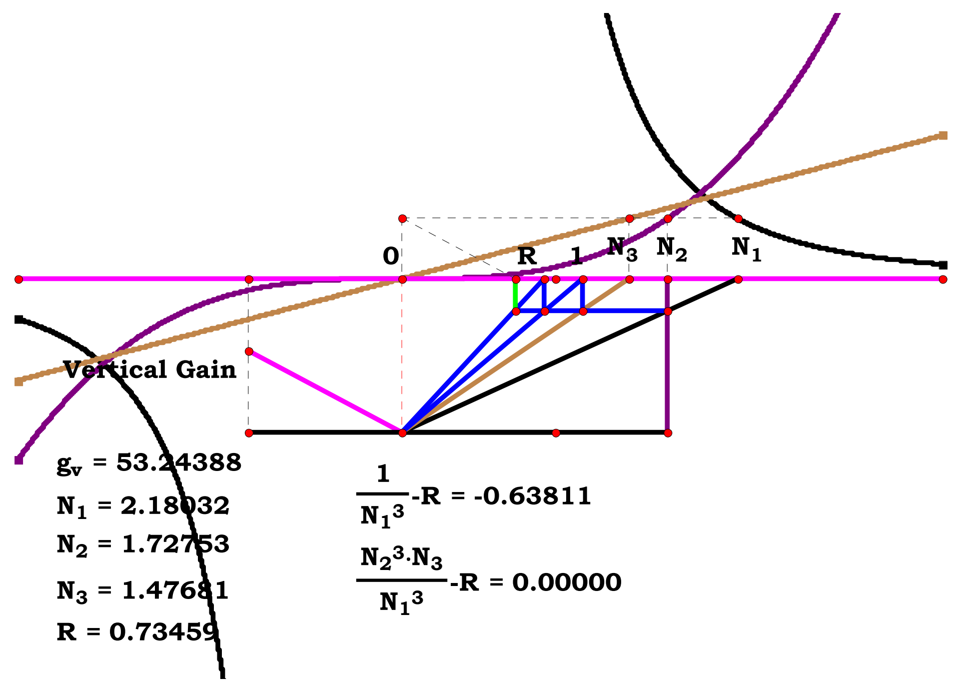
$$AR - \frac{1}{AN^3} = 0 \qquad AR - AN^{-3} = 0$$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$



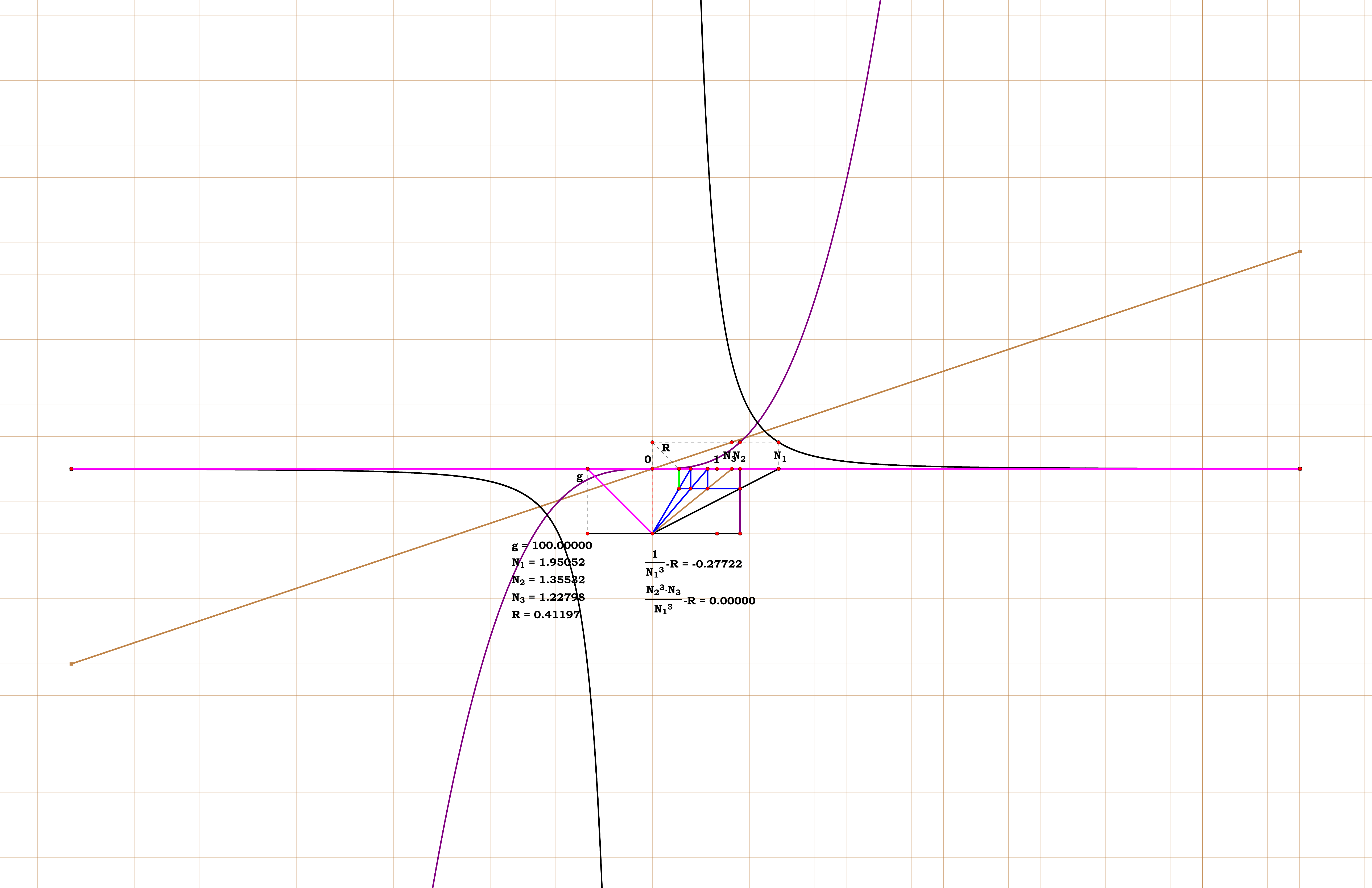
$$ab := 1 \qquad ad := \frac{N_2^{\,2} \cdot N_3}{N_1^{\,2}} \qquad cr := \frac{N_1 - N_2}{N_1} \qquad dr := \frac{ad \cdot cr}{ab} \qquad ar := ad - dr$$

Definitions.

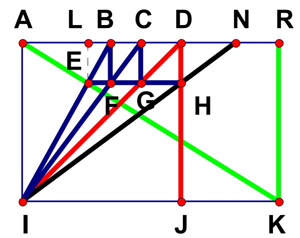
$$dr - \frac{N_2^{\,2} \cdot N_3 \cdot (N_1 - N_2)}{N_1^{\,3}} = 0 \qquad ar - \frac{N_2^{\,3} \cdot N_3}{N_1^{\,3}} = 0$$

Three Transforms.

0, 0, 0.	1	1, 2, 0.	$\dfrac{N_2^{\,3}}{N_1^{\,3}}$
1, 0, 0.	$\dfrac{1}{N_1^{\,3}}$	1, 0, 3.	$\dfrac{N_3}{N_1^{\,3}}$
0, 2, 0.	$N_2^{\,3}$	0, 2, 3.	$N_2^{\,3} \cdot N_3$
0, 0, 3.	N_3	1, 2, 3.	$\dfrac{N_2^{\,3} \cdot N_3}{N_1^{\,3}}$



R
0
1
N_3 N_2
N_1
g
g = 100.00000
N_1 = 1.95052
N_2 = 1.35532
N_3 = 1.22798
R = 0.41197
$\frac{1}{N_1^3}$-R = -0.27722
$\frac{N_2^3 \cdot N_3}{N_1^3}$-R = 0.00000



1CST3R3

Unit.

$AD := 1$

Given.

$AN := 3$

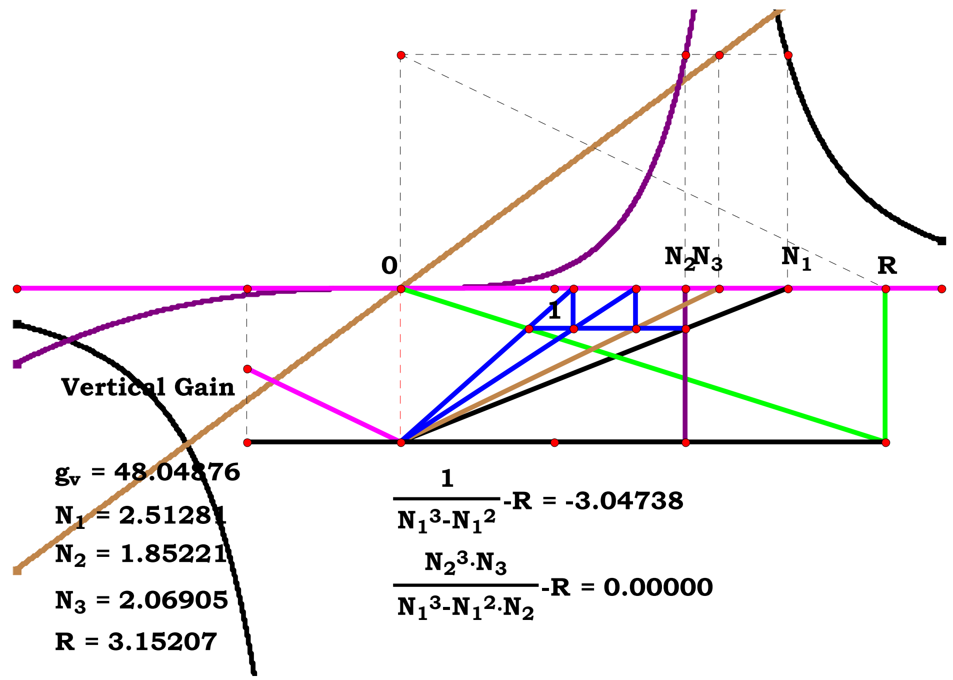
Descriptions.

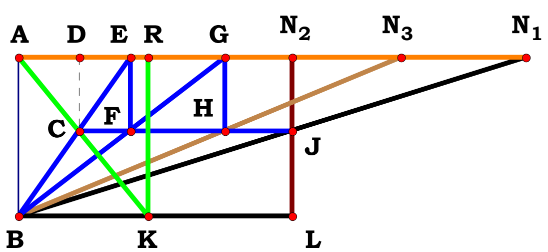
$$AL := \frac{1}{AN^3} \quad DH := \frac{AN-1}{AN} \quad EL := DH$$

$$IK := \frac{AL \cdot AD}{EL} \quad AR := IK$$

Definitions.

$$AR - \frac{1}{AN^3 - AN^2} = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

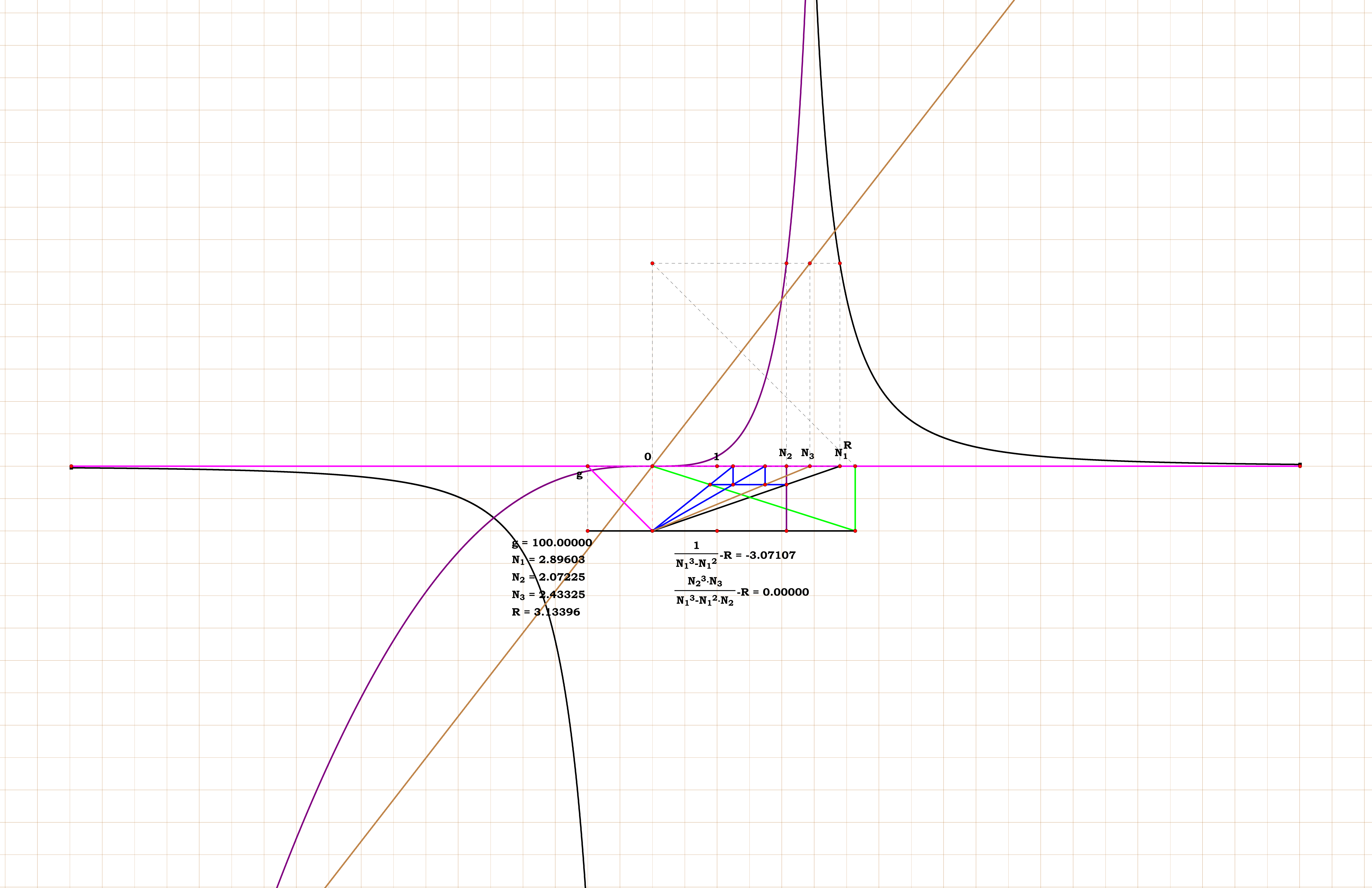
$$ab := 1 \quad ad := \frac{N_2^{\ 3} \cdot N_3}{N_1^{\ 3}} \quad cd := \frac{N_1 - N_2}{N_1} \quad ar := \frac{ad \cdot ab}{cd}$$

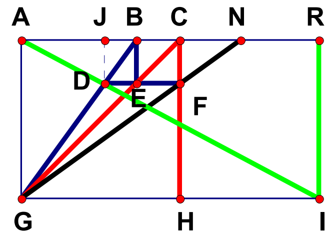
Definitions.

$$ar - \frac{N_2^{\ 3} \cdot N_3}{N_1^{\ 2} \cdot (N_1 - N_2)} = 0$$

Three Transforms.

0, 0, 0.	undefined	1, 2, 0.	$\frac{N_2^{\,3}}{N_1^{\,2}\cdot(N_1-N_2)}$
1, 0, 0.	$\frac{1}{N_1^{\,2}\cdot(N_1-1)}$	1, 0, 3.	$\frac{N_3}{N_1^{\,2}\cdot(N_1-1)}$
0, 2, 0.	$-\frac{N_2^{\,3}}{N_2-1}$	0, 2, 3.	$-\frac{N_2^{\,3}\cdot N_3}{N_2-1}$
0, 0, 3.	undefined	1, 2, 3.	$\frac{N_2^{\,3}\cdot N_3}{N_1^{\,2}\cdot(N_1-N_2)}$


0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 2.89603
N_2 = 2.07225
N_3 = 2.43325
R = 3.13396
$\frac{1}{N_1^3-N_1^2}$-R = -3.07107
$\frac{N_2^3 \cdot N_3}{N_1^3-N_1^2 \cdot N_2}$-R = 0.00000



1CST3R4

Unit.

$AC := 1$

Given.

$AN := 3$

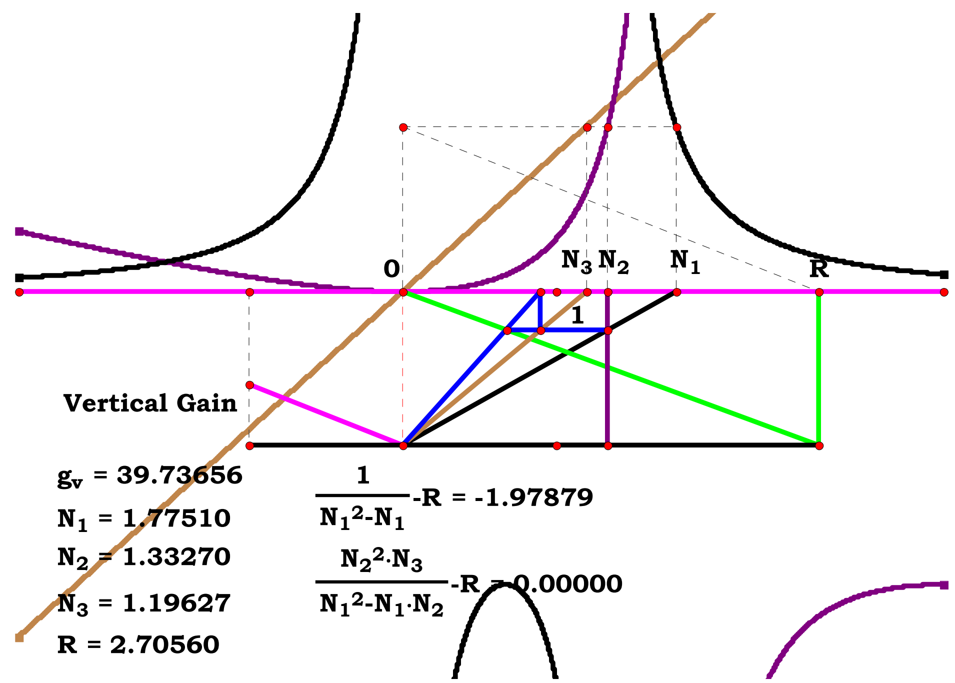
Descriptions.

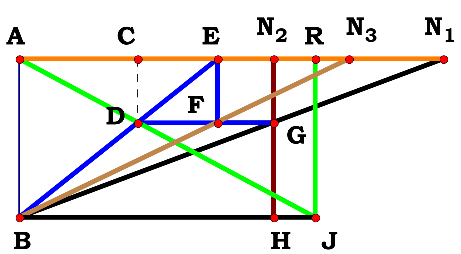
$$AJ := \frac{1}{AN^2} \quad CF := \frac{AN - 1}{AN} \quad DJ := CF$$

$$GI := \frac{AJ \cdot AC}{DJ} \quad AR := GI$$

Definitions.

$$AR - \frac{1}{AN^2 - AN} = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$ab := 1 \quad cd := \frac{N_1 - N_2}{N_1} \quad ac := \frac{{N_2}^2 \cdot N_3}{{N_1}^2} \quad ar := \frac{ac \cdot ab}{cd}$$

Definitions.

$$ar - \frac{{N_2}^2 \cdot N_3}{N_1 \cdot (N_1 - N_2)} = 0$$

Three Transforms.

0, 0, 0. undefined

1, 0, 0. $$\frac{1}{N_1 \cdot (N_1 - 1)}$$

0, 2, 0. $$-\frac{{N_2}^2}{N_2 - 1}$$

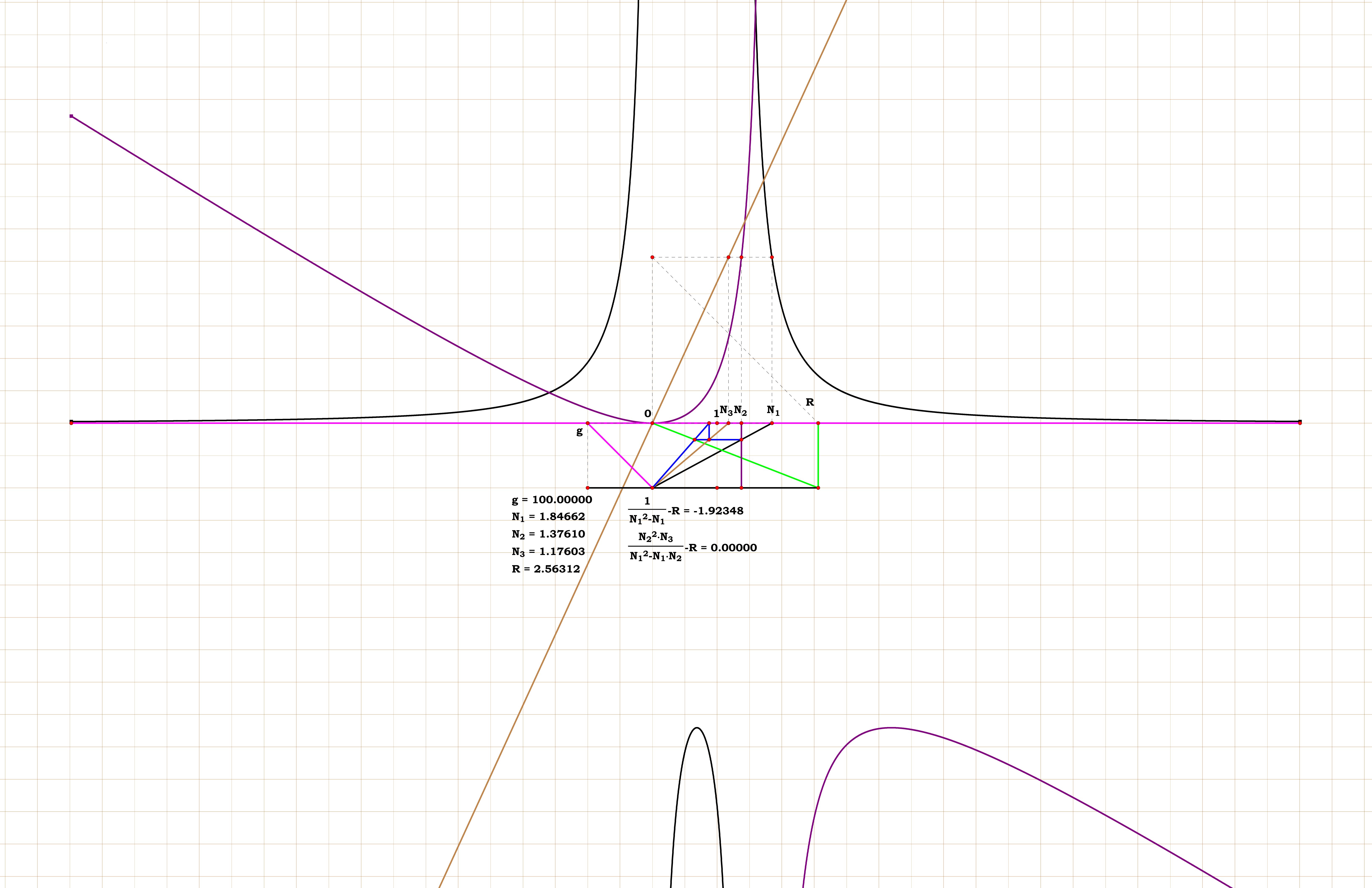
0, 0, 3. undefined

1, 2, 0. $$\frac{{N_2}^2}{{N_1}^2 - N_1 \cdot N_2}$$

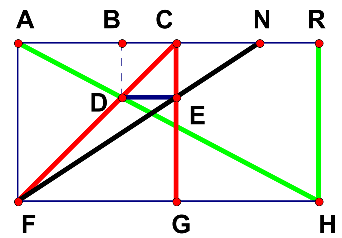
1, 0, 3. $$\frac{N_3}{N_1 \cdot (N_1 - 1)}$$

0, 2, 3. $$-\frac{{N_2}^2 \cdot N_3}{N_2 - 1}$$

1, 2, 3. $$\frac{{N_2}^2 \cdot N_3}{N_1 \cdot (N_1 - N_2)}$$



0
1
N_3
N_2
N_1
R
g
g = 100.00000
N_1 = 1.84662
N_2 = 1.37610
N_3 = 1.17603
R = 2.56312
$\frac{1}{N_1^2 - N_1} - R = -1.92348$
$\frac{N_2^2 \cdot N_3}{N_1^2 - N_1 \cdot N_2} - R = 0.00000$



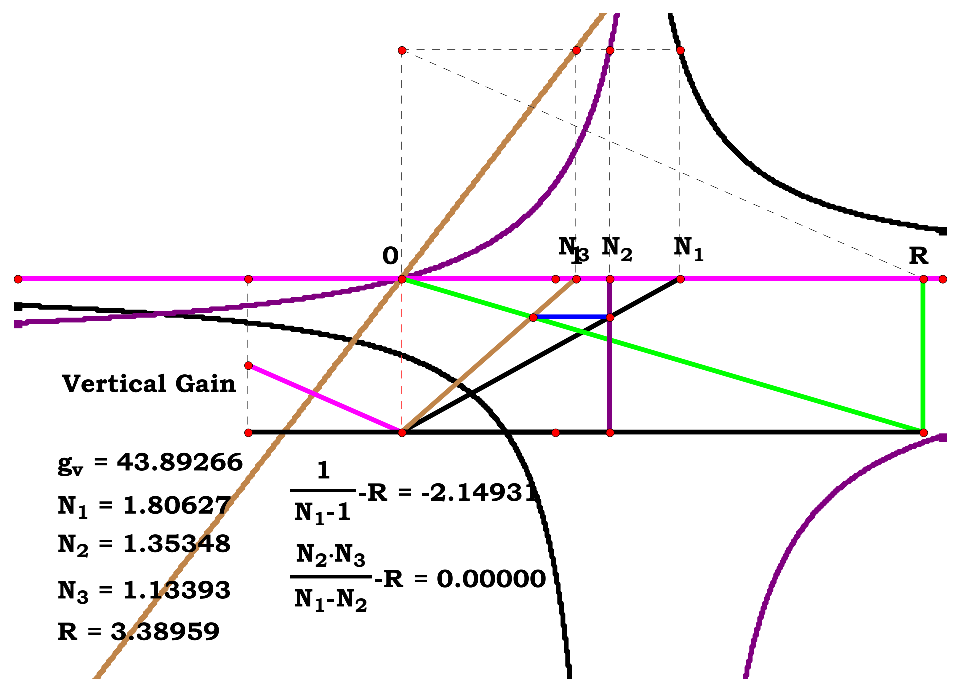
1CST3R5

Unit.

$AC := 1$

Given.

$AN := 3$



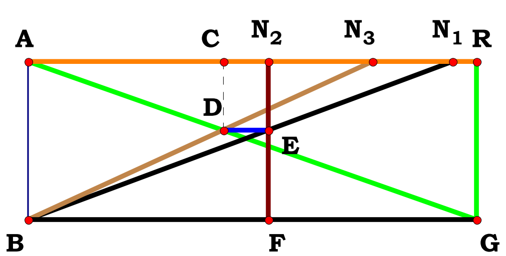
Descriptions.

$$AB := \frac{1}{AN} \qquad BD := \frac{AN-1}{AN}$$

$$FH := \frac{AB \cdot AC}{BD} \qquad AR := FH$$

Definitions.

$$AR - \frac{1}{AN-1} = 0$$



$$N_1 := 5$$

$$N_2 := 3$$

$$N_3 := 2$$

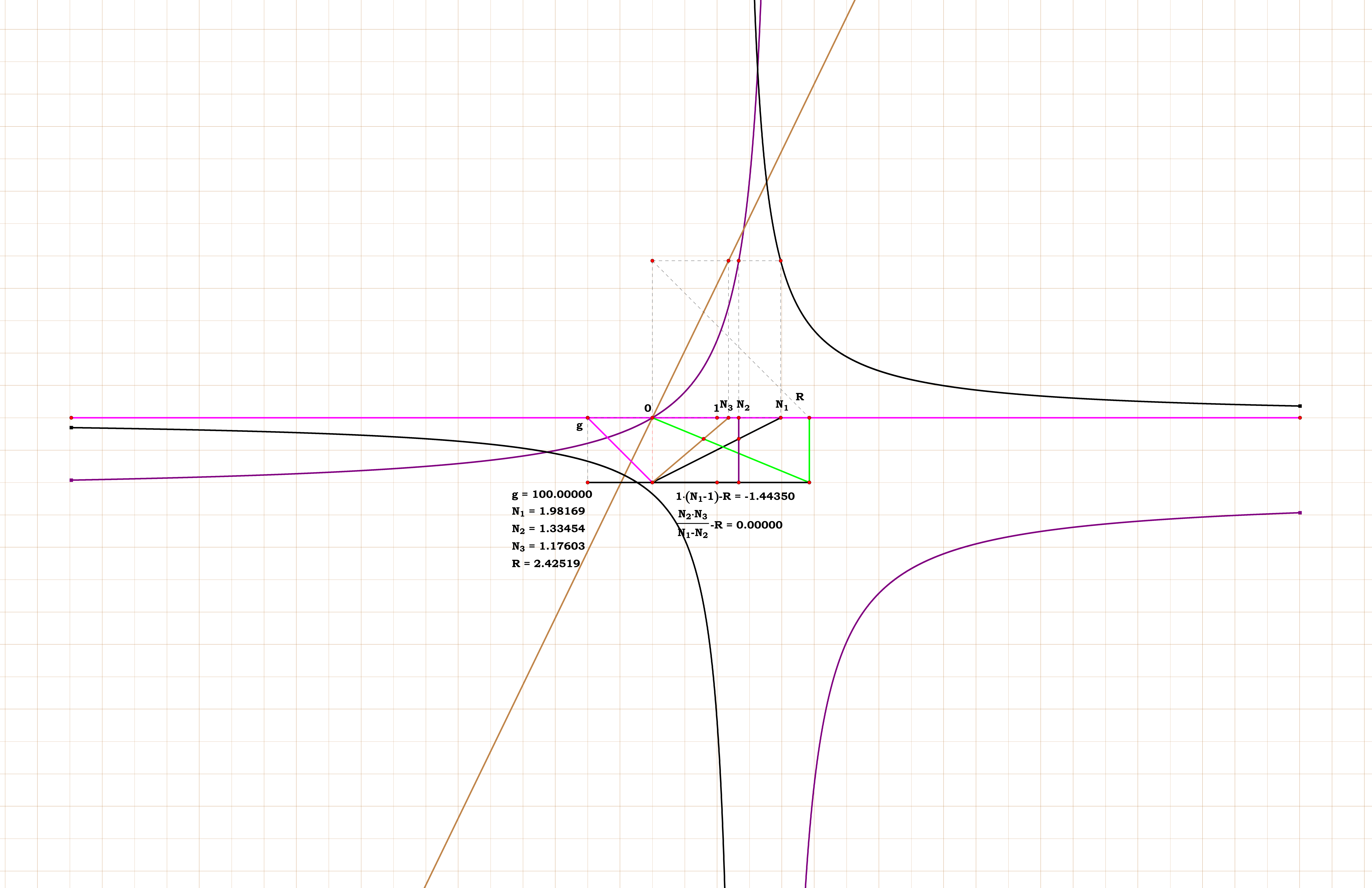
$$ab := 1 \qquad ac := \frac{N_2 \cdot N_3}{N_1} \qquad cd := \frac{N_1 - N_2}{N_1} \qquad ar := \frac{ac}{cd}$$

Definitions.

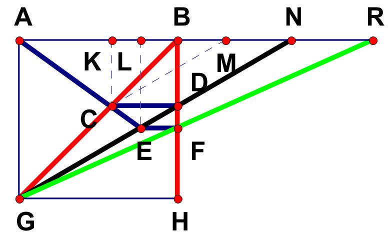
$$ar - \frac{N_2 \cdot N_3}{N_1 - N_2} = 0$$

Three Transforms.

0, 0, 0.	**undefined**	**1, 2, 0.**	$\frac{N_2}{N_1 - N_2}$
1, 0, 0.	$\frac{1}{N_1 - 1}$	**1, 0, 3.**	$\frac{N_3}{N_1 - 1}$
0, 2, 0.	$-\frac{N_2}{N_2 - 1}$	**0, 2, 3.**	$-\frac{N_2 \cdot N_3}{N_2 - 1}$
0, 0, 3.	**undefined**	**1, 2, 3.**	$\frac{N_2 \cdot N_3}{N_1 - N_2}$



0
1
N_3
N_2
N_1
R
g
g = 100.00000
N_1 = 1.98169
N_2 = 1.33454
N_3 = 1.17603
R = 2.42519
$1\cdot(N_1-1)-R = -1.44350$
$\frac{N_2\cdot N_3}{N_1-N_2}-R = 0.00000$



1CST3R6

Unit.

$AB := 1$

Given.

$AN := 2$

Descriptions.

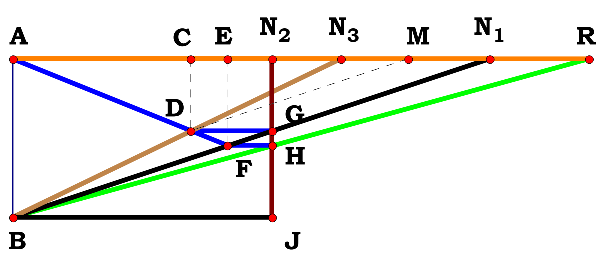
$$AK := \frac{1}{AN} \quad KM := AN - AB \quad AM := AK + KM$$

$$AL := \frac{AK \cdot AN}{AM} \quad LN := AN - AL \quad EL := \frac{AB \cdot LN}{AN}$$

$$BF := EL \quad AR := \frac{AB^2}{AB - BF}$$

Definitions.

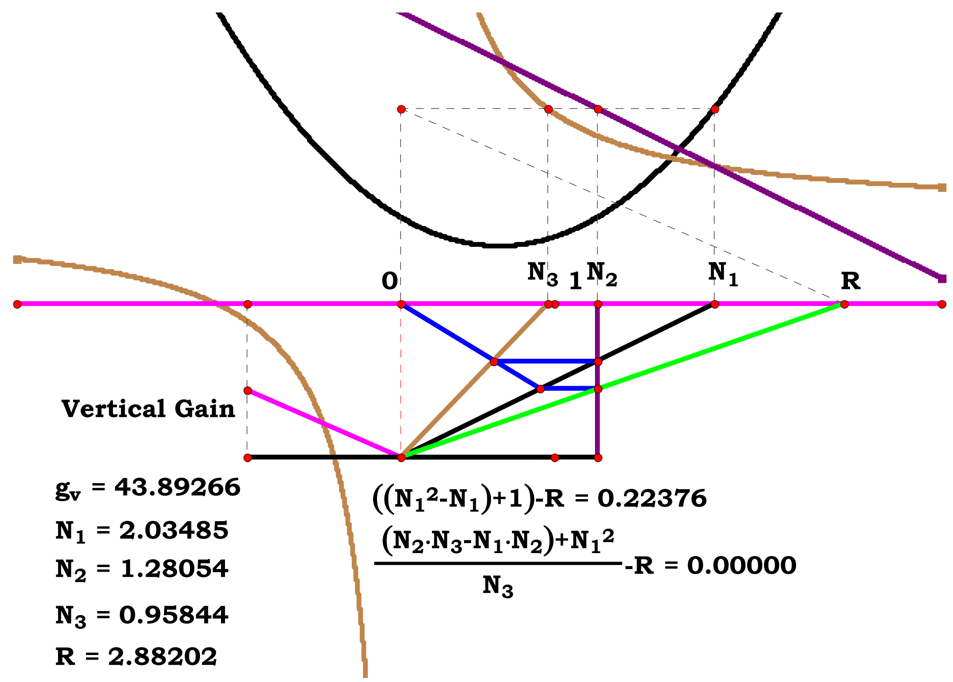
$$AR - \left(AN^2 - AN + 1\right) = 0$$



$$N_1 := 5$$

$$N_2 := 3$$

$$N_3 := 2$$



$$ab := 1 \quad cd := \frac{N_1 - N_2}{N_1} \quad ac := N_2 \cdot \frac{N_3}{N_1} \quad bk := \frac{ac \cdot ab}{cd} \quad ef := \frac{ab \cdot bk}{bk + N_1}$$

$$hj := ef \quad ar := \frac{N_2 \cdot ab}{hj}$$

Definitions.

$$bk - \frac{N_2 \cdot N_3}{N_1 - N_2} = 0 \qquad ef - \frac{N_2 \cdot N_3}{N_1^2 - N_2 \cdot N_1 + N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1^2 - N_2 \cdot N_1 + N_2 \cdot N_3}{N_3} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $N_1^{\,2} - N_1 + 1$

0, 2, 0. 1

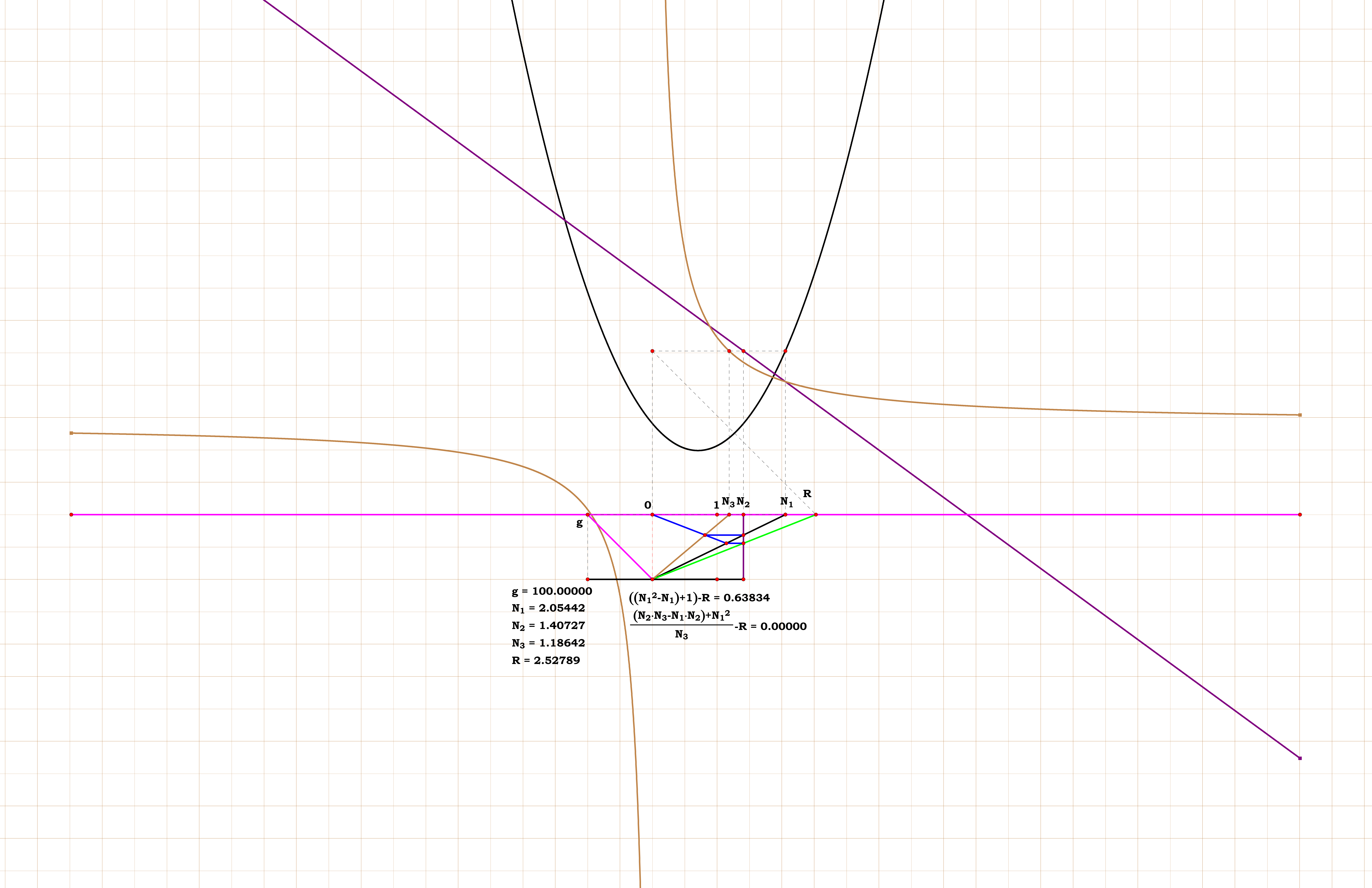
0, 0, 3. 1

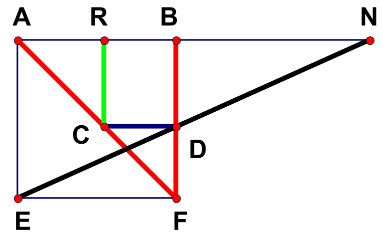
1, 2, 0. $N_1^{\,2} - N_2 \cdot N_1 + N_2$

1, 0, 3. $\dfrac{N_1^{\,2} - N_1 + N_3}{N_3}$

0, 2, 3. $\dfrac{N_2 \cdot N_3 - N_2 + 1}{N_3}$

1, 2, 3. $\dfrac{N_2 \cdot N_3 - N_1 \cdot N_2 + N_1^{\,2}}{N_3}$


0
1
N_3 N_2
N_1
R
g
g = 100.00000
N_1 = 2.05442
N_2 = 1.40727
N_3 = 1.18642
R = 2.52789
$((N_1^2 - N_1) + 1) - R = 0.63834$
$\dfrac{(N_2 \cdot N_3 - N_1 \cdot N_2) + N_1^2}{N_3} - R = 0.00000$



1CST4R0

Unit.

$AB := 1$

Given.

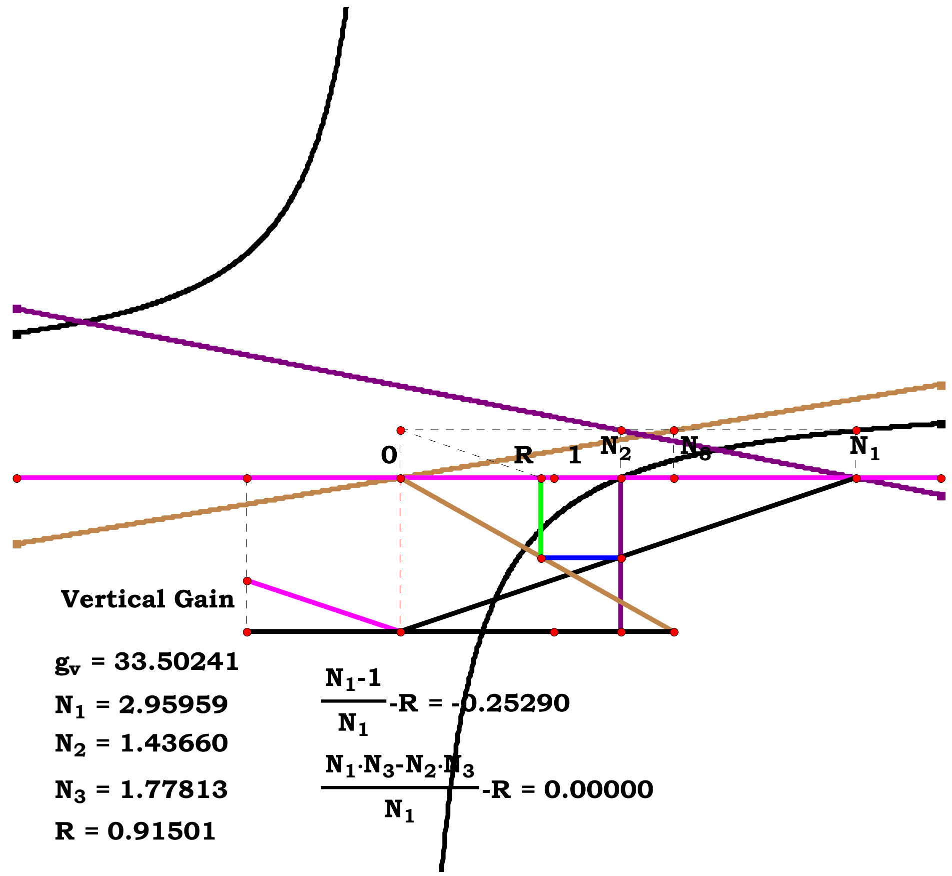
$AN := 3$

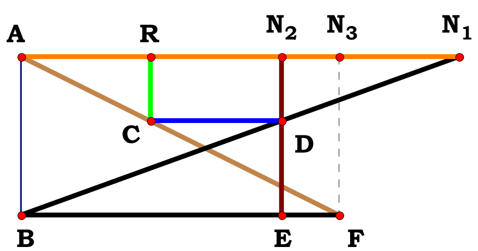
Descriptions.

$$BD := \frac{AN-1}{AN} \quad AR := BD$$

Definitions.

$$AR - \frac{AN-1}{AN} = 0$$





$$N_1 := 4$$

$$N_2 := 3$$

$$N_3 := 2$$

$$ab := 1 \quad dn := 1 - \frac{N_2}{N_1} \quad ar := \frac{N_3 \cdot dn}{ab}$$

Definitions.

$$dn - \frac{N_1 - N_2}{N_1} = 0 \quad ar - \frac{N_3 \cdot (N_1 - N_2)}{N_1} = 0$$

Three Transforms.

0, 0, 0. 0

1, 0, 0. $\frac{N_1 - 1}{N_1}$

0, 2, 0. $1 - N_2$

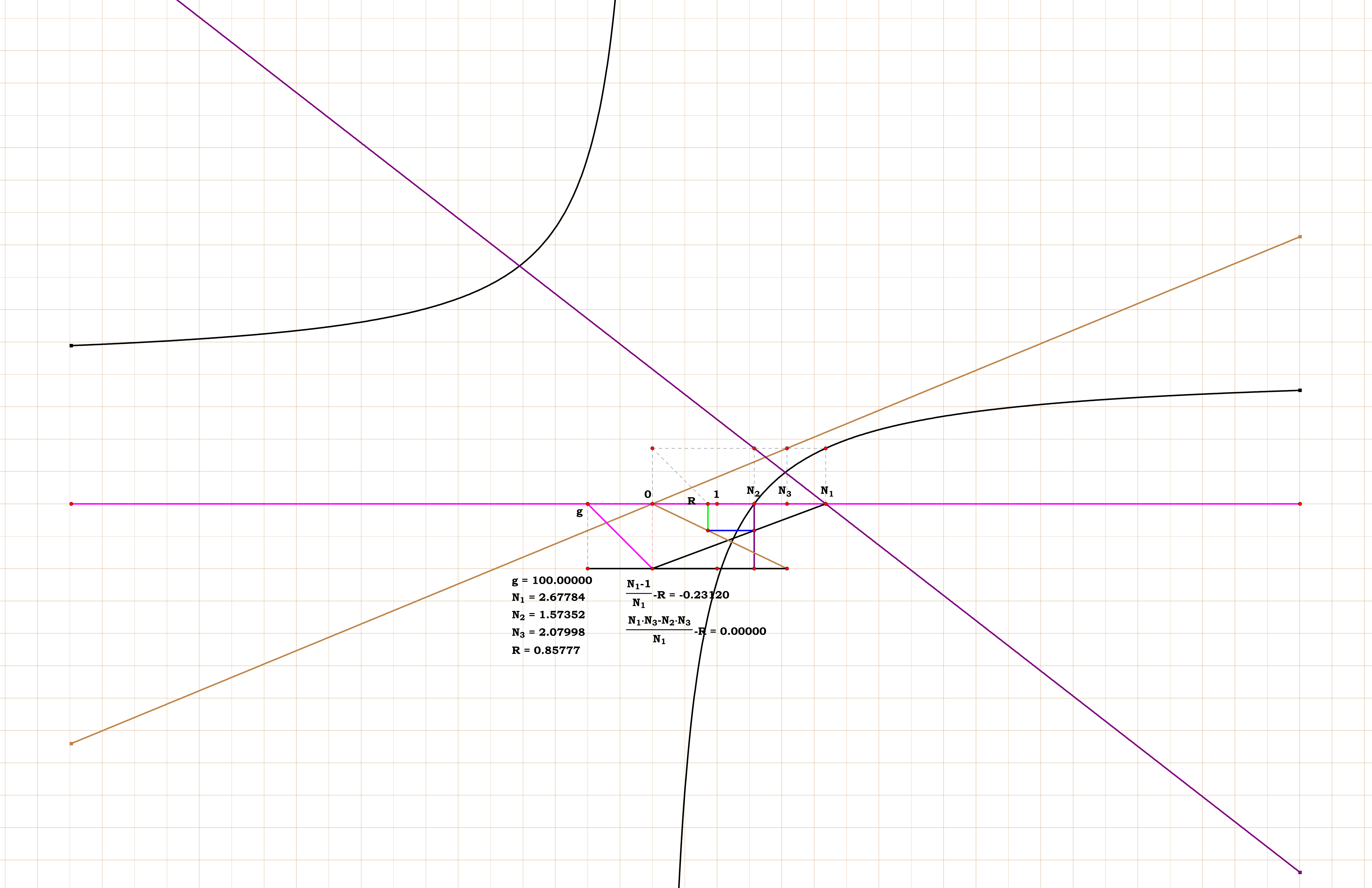
0, 0, 3. 0

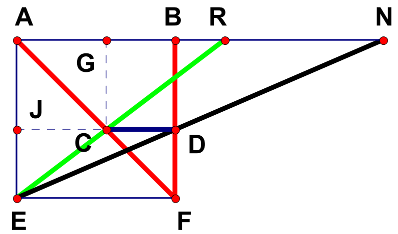
1, 2, 0. $\frac{N_1 - N_2}{N_1}$

1, 0, 3. $\frac{N_1 \cdot N_3 - N_3}{N_1}$

0, 2, 3. $N_3 - N_2 \cdot N_3$

1, 2, 3. $\frac{N_3 \cdot (N_1 - N_2)}{N_1}$


0
R
1
N_2
N_3
N_1
g
g = 100.00000
N_1 = 2.67784
N_2 = 1.57352
N_3 = 2.07998
R = 0.85777
$\frac{N_1-1}{N_1}-R = -0.23120$
$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1}-R = 0.00000$



1CST4R1

Unit.

$AB := 1$

Given.

$AN := 3$

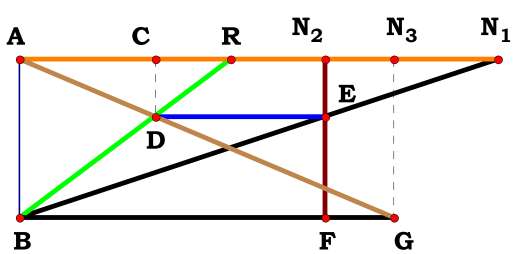
Descriptions.

$$BD := \frac{AN - 1}{AN} \qquad CJ := BD$$

$$EJ := AB - BD \qquad AR := \frac{CJ \cdot AB}{EJ}$$

Definitions.

$$AR - (AN - 1) = 0$$

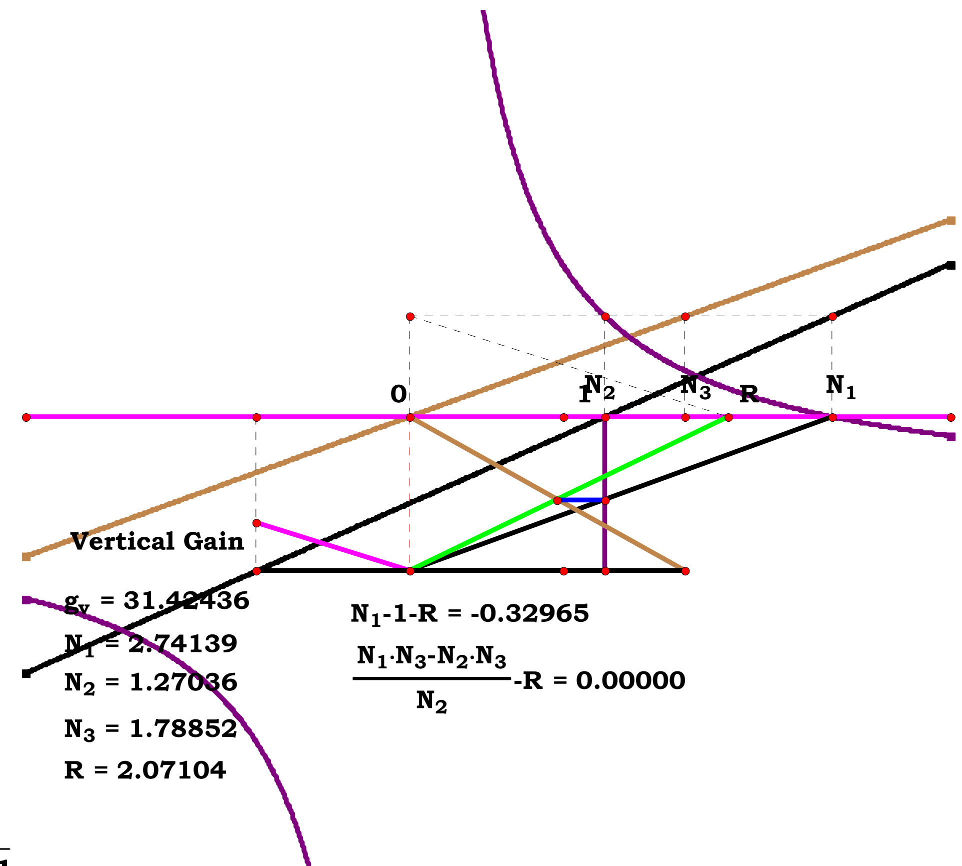


$N_1 := 4$

$N_2 := 3$

$N_3 := 2$

$$ab := 1 \qquad cd := \frac{N_1 - N_2}{N_1} \qquad ac := \frac{N_3 \cdot (N_1 - N_2)}{N_1} \qquad ar := \frac{ac}{ab - cd}$$



Definitions.

$$ar - \frac{N_3 \cdot (N_1 - N_2)}{N_2} = 0$$

Three Transforms.

0, 0, 0. 0

1, 0, 0. $N_1 - 1$

0, 2, 0. $-\dfrac{N_2 - 1}{N_2}$

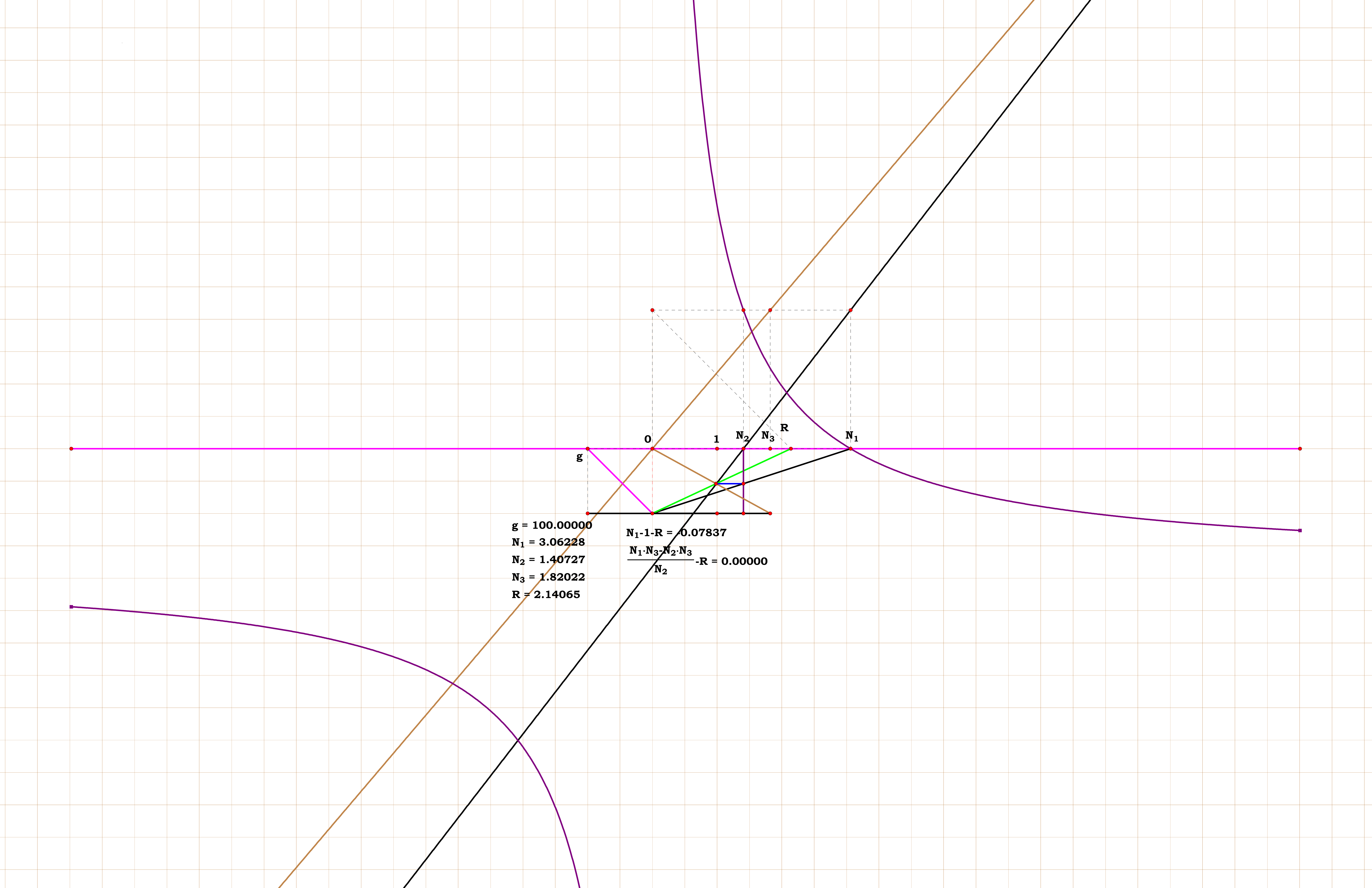
0, 0, 3. 0

1, 2, 0. $\dfrac{N_1 - N_2}{N_2}$

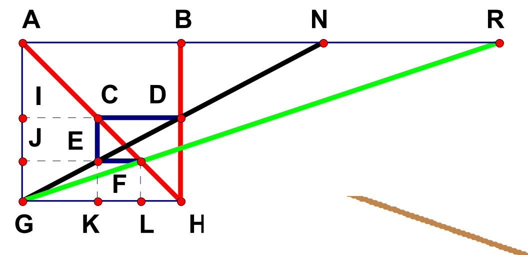
1, 0, 3. $N_3 \cdot (N_1 - 1)$

0, 2, 3. $-\dfrac{N_3 \cdot (N_2 - 1)}{N_2}$

1, 2, 3. $\dfrac{N_3 \cdot (N_1 - N_2)}{N_2}$



0
1
N2
N3
R
N1
g
g = 100.00000
N1 = 3.06228
N2 = 1.40727
N3 = 1.82022
R = 2.14065
N1-1-R = -0.07837
(N1·N3-N2·N3)/N2 -R = 0.00000



1CST4R2

Unit.

$AB := 1$

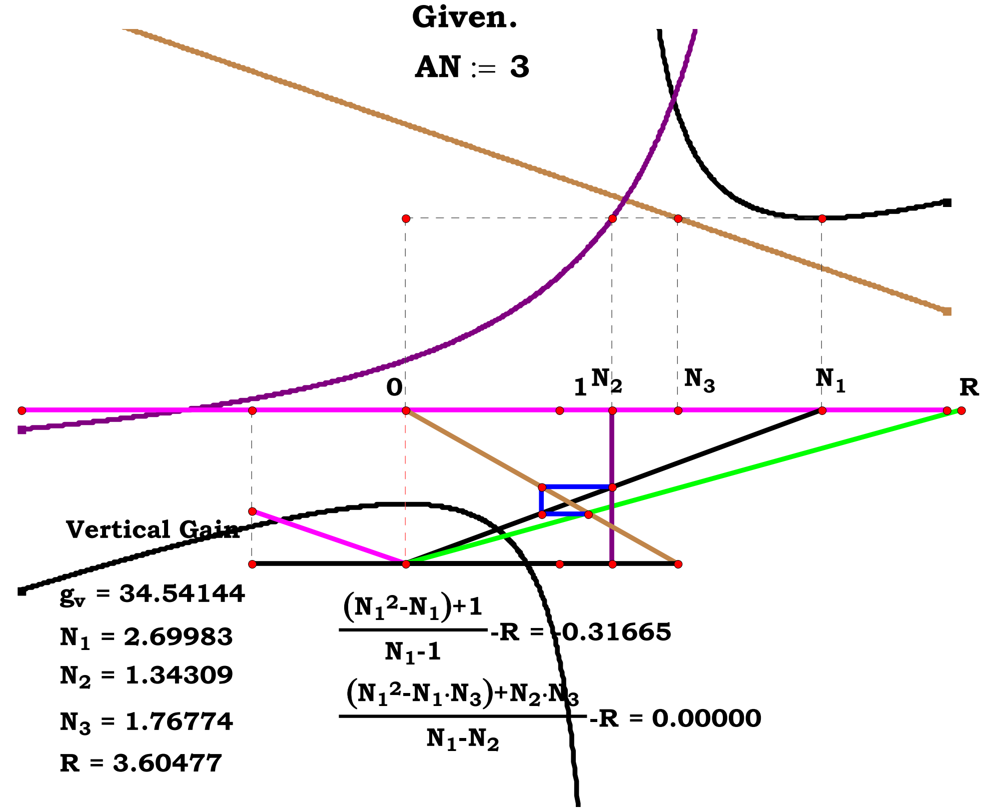
Given.

$AN := 3$

Descriptions.

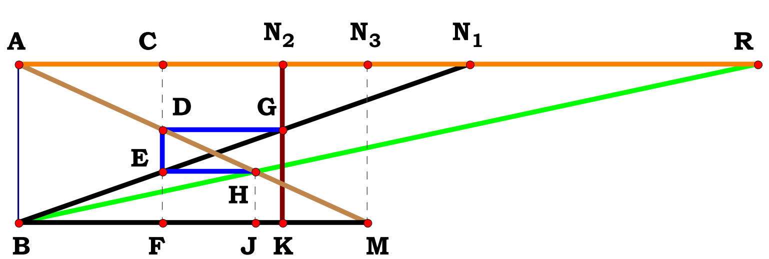
$BD := \frac{AN - 1}{AN} \quad EK := \frac{AB \cdot BD}{AN}$

$GL := AB - EK \quad AR := \frac{GL \cdot AB}{EK}$



Definitions.

$$AR - \frac{AN^2 - AN + 1}{AN - 1} = 0$$



$N_1 := 2.94425$

$N_2 := 1.30995$

$N_3 := 1.4746$

$ab := 1 \quad gk := \frac{N_2}{N_1} \quad fm := N_3 \cdot gk \quad bf := N_3 - fm \quad ef := \frac{bf}{N_1} \quad jm := N_3 \cdot ef \quad bj := N_3 - jm \quad ar := \frac{bj}{ef}$

Definitions.

$$fm - \frac{N_2 \cdot N_3}{N_1} = 0 \quad bf - \frac{N_3 \cdot (N_1 - N_2)}{N_1} = 0 \quad ef - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1^2} = 0 \quad jm - \frac{N_3^2 \cdot (N_1 - N_2)}{N_1^2} = 0 \quad bj - \frac{N_3 \cdot (N_1^2 - N_3 \cdot N_1 + N_2 \cdot N_3)}{N_1^2} = 0 \quad ar - \frac{N_1^2 - N_1 \cdot N_3 + N_2 \cdot N_3}{N_1 - N_2} = 0$$

Three Transforms.

0, 0, 0. undefined

1, 0, 0. $\dfrac{N_1^{\,2} - N_1 + 1}{N_1 - 1}$

0, 2, 0. $-\dfrac{N_2}{N_2 - 1}$

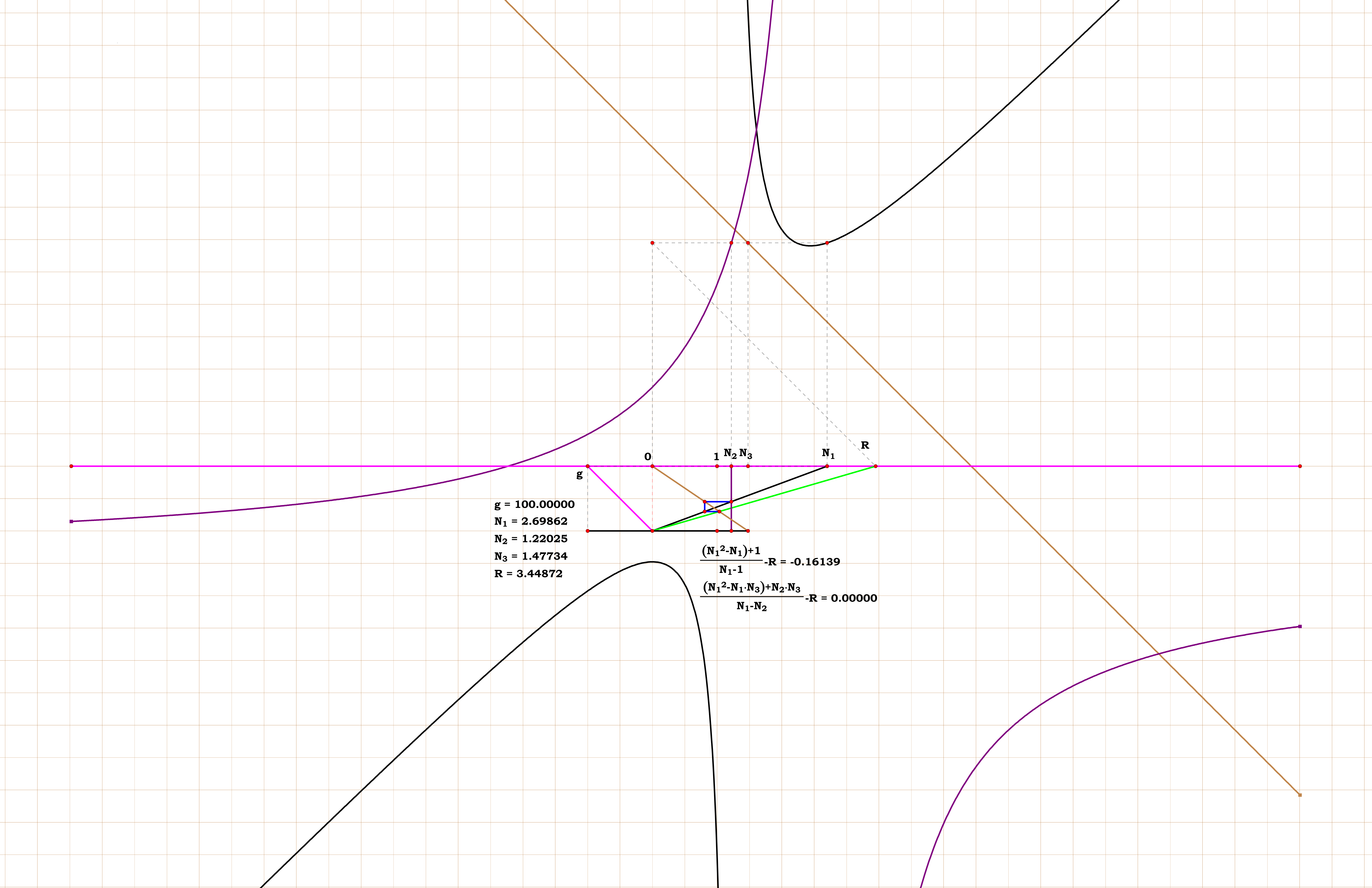
0, 0, 3. undefined

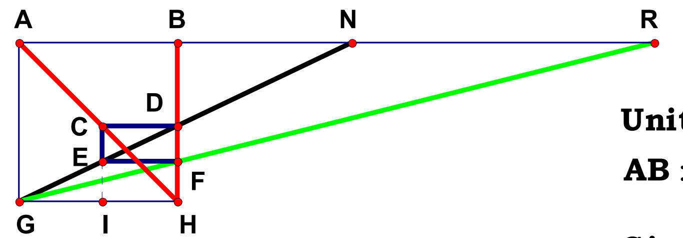
1, 2, 0. $\dfrac{N_1^{\,2} - N_1 + N_2}{N_1 - N_2}$

1, 0, 3. $\dfrac{N_1^{\,2} - N_3 \cdot N_1 + N_3}{N_1 - 1}$

0, 2, 3. $\dfrac{N_3 - N_2 \cdot N_3 - 1}{N_2 - 1}$

1, 2, 3. $\dfrac{N_1^{\,2} - N_1 \cdot N_3 + N_2 \cdot N_3}{N_1 - N_2}$


0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 2.69862
N_2 = 1.22025
N_3 = 1.47734
R = 3.44872
$\frac{(N_1^2-N_1)+1}{N_1-1}$-R = -0.16139
$\frac{(N_1^2-N_1 \cdot N_3)+N_2 \cdot N_3}{N_1-N_2}$-R = 0.00000



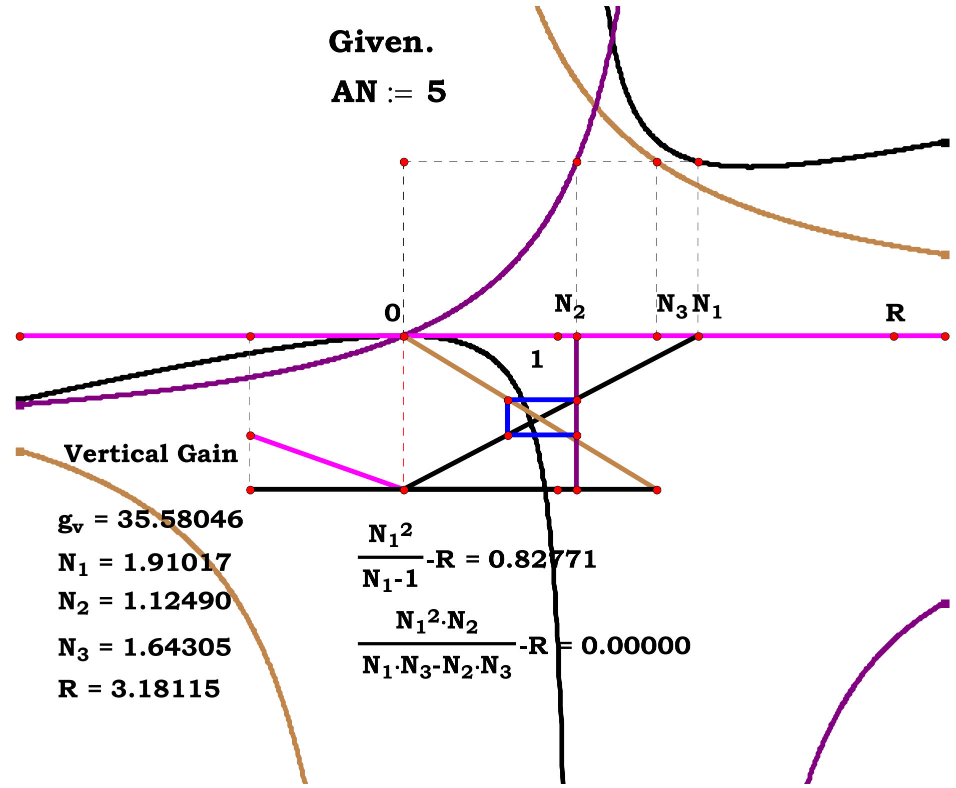
1CST4R3

Unit.

$AB := 1$

Given.

$AN := 5$



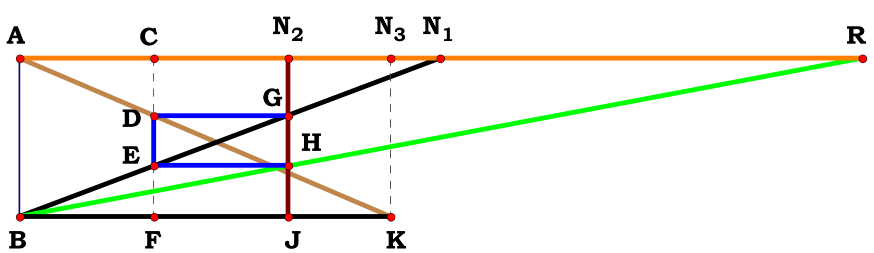
Descriptions.

$$BD := \frac{AN - 1}{AN} \quad EI := \frac{AB \cdot BD}{AN}$$

$$AR := \frac{AB^2}{EI}$$

Definitions.

$$AR - \frac{AN^2}{AN - 1} = 0$$



$N_1 := 2.5665$

$N_2 := 1.3631$

$N_3 := 2$

$$gj := \frac{N_2}{N_1} \quad fk := N_3 \cdot gj \quad bf := N_3 - fk \quad ef := \frac{bf}{N_1} \quad ar := \frac{N_2}{ef}$$

Definitions.

$$ar - \frac{{N_1}^2 \cdot N_2}{N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. undefined

1, 0, 0. $\dfrac{N_1^{\,2}}{N_1 - 1}$

0, 2, 0. $-\dfrac{N_2}{N_2 - 1}$

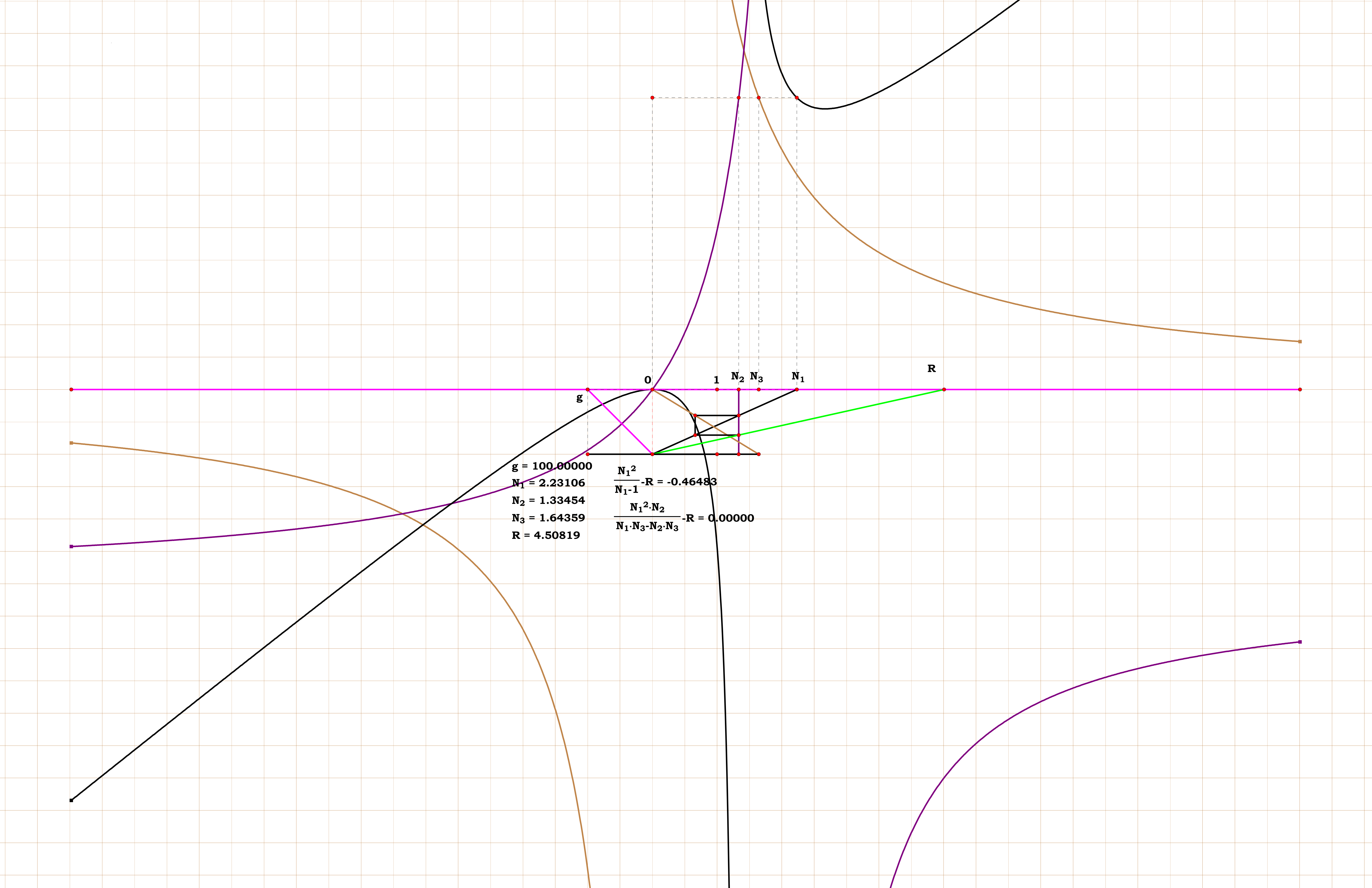
0, 0, 3. undefined

1, 2, 0. $\dfrac{N_1^{\,2} \cdot N_2}{N_1 - N_2}$

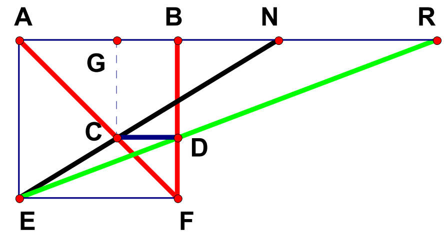
1, 0, 3. $-\dfrac{N_1^{\,2}}{N_3 - N_1 \cdot N_3}$

0, 2, 3. $\dfrac{N_2}{N_3 - N_2 \cdot N_3}$

1, 2, 3. $\dfrac{N_1^{\,2} \cdot N_2}{N_1 \cdot N_3 - N_2 \cdot N_3}$



0
1
N2
N3
N1
R
g
g = 100.00000
N1 = 2.23106
N2 = 1.33454
N3 = 1.64359
R = 4.50819
N1^2/(N1-1) - R = -0.46483
N1^2·N2/(N1·N3-N2·N3) - R = 0.00000



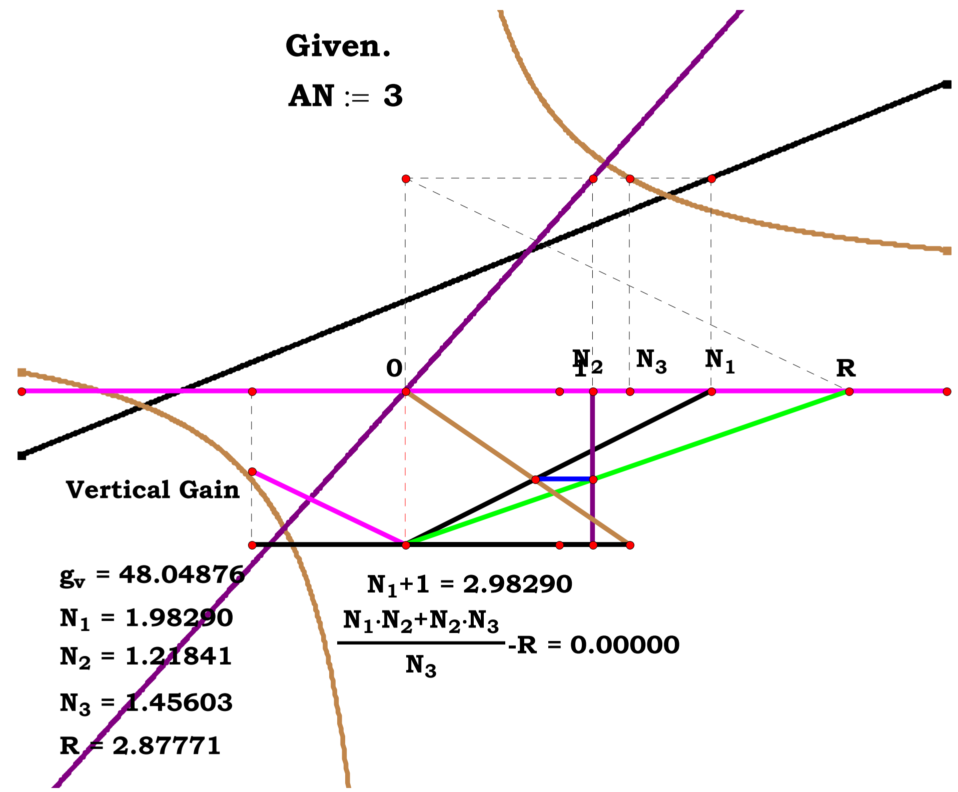
1CST4R4

Unit.

$$AB := 1$$

Given.

$$AN := 3$$



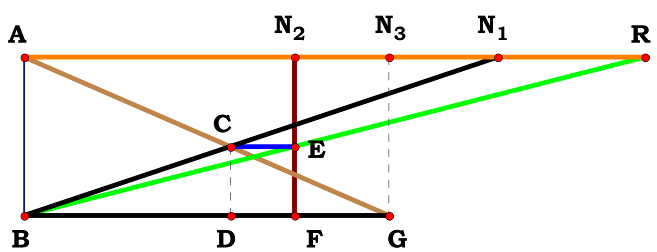
Descriptions.

$$AG := \frac{AB \cdot AN}{AB + AN} \qquad BD := AG$$

$$AR := \frac{AB^2}{AB - BD}$$

Definitions.

$$AR - (AN + 1) = 0$$



$$N_1 := 4$$

$$N_2 := 3$$

$$N_3 := 2$$

$$cd := \frac{N_3}{N_1 + N_3} \qquad ar := \frac{N_2}{cd}$$

Definitions.

$$ar - \frac{N_2 \cdot (N_1 + N_3)}{N_3} = 0$$

Three Transforms.

0, 0, 0. 2

1, 0, 0. $N_1 + 1$

0, 2, 0. $2 \cdot N_2$

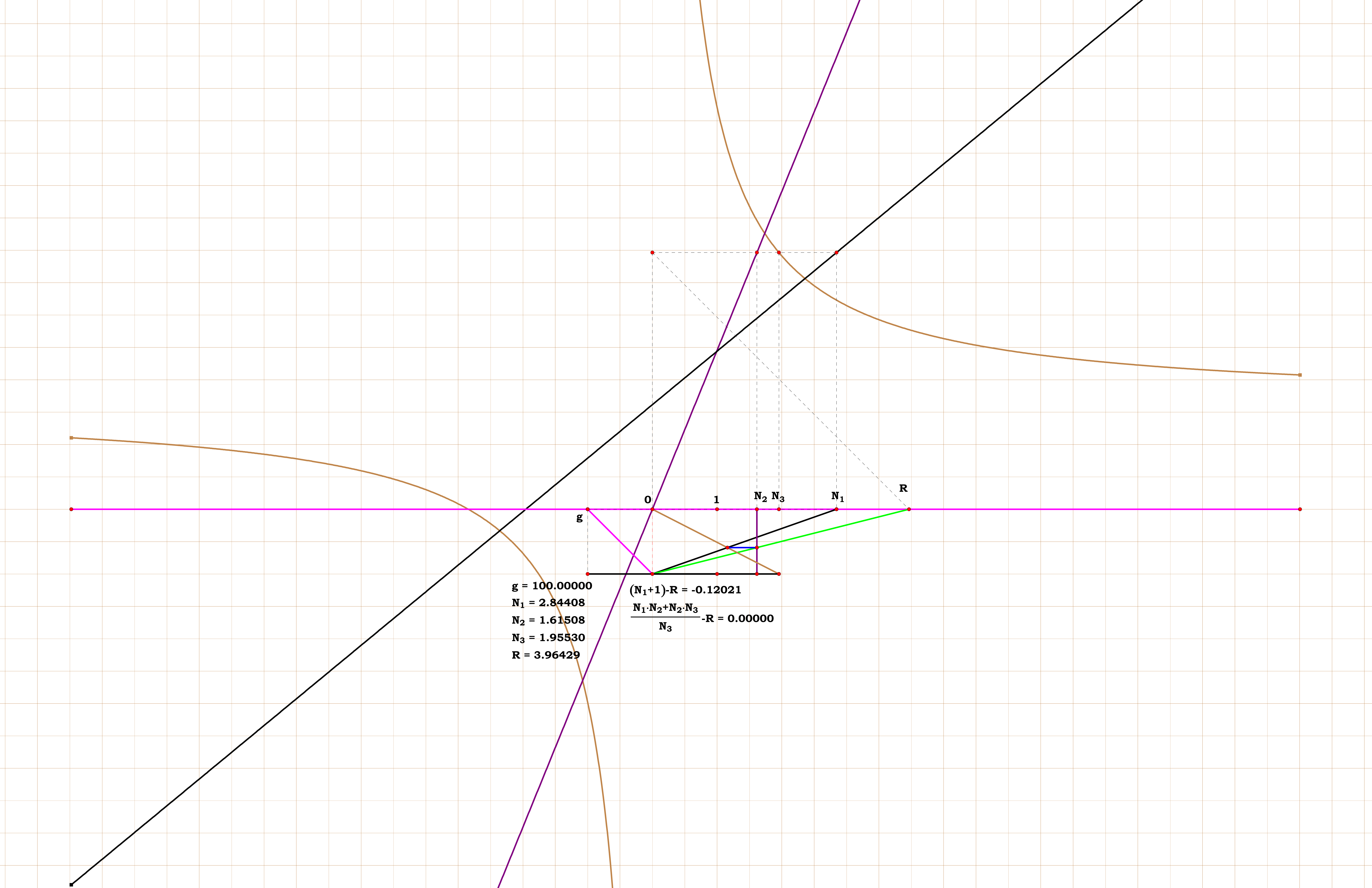
0, 0, 3. $\dfrac{N_3 + 1}{N_3}$

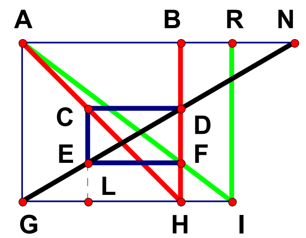
1, 2, 0. $N_2 \cdot \left(N_1 + 1\right)$

1, 0, 3. $\dfrac{N_1 + N_3}{N_3}$

0, 2, 3. $\dfrac{N_2 \cdot \left(N_3 + 1\right)}{N_3}$

1, 2, 3. $\dfrac{N_2 \cdot \left(N_1 + N_3\right)}{N_3}$


0
1
N_2 N_3
N_1
R
g
g = 100.00000
N_1 = 2.84408
N_2 = 1.61508
N_3 = 1.95530
R = 3.96429
(N_1+1)-R = -0.12021
$\frac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_3}$-R = 0.00000



1CST4R5

Unit.

$AB := 1$

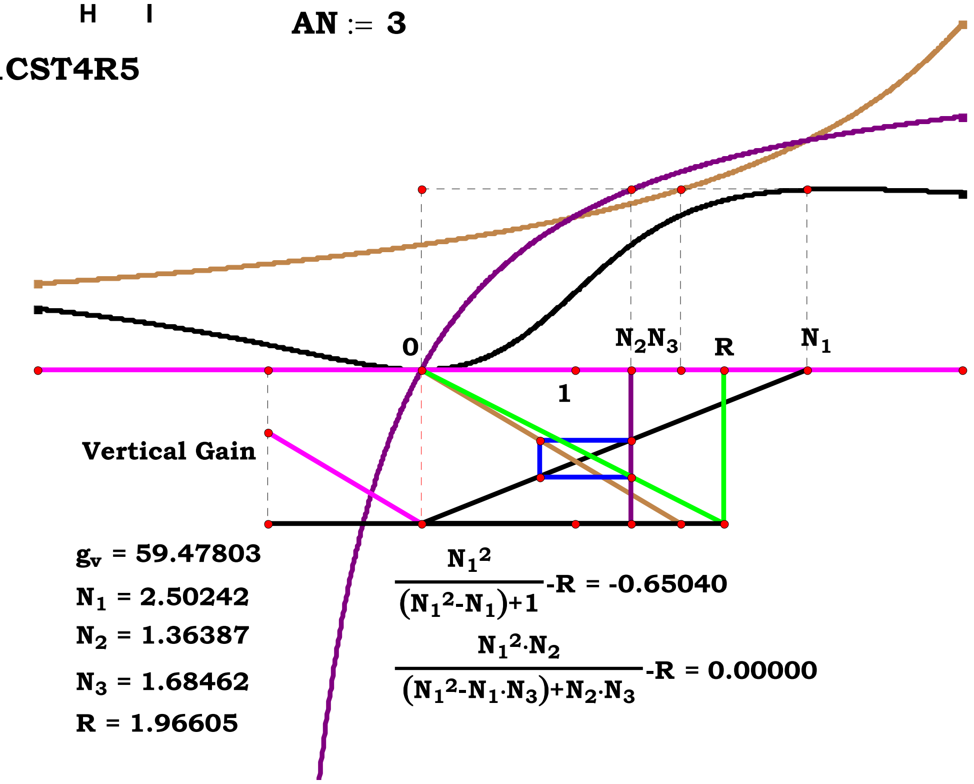
Given.

$AN := 3$

Descriptions.

$$BD := \frac{AN - 1}{AN} \qquad EL := \frac{AB \cdot BD}{AN}$$

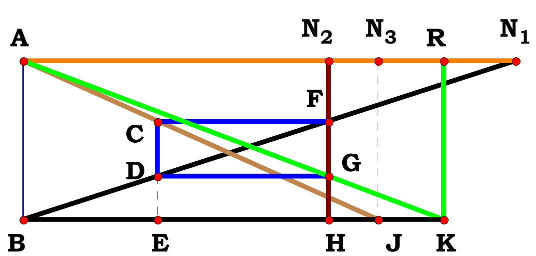
$$GI := \frac{AB^2}{AB - EL} \qquad AR := GI$$



Definitions.

$$AR - \frac{AN^2}{AN^2 - AN + 1} = 0$$

Descriptions.



$N_1 := 4$

$N_2 := 3$

$N_3 := 2$

$$ab := 1 \qquad gn := 1 - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1^2} \qquad ar := \frac{N_2}{gn}$$

Definitions.

$$gn - \frac{N_1^2 - N_1 \cdot N_3 + N_2 \cdot N_3}{N_1^2} = 0 \qquad ar - \frac{N_1^2 \cdot N_2}{N_1^2 - N_1 \cdot N_3 + N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_1 + 1}$

0, 2, 0. 1

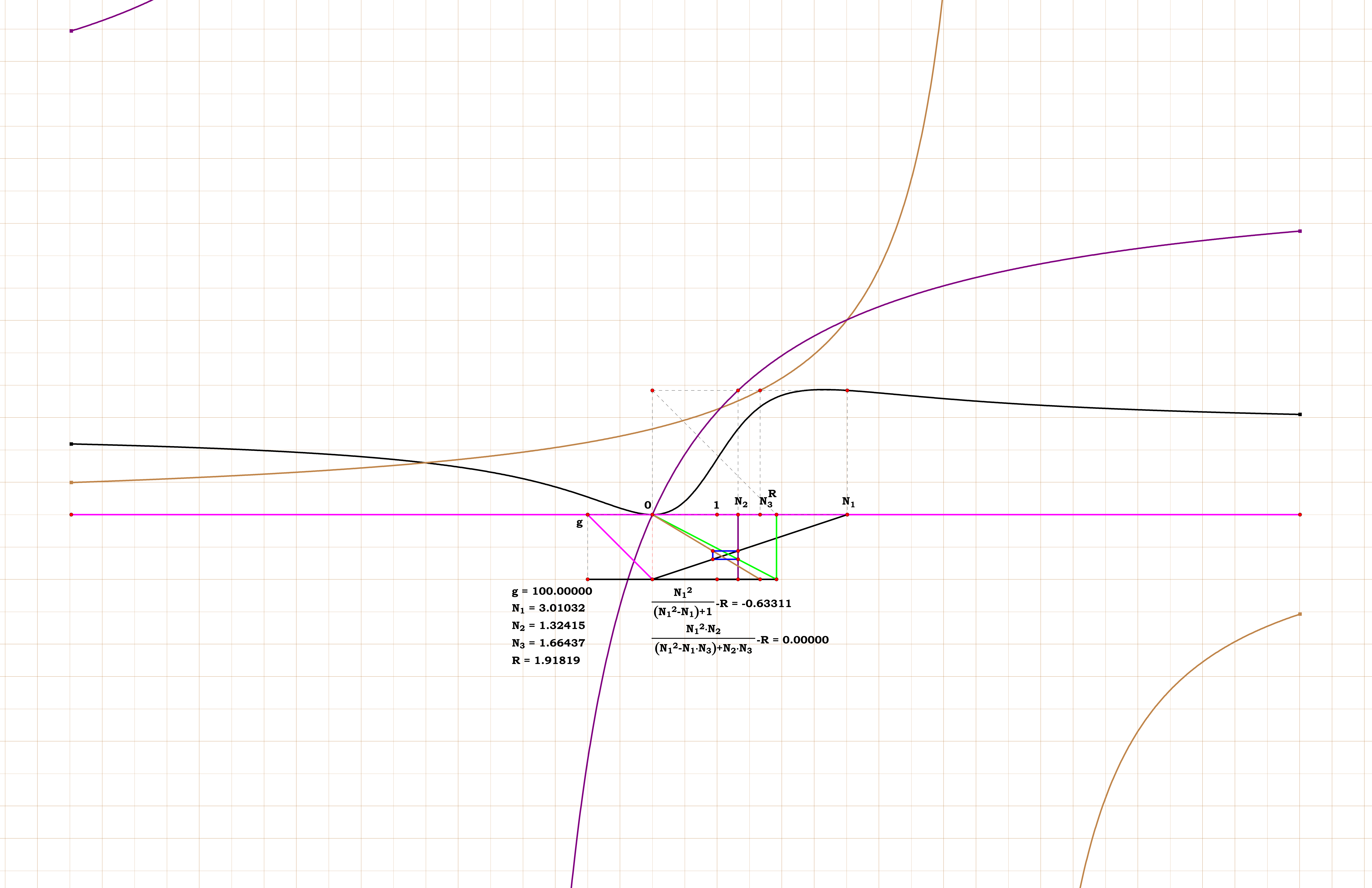
0, 0, 3. 1

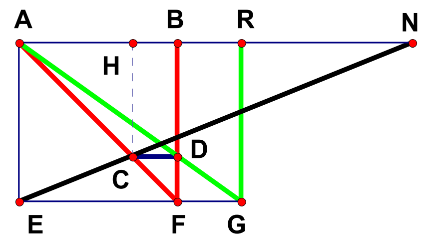
1, 2, 0. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_1 + N_2}$

1, 0, 3. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_3 \cdot N_1 + N_3}$

0, 2, 3. $\dfrac{N_2}{N_2 \cdot N_3 - N_3 + 1}$

1, 2, 3. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_1 \cdot N_3 + N_2 \cdot N_3}$


0
1
N_2
N_3
R
N_1
g
g = 100.00000
N_1 = 3.01032
N_2 = 1.32415
N_3 = 1.66437
R = 1.91819
$\frac{N_1^2}{(N_1^2 - N_1) + 1} - R = -0.63311$
$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_3) + N_2 \cdot N_3} - R = 0.00000$



1CST4R6

Unit.

$AB := 1$

Given.

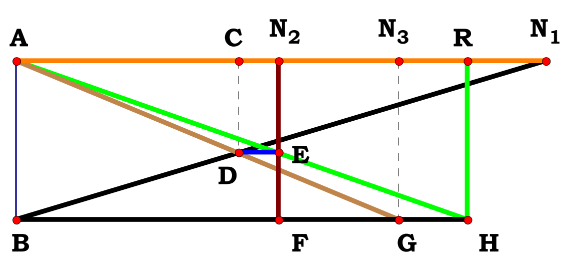
$AN := 3$

Descriptions.

$$AH := \frac{AB \cdot AN}{AB + AN} \qquad EG := \frac{AB^2}{AH} \qquad AR := EG$$

Definitions.

$$AR - \frac{AN + 1}{AN} = 0$$



$N_1 := 4$

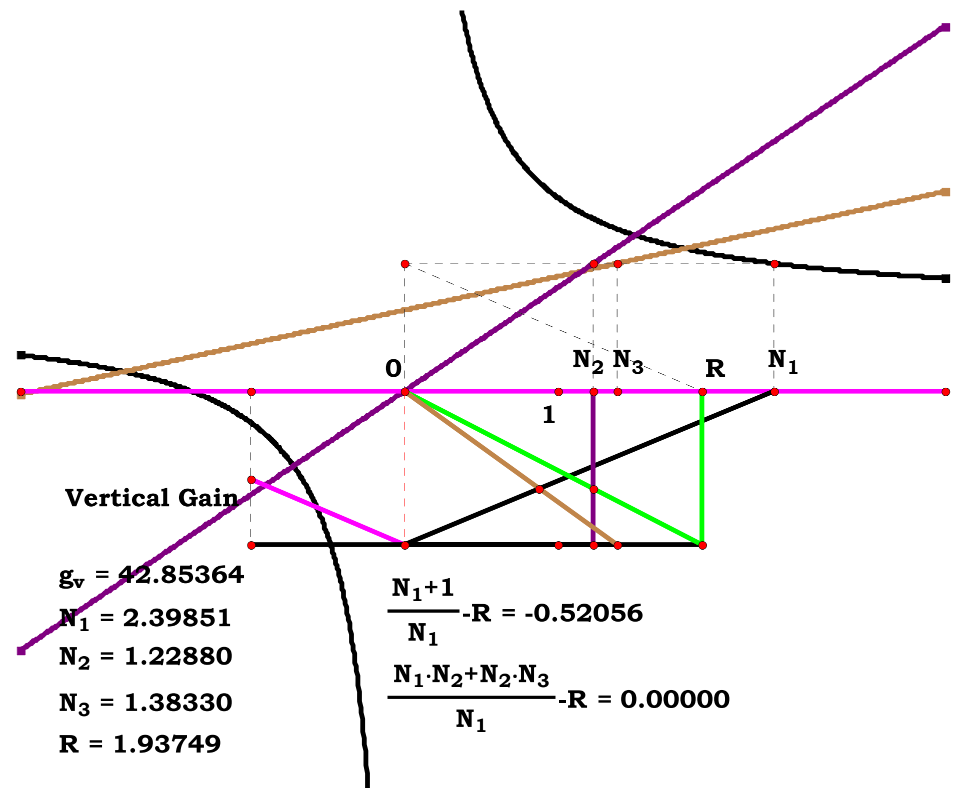
$N_2 := 3$

$N_3 := 2$

$$cd := \frac{N_1}{N_1 + N_3} \qquad ar := \frac{N_2}{cd}$$

Definitions.

$$ar - \frac{N_2 \cdot (N_1 + N_3)}{N_1} = 0$$



Three Transforms.

0, 0, 0. 2

1, 0, 0. $\dfrac{N_1 + 1}{N_1}$

0, 2, 0. $2 \cdot N_2$

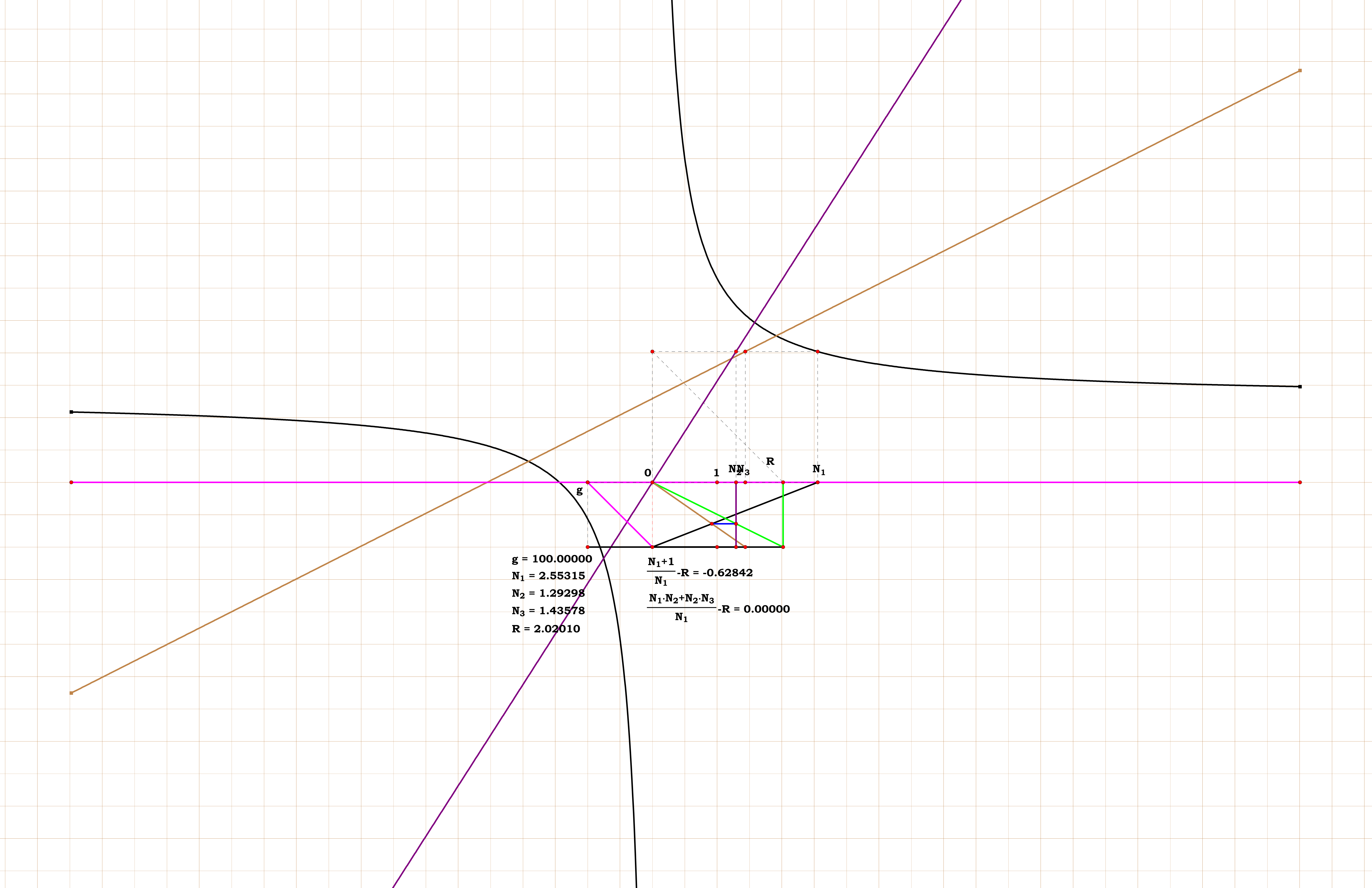
0, 0, 3. $N_3 + 1$

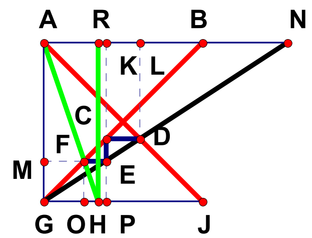
1, 2, 0. $\dfrac{N_2 \cdot (N_1 + 1)}{N_1}$

1, 0, 3. $\dfrac{N_1 + N_3}{N_1}$

0, 2, 3. $N_2 \cdot (N_3 + 1)$

1, 2, 3. $\dfrac{N_2 \cdot (N_1 + N_3)}{N_1}$


0
1
N_2 N_3
R
N_1
g
g = 100.00000
N_1 = 2.55315
N_2 = 1.29298
N_3 = 1.43578
R = 2.02010
$\frac{N_1+1}{N_1}$-R = -0.62842
$\frac{N_1 \cdot N_2+N_2 \cdot N_3}{N_1}$-R = 0.00000



1CST5R0

Unit.

$AB := 1$

Given.

$AN := -2$

Descriptions.

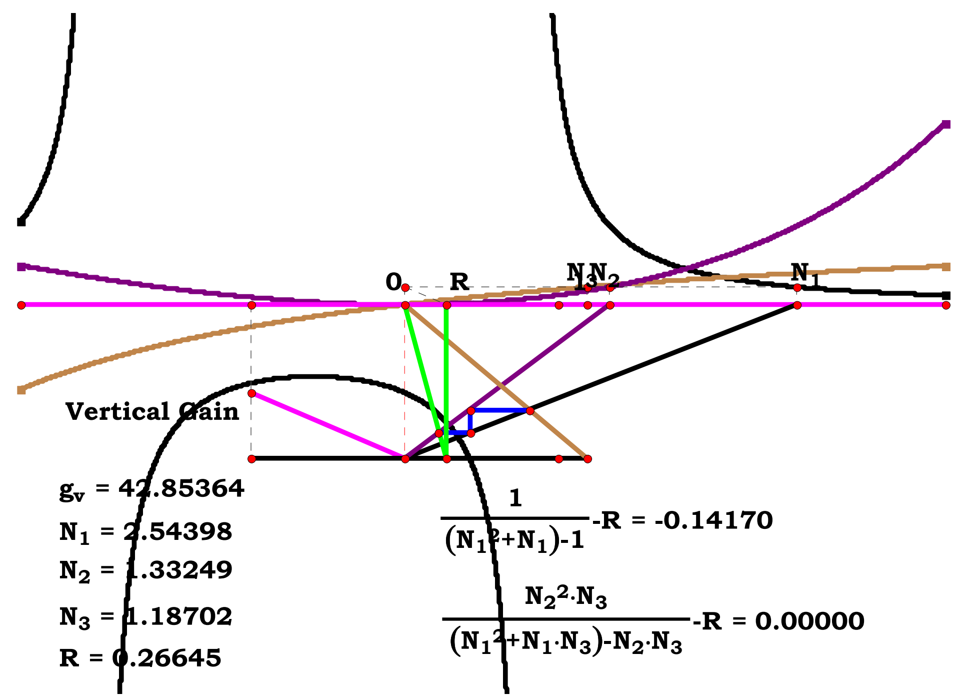
$$AL := \frac{AB \cdot AN}{AB + AN} \quad GP := AB - AL \quad EP := \frac{AB \cdot GP}{AN}$$

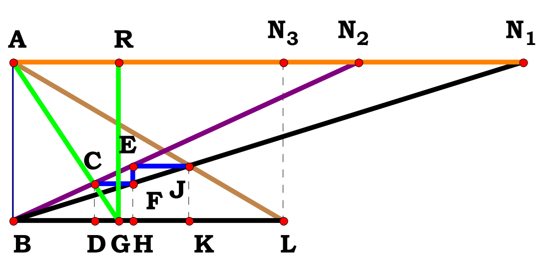
$$AM := AB - EP \quad FM := EP \quad GH := \frac{FM \cdot AB}{AM}$$

$$AR := GH$$

Definitions.

$$AR - \frac{1}{AN^2 + AN - 1} = 0$$





$N_1 := 7$

$N_2 := 3$

$$N_3 := 2 \quad ab := 1 \quad jk := \frac{N_3}{N_1 + N_3} \quad bh := N_2 \cdot jk \quad fh := \frac{bh}{N_1} \quad bd := N_2 \cdot fh \quad ar := \frac{bd}{ab - fh}$$

Definitions.

$$bh - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0 \quad fh - \frac{N_2 \cdot N_3}{N_1^2 + N_1 \cdot N_3} = 0 \quad bd - \frac{N_2^2 \cdot N_3}{N_1^2 + N_1 \cdot N_3} = 0 \quad ar - \frac{N_2^2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\dfrac{1}{N_1^{\,2} + N_1 - 1}$

0, 2, 0. $\dfrac{N_2^{\,2}}{2 - N_2}$

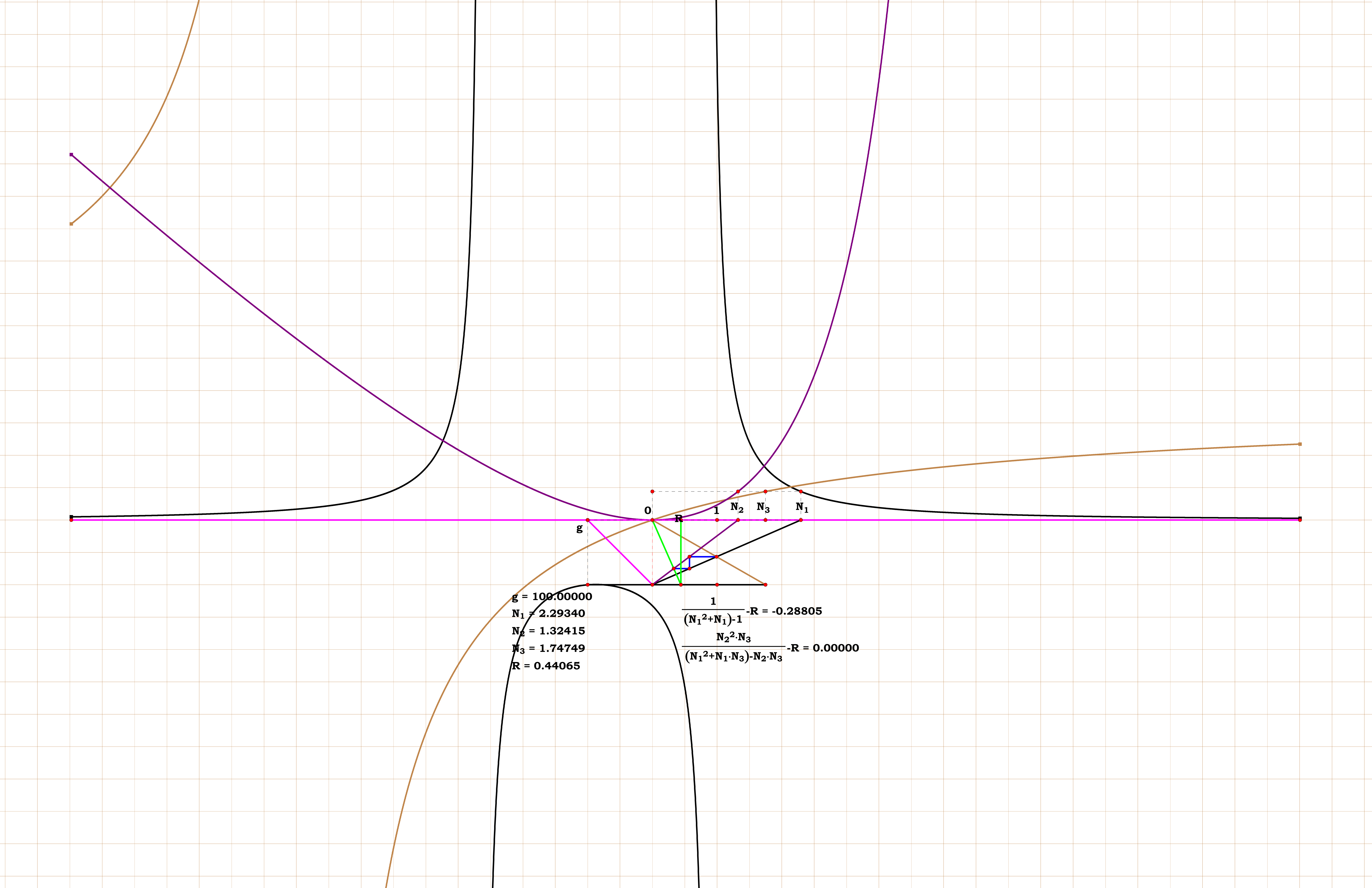
0, 0, 3. N_3

1, 2, 0. $\dfrac{N_2^{\,2}}{N_1^{\,2} + N_1 - N_2}$

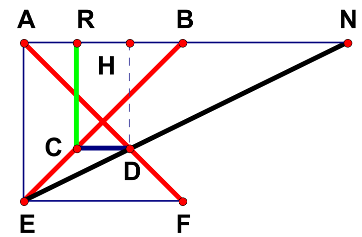
1, 0, 3. $\dfrac{N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_3}$

0, 2, 3. $N_2^{\,2} \cdot \dfrac{N_3}{1 + N_3 - N_2 \cdot N_3}$

1, 2, 3. $\dfrac{N_2^{\,2} \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3}$



0
R
1
N2
N3
N1
g
1
g = 100.00000
N1 = 2.29340
N2 = 1.32415
N3 = 1.74749
R = 0.44065
1/((N1^2+N1)-1) - R = -0.28805
N2^2·N3/((N1^2+N1·N3)-N2·N3) - R = 0.00000



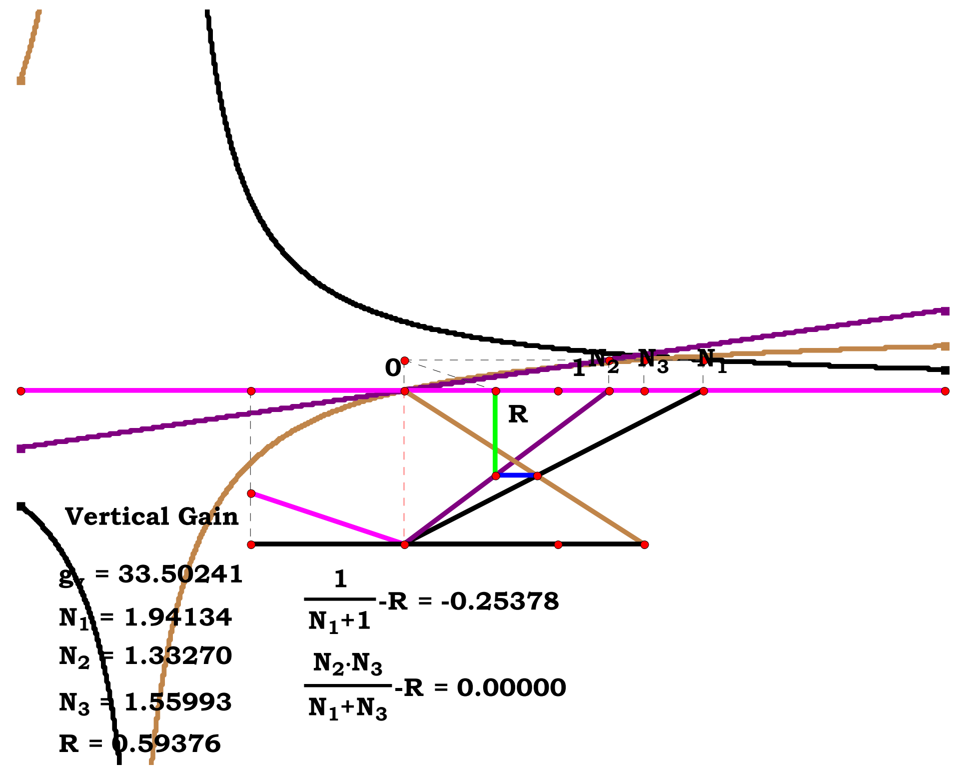
1CST5R1

Unit.

$AB := 1$

Given.

$AN := 3$

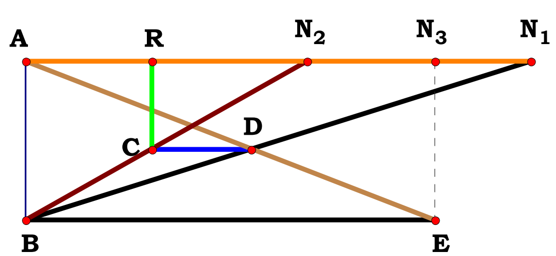


Descriptions.

$$AH := \frac{AB \cdot AN}{AB + AN} \qquad AR := AB - AH$$

Definitions.

$$AR - \frac{1}{AN + 1} = 0$$



$N_1 := 7$

$N_2 := 3$

$N_3 := 2 \qquad ch := \dfrac{N_1}{N_1 + N_3} \qquad rn := N_2 \cdot ch \qquad ar := N_2 - rn$

Definitions.

$$rn - \frac{N_1 \cdot N_2}{N_1 + N_3} = 0 \qquad ar - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0$$

Three Transforms.

0, 0, 0. $\frac{1}{2}$

1, 0, 0. $\frac{1}{N_1 + 1}$

0, 2, 0. $\frac{N_2}{2}$

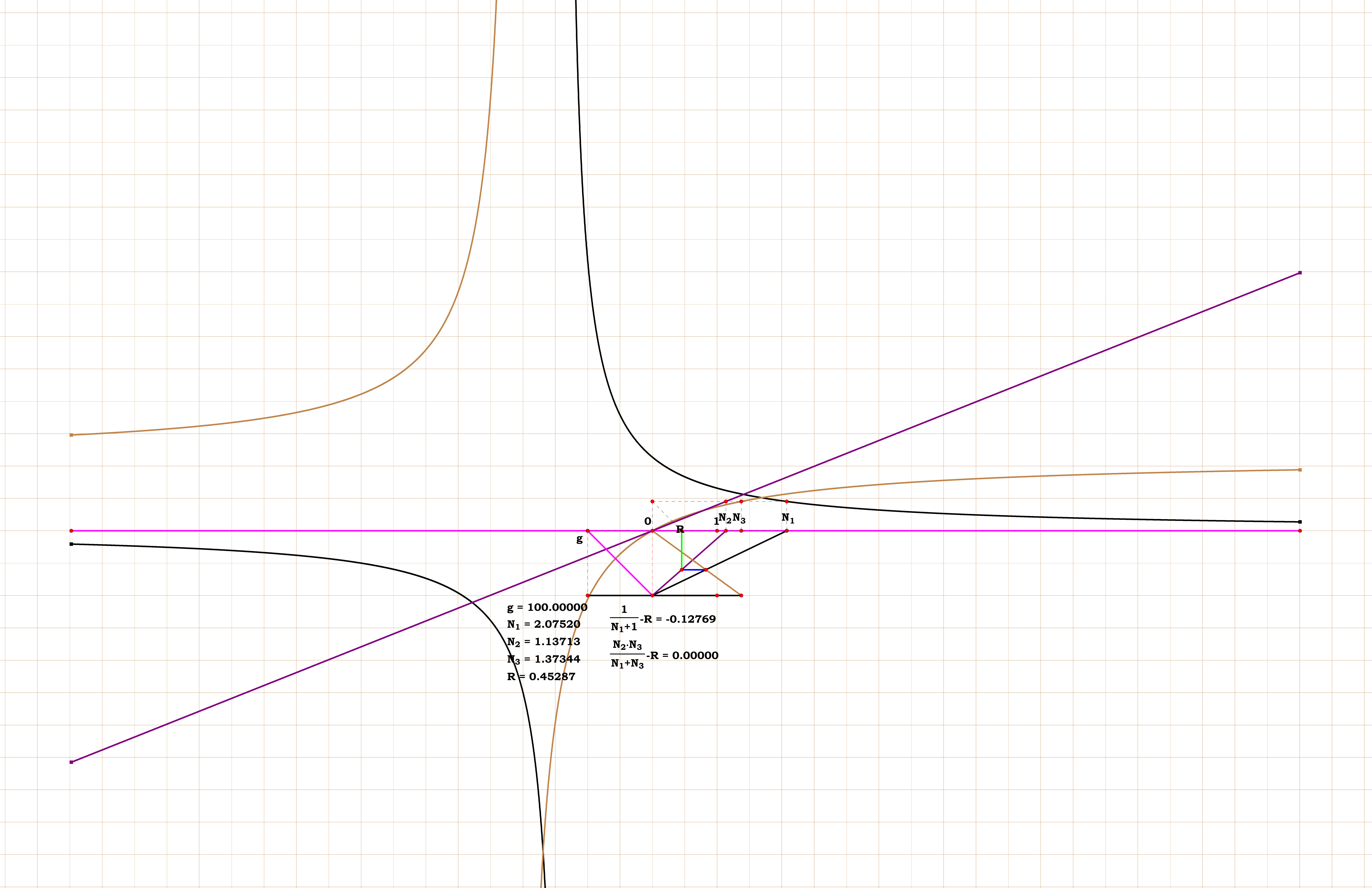
0, 0, 3. $\frac{N_3}{N_3 + 1}$

1, 2, 0. $\frac{N_2}{N_1 + 1}$

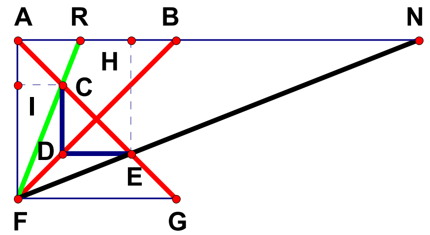
1, 0, 3. $\frac{N_3}{N_1 + N_3}$

0, 2, 3. $\frac{N_2 \cdot N_3}{N_3 + 1}$

1, 2, 3. $\frac{N_2 \cdot N_3}{N_1 + N_3}$



0
R
1
N_2 N_3
N_1
g
g = 100.00000
N_1 = 2.07520
N_2 = 1.13713
N_3 = 1.37344
R = 0.45287
$\frac{1}{N_1+1}$-R = -0.12769
$\frac{N_2 \cdot N_3}{N_1+N_3}$-R = 0.00000



1CST5R2

Unit.

$AB := 1$

Given.

$AN := 3$

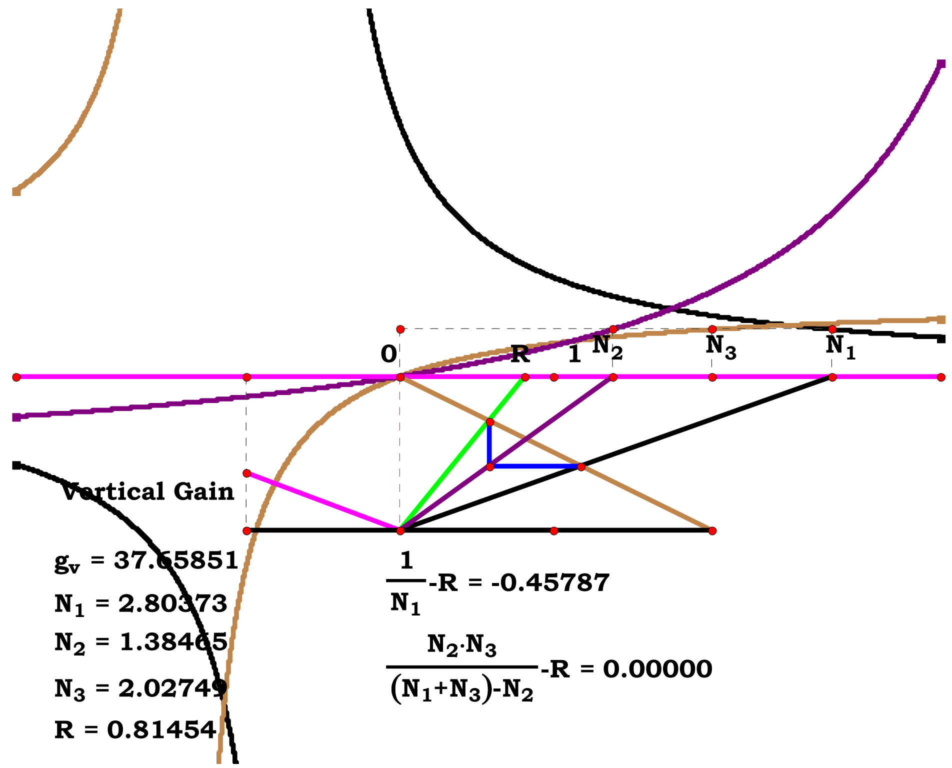
Descriptions.

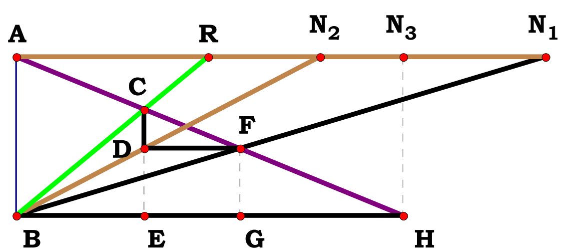
$$AH := \frac{AB \cdot AN}{AB + AN} \quad BH := AB - AH \quad CI := BH$$

$$FI := AH \qquad AR := \frac{CI \cdot AB}{FI}$$

Definitions.

$$AR - \frac{1}{AN} = 0$$





$N_1 := 7$

$N_2 := 3$

$$N_3 := 2 \quad fg := \frac{N_3}{N_1 + N_3} \quad be := N_2 \cdot fg \quad eh := N_3 - be \quad ce := \frac{eh}{N_3} \quad ar := \frac{be}{ce}$$

Definitions.

$$be - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0 \qquad eh - \frac{N_1 \cdot N_3 + N_3^{\,2} - N_2 \cdot N_3}{N_1 + N_3} = 0 \qquad ce - \frac{N_1 + N_3 - N_2}{N_1 + N_3} = 0 \qquad ar - \frac{N_2 \cdot N_3}{N_1 + N_3 - N_2} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\dfrac{1}{N_1}$

0, 2, 0. $-\dfrac{N_2}{N_2 - 2}$

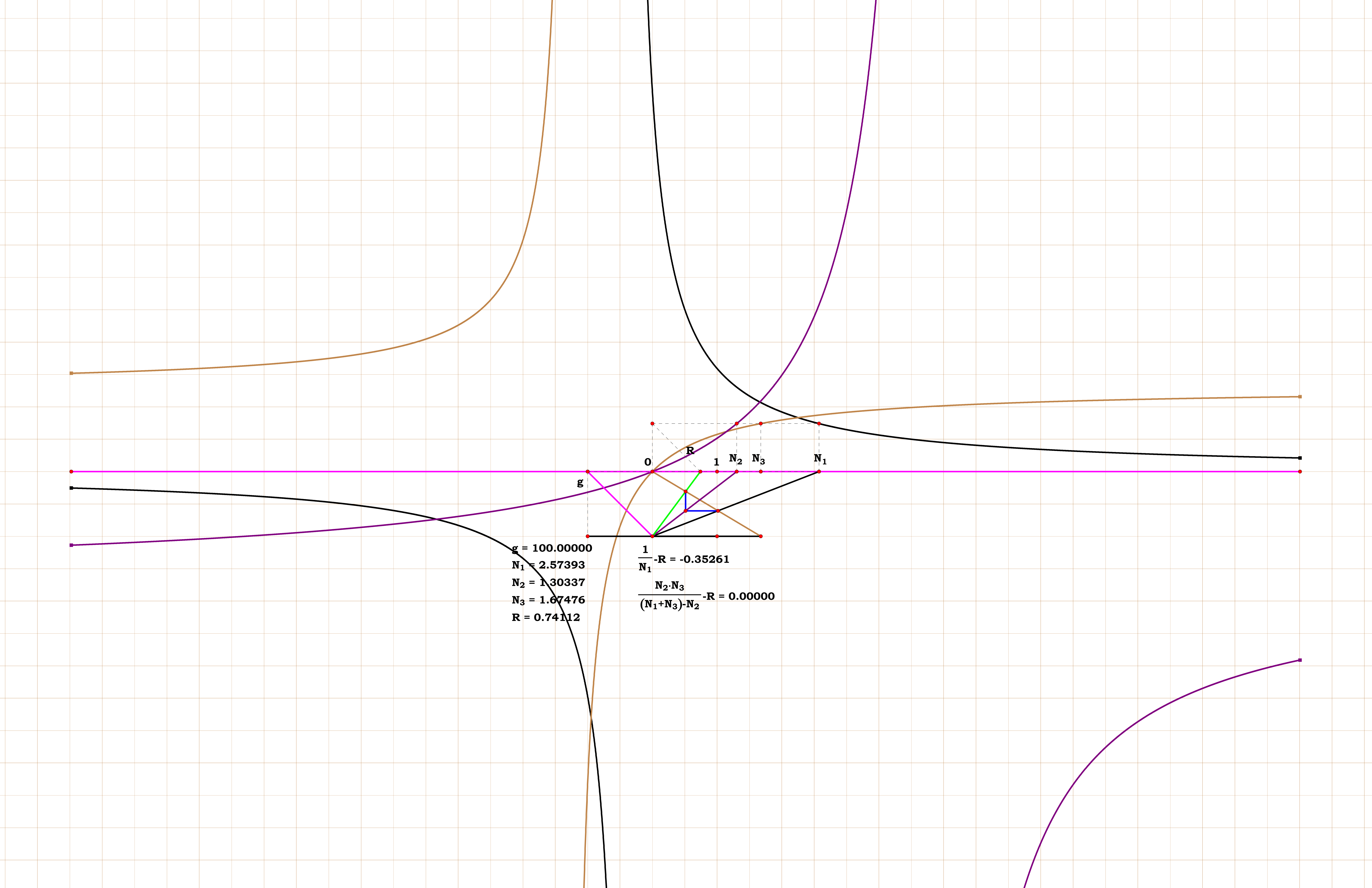
0, 0, 3. 1

1, 2, 0. $\dfrac{N_2}{N_1 - N_2 + 1}$

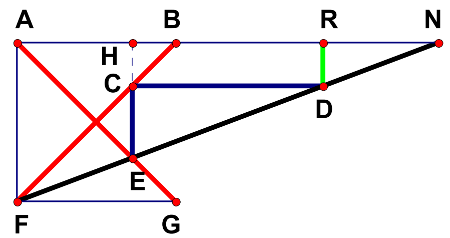
1, 0, 3. $\dfrac{N_3}{N_1 + N_3 - 1}$

0, 2, 3. $\dfrac{N_2 \cdot N_3}{N_3 - N_2 + 1}$

1, 2, 3. $\dfrac{N_2 \cdot N_3}{N_1 + N_3 - N_2}$



R
0
1
N_2
N_3
N_1
g
g = 100.00000
N_1 = 2.57393
N_2 = 1.30337
N_3 = 1.67476
R = 0.74112
$\frac{1}{N_1}$-R = -0.35261
$\frac{N_2 \cdot N_3}{(N_1+N_3)-N_2}$-R = 0.00000



1CST5R3

Unit.

$AB := 1$

Given.

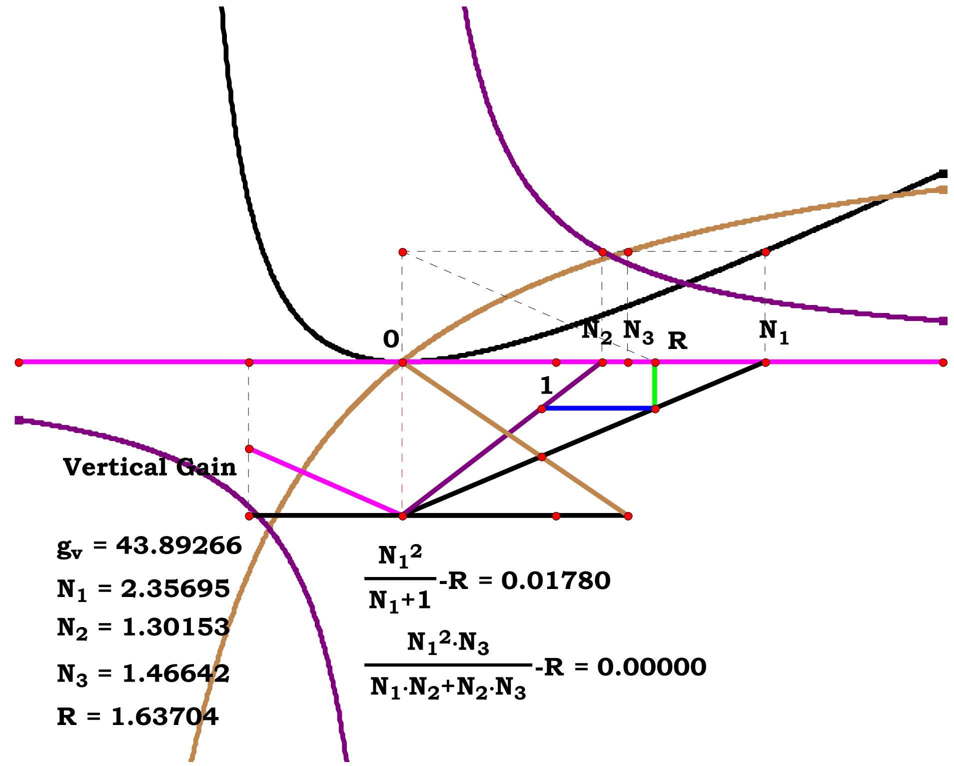
$AN := 5$

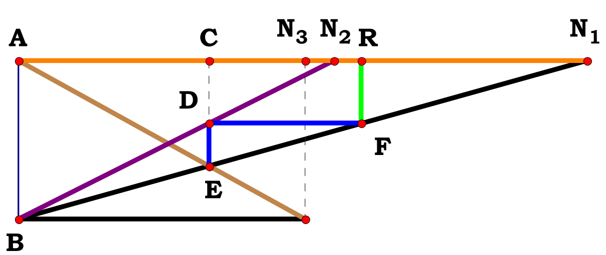
Descriptions.

$$AH := \frac{AB \cdot AN}{AB + AN} \quad AR := AN - AH$$

Definitions.

$$AR - \frac{AN^2}{AN + 1} = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$ce := \frac{N_1}{N_1 + N_3} \quad ac := N_3 \cdot ce \quad cd := \frac{N_2 - ac}{N_2} \quad ar := N_1 - N_1 \cdot cd$$

Definitions.

$$ac - \frac{N_1 \cdot N_3}{N_1 + N_3} = 0 \quad cd - \frac{N_1 \cdot N_2 + N_2 \cdot N_3 - N_1 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0 \quad ar - \frac{N_1^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. $\frac{1}{2}$

1, 0, 0. $\frac{N_1^{\,2}}{N_1 + 1}$

0, 2, 0. $\frac{1}{2 \cdot N_2}$

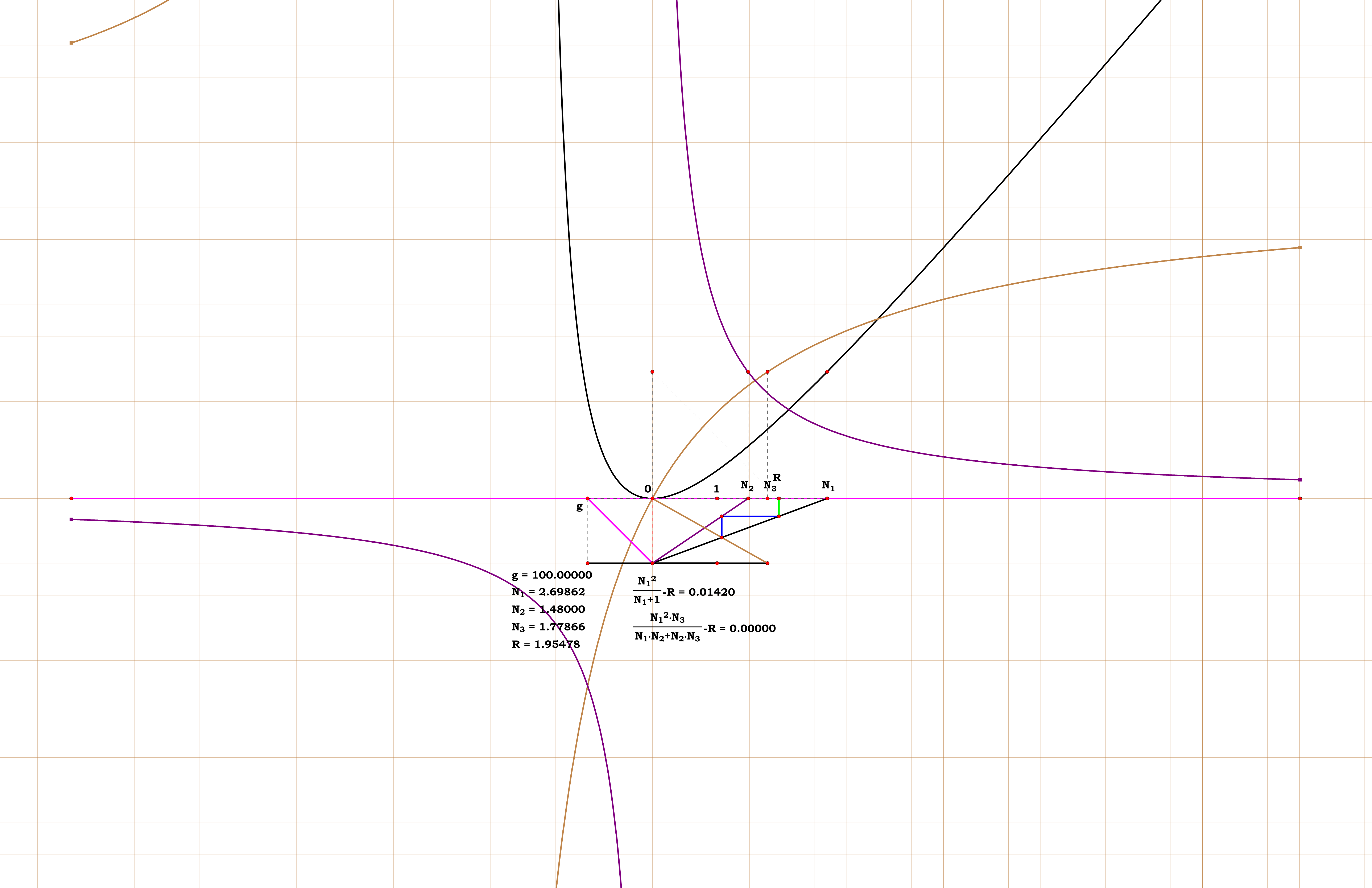
0, 0, 3. $\frac{N_3}{1 + N_3}$

1, 2, 0. $\frac{N_1^{\,2}}{N_1 \cdot N_2 + N_2}$

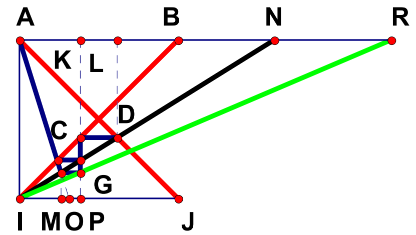
1, 0, 3. $N_1^{\,2} \cdot \frac{N_3}{N_1 + N_3}$

0, 2, 3. $\frac{N_3}{N_2 + N_2 \cdot N_3}$

1, 2, 3. $\frac{N_1^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3}$



0
1
N2
N3
R
N1
g
g = 100.00000
N1 = 2.69862
N2 = 1.48000
N3 = 1.77866
R = 1.95478
$\frac{N_1^2}{N_1+1}$-R = 0.01420
$\frac{N_1^2 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3}$-R = 0.00000



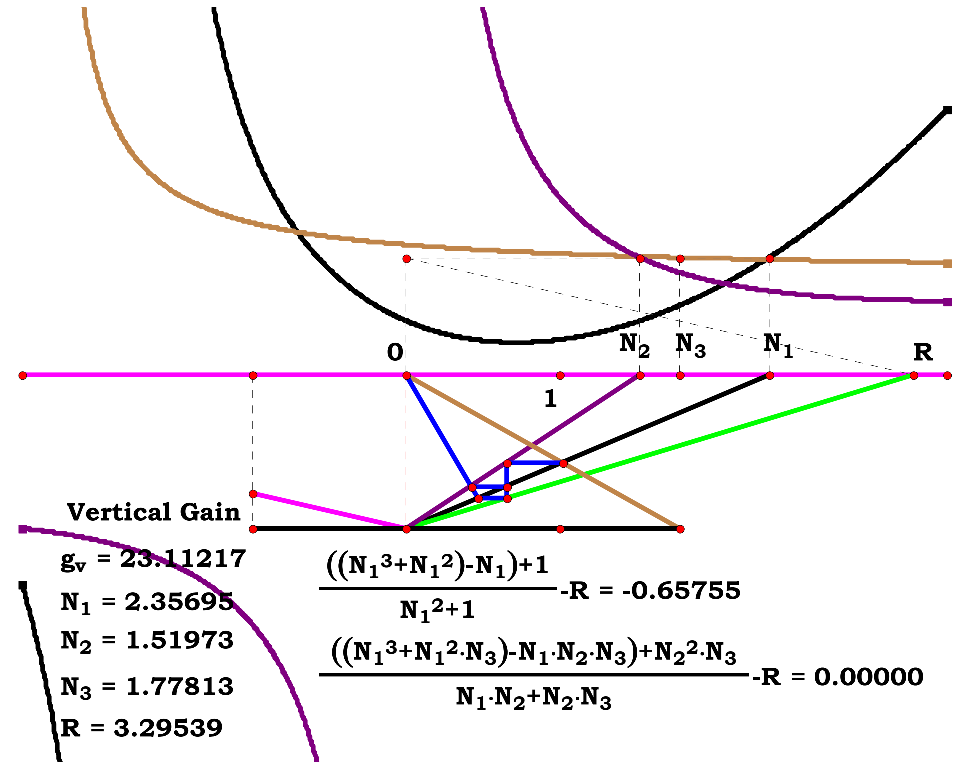
1CST5R4

Unit.

$AB := 1$

Given.

$AN := 3$



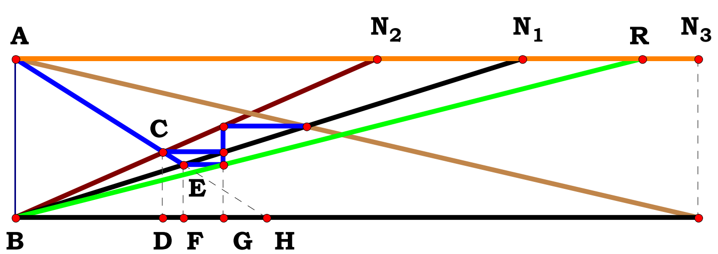
Descriptions.

$$AL := \frac{AB \cdot AN}{AB + AN} \qquad IO := \frac{1}{AN^2 + AN - 1} \qquad IM := \frac{IO \cdot AN}{IO + AN} \qquad HM := \frac{AB \cdot IM}{AN}$$

$$BL := AB - AL \qquad IP := BL \qquad GP := HM \qquad AR := \frac{IP \cdot AB}{GP}$$

Definitions.

$$AR - \frac{AN^3 + AN^2 - AN + 1}{AN + 1} = 0$$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$bh := \frac{N_2^2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} \qquad ef := \frac{bh}{N_1 + bh} \qquad bg := \frac{N_2 \cdot N_3}{N_1 + N_3} \qquad ar := \frac{bg}{ef}$$

Definitions.

$$ef - \frac{N_2^2 \cdot N_3}{N_1^3 + N_1^2 \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^2 \cdot N_3} = 0 \qquad ar - \frac{N_1^3 + N_1^2 \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^2 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $$\frac{N_1^{\,3} + N_1^{\,2} - N_1 + 1}{N_1 + 1}$$

0, 2, 0. $$\frac{N_2^{\,2} - N_2 + 2}{2 \cdot N_2}$$

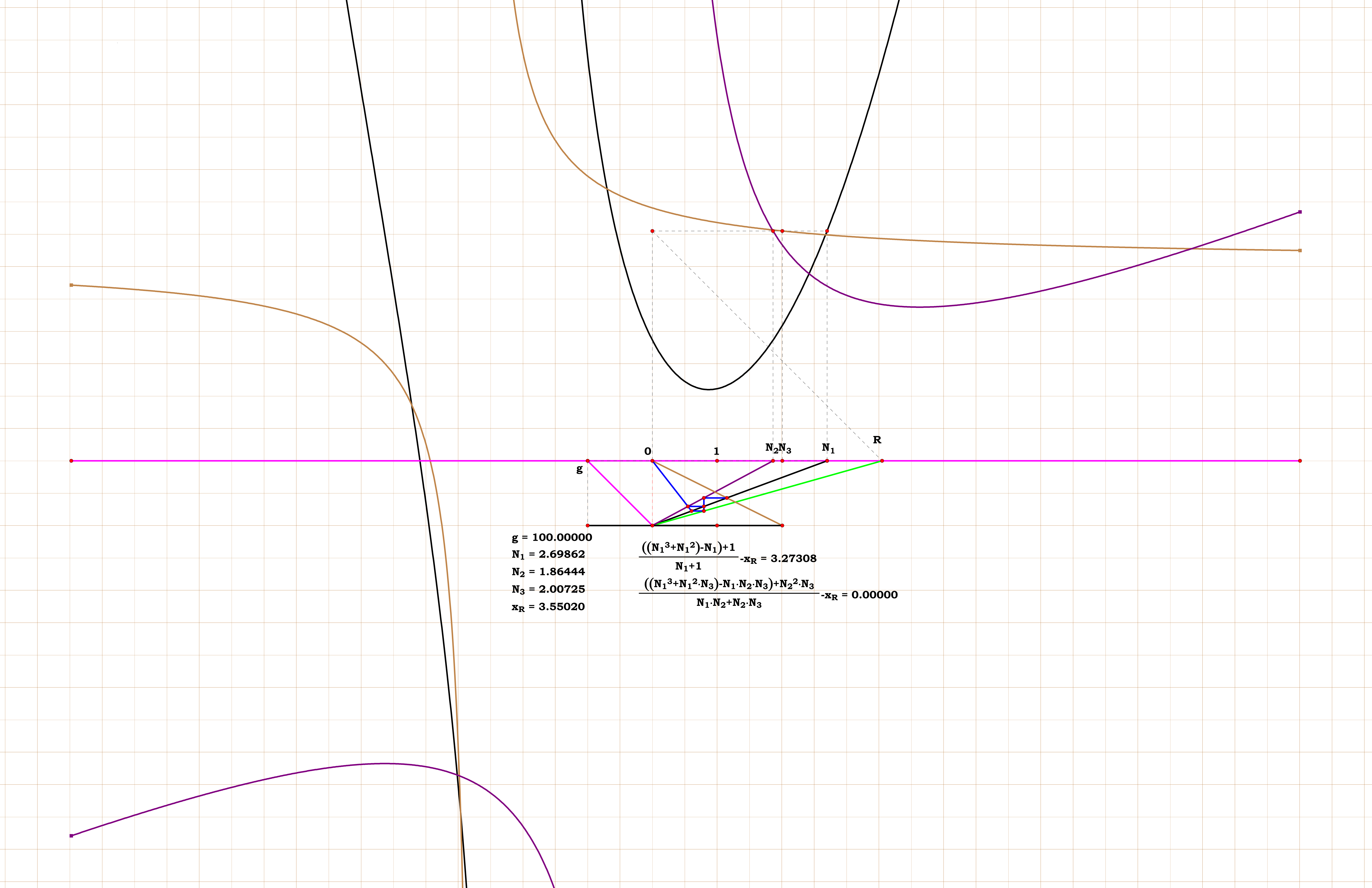
0, 0, 3. 1

1, 2, 0. $$\frac{N_1^{\,3} + N_1^{\,2} - N_1 \cdot N_2 + N_2^{\,2}}{N_2 + N_1 \cdot N_2}$$

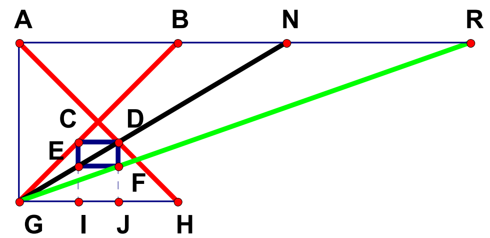
1, 0, 3. $$\frac{N_1^{\,3} + N_3 \cdot N_1^{\,2} - N_3 \cdot N_1 + N_3}{N_1 + N_3}$$

0, 2, 3. $$\frac{N_3 \cdot N_2^{\,2} - N_3 \cdot N_2 + N_3 + 1}{N_2 + N_2 \cdot N_3}$$

1, 2, 3. $$\frac{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3}$$



0
1
N_2N_3
N_1
R
g
g = 100.00000
N_1 = 2.69862
N_2 = 1.86444
N_3 = 2.00725
x_R = 3.55020
$\frac{((N_1^3+N_1^2)-N_1)+1}{N_1+1}-x_R = 3.27308$
$\frac{((N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_2 \cdot N_3)+N_2^2 \cdot N_3}{N_1 \cdot N_2+N_2 \cdot N_3}-x_R = 0.00000$



1CST5R5

Unit.

$AB := 1$

Given.

$AN := 3$

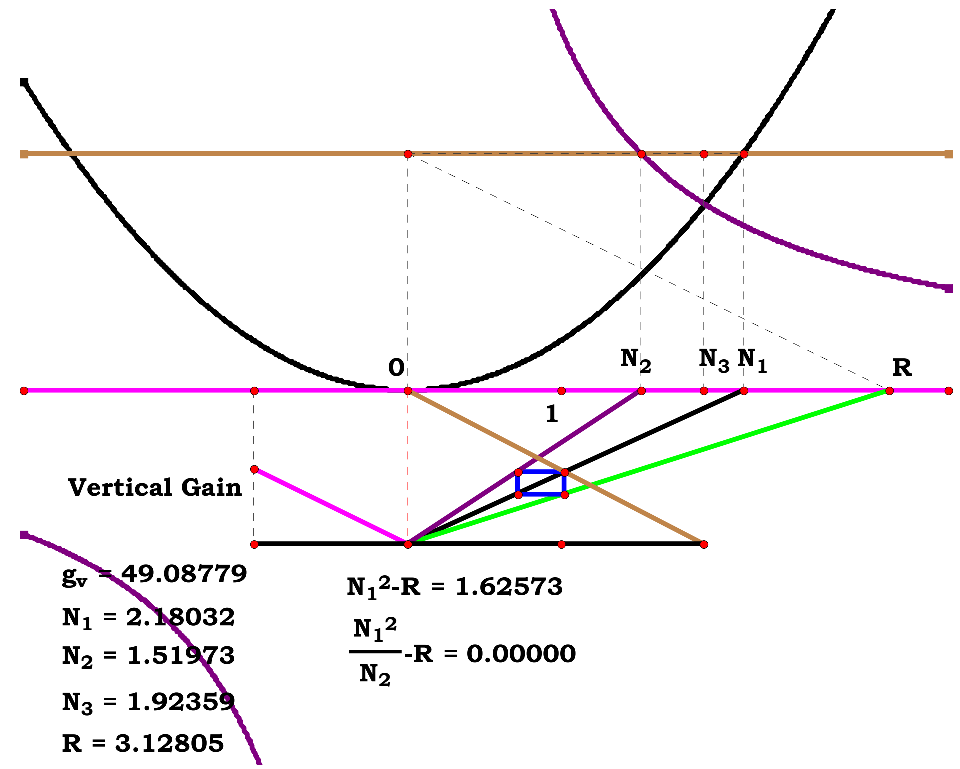
Descriptions.

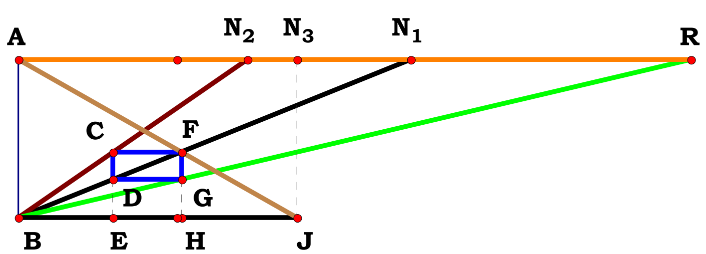
$GJ := \dfrac{AB \cdot AN}{AB + AN}$ $\quad GI := AB - GJ$

$EI := \dfrac{AB \cdot GI}{AN}$ $\quad AR := \dfrac{GJ \cdot AB}{EI}$

Definitions.

$AR - AN^2 = 0$





$N_1 := 5$

$N_2 := 3$

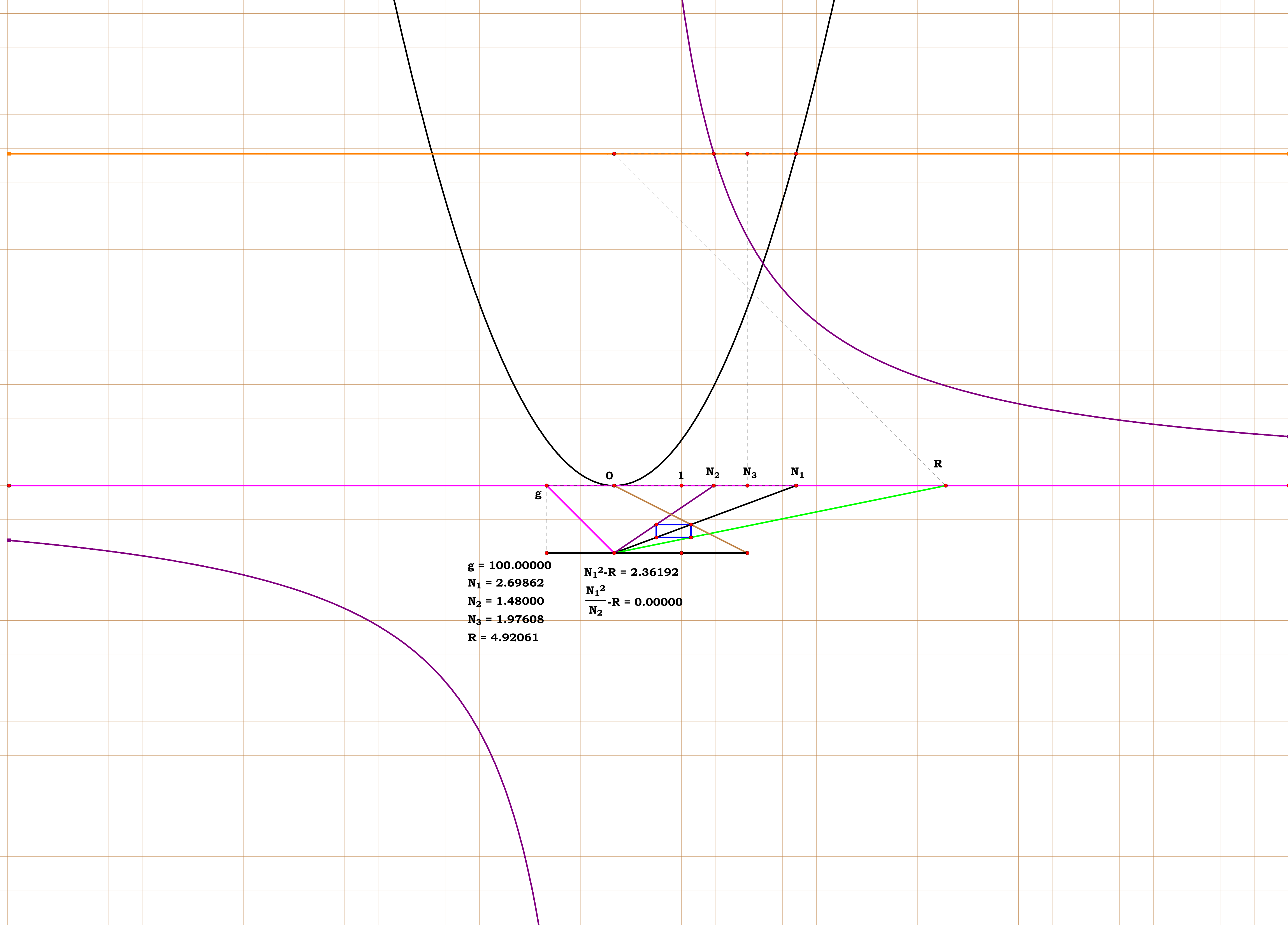
$N_3 := 2$ $\quad fh := \dfrac{N_3}{N_1 + N_3}$ $\quad be := N_2 \cdot fh$ $\quad de := \dfrac{be}{N_1}$ $\quad bh := N_1 \cdot fh$ $\quad ar := \dfrac{bh}{de}$

Definitions.

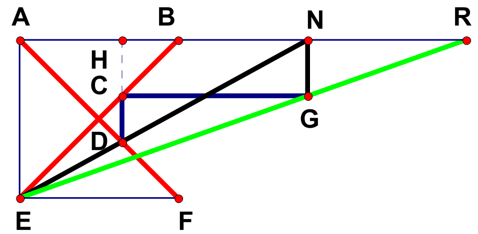
$be - \dfrac{N_2 \cdot N_3}{N_1 + N_3} = 0$ $\quad de - \dfrac{N_2 \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3} = 0$ $\quad bh - \dfrac{N_1 \cdot N_3}{N_1 + N_3} = 0$ $\quad ar - \dfrac{N_1^{\,2}}{N_2} = 0$

Three Transforms.

0, 0, 0.	1	1, 2, 0.	$\frac{N_1^2}{N_2}$
1, 0, 0.	N_1^2	1, 0, 3.	N_1^2
0, 2, 0.	$\frac{1}{N_2}$	0, 2, 3.	$\frac{1}{N_2}$
0, 0, 3.	1	1, 2, 3.	$\frac{N_1^2}{N_2}$



0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 2.69862
N_2 = 1.48000
N_3 = 1.97608
R = 4.92061
N_1^2-R = 2.36192
$\frac{N_1^2}{N_2}$-R = 0.00000



1CST5R6

Unit.

$AB := 1$

Given.

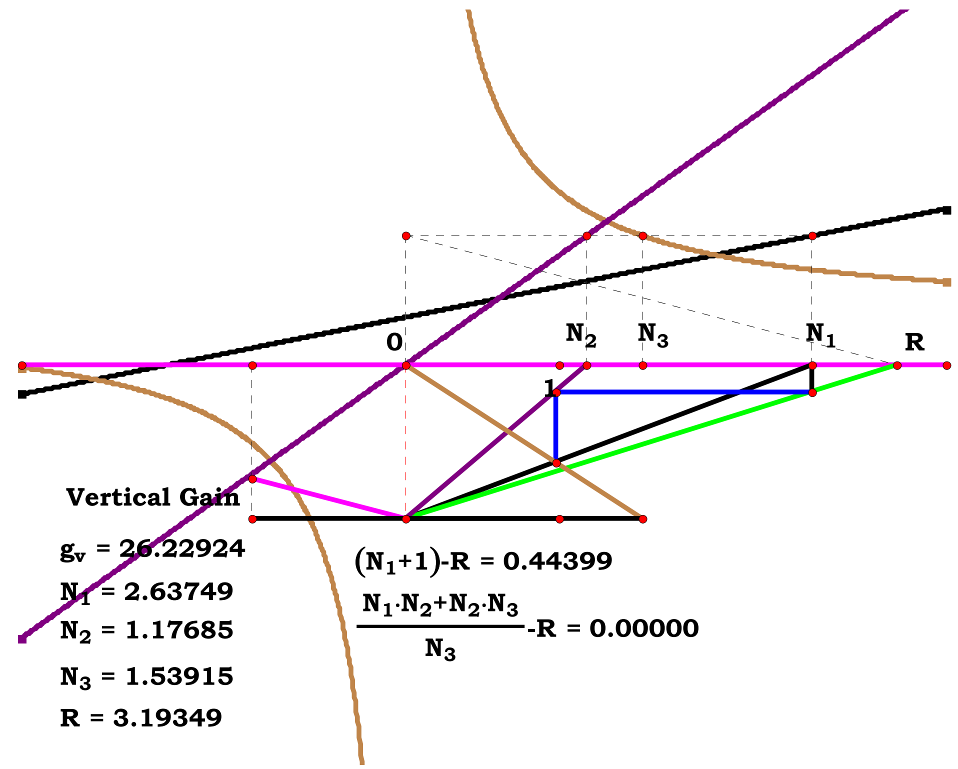
$AN := 3$

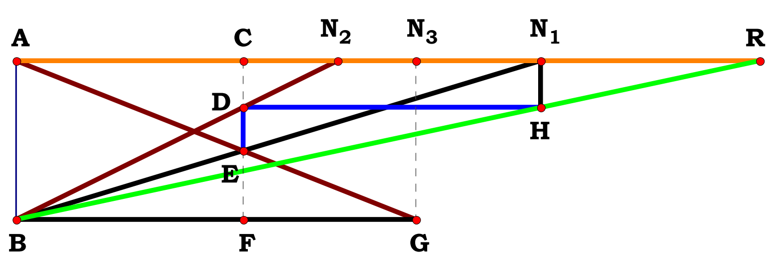
Descriptions.

$$AH := \frac{AB \cdot AN}{AB + AN} \qquad NG := AB - AH \qquad AR := \frac{AN \cdot AB}{AH}$$

Definitions.

$$AR - (AN + 1) = 0$$





$N_1 := 5$

$N_2 := 3$

$$N_3 := 2 \qquad ab := 1 \qquad bf := \frac{N_1 \cdot N_3}{N_1 + N_3} \qquad cd := ab - \frac{bf}{N_2} \qquad ar := \frac{N_1}{ab - cd}$$

Definitions.

$$cd - \frac{N_1 \cdot N_2 + N_2 \cdot N_3 - N_1 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_3} = 0$$

Three Transforms.

0, 0, 0. 2

1, 0, 0. $N_1 + 1$

0, 2, 0. $2 \cdot N_2$

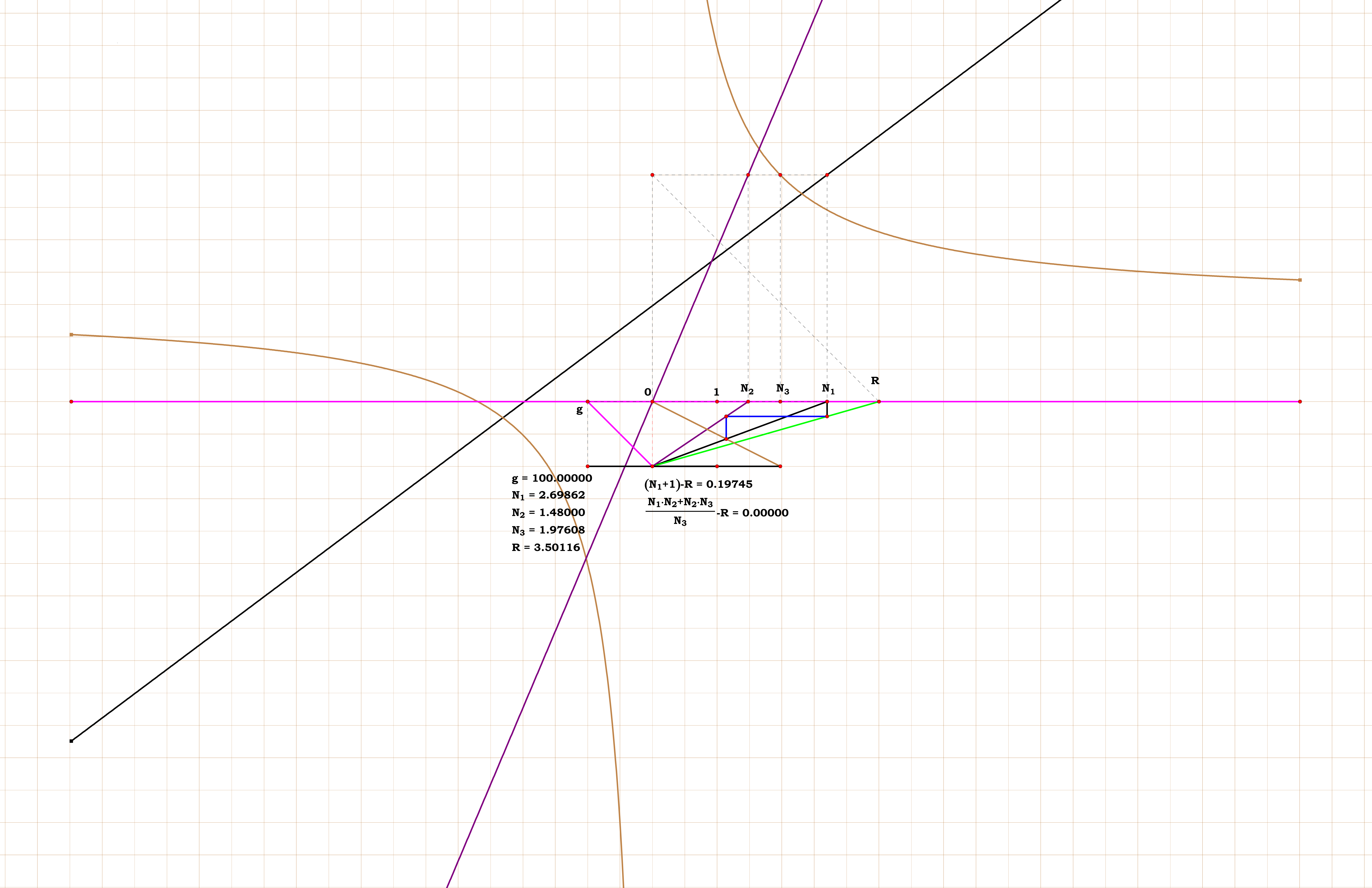
0, 0, 3. $\dfrac{N_3 + 1}{N_3}$

1, 2, 0. $N_2 \cdot \left(N_1 + 1\right)$

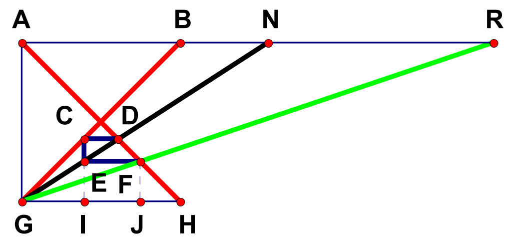
1, 0, 3. $\dfrac{N_1 + N_3}{N_3}$

0, 2, 3. $\dfrac{N_2 \cdot \left(N_3 + 1\right)}{N_3}$

1, 2, 3. $\dfrac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_3}$



0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 2.69862
N_2 = 1.48000
N_3 = 1.97608
R = 3.50116
$(N_1+1)-R = 0.19745$
$\frac{N_1 \cdot N_2 + N_2 \cdot N_3}{N_3} - R = 0.00000$



1CST5R7

Unit.

$AB := 1$

Given.

$AN := 3$

Descriptions.

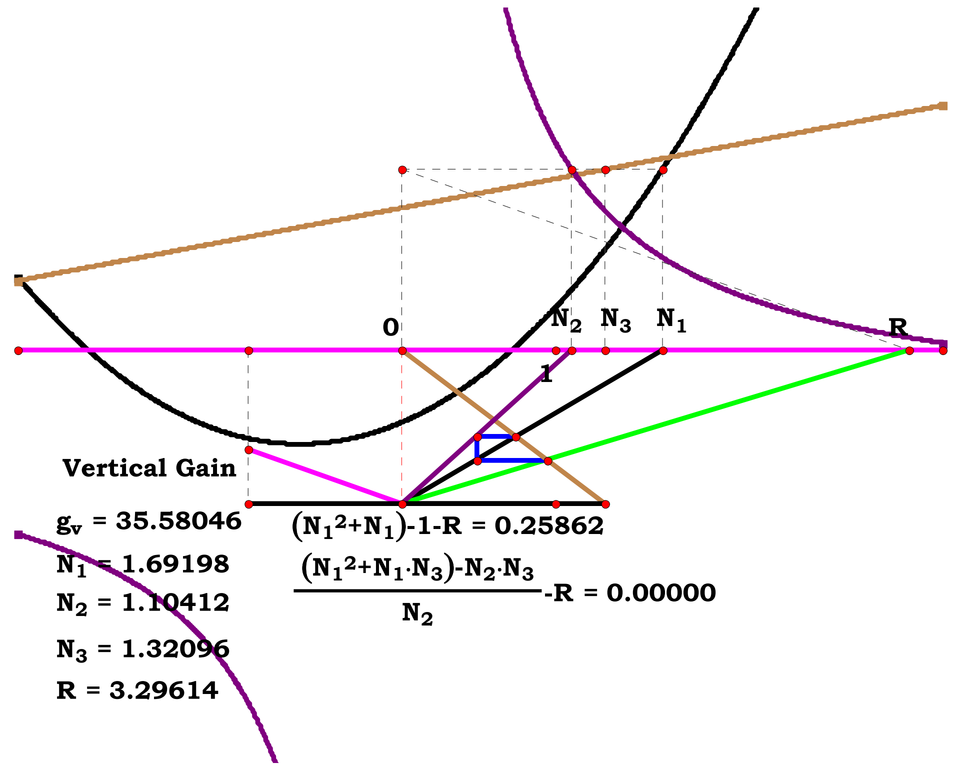
$$HI := \frac{AB \cdot AN}{AB + AN} \qquad GI := AB - HI \qquad EI := \frac{AB \cdot GI}{AN}$$

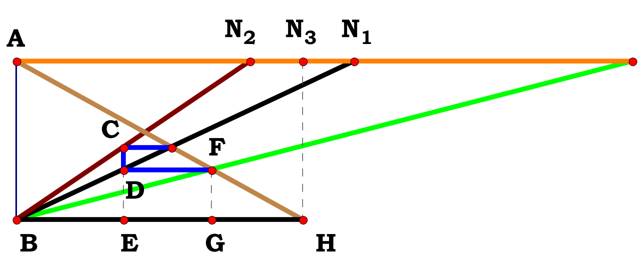
$$GJ := AB - EI \qquad AR := \frac{GJ \cdot AB}{EI}$$

Definitions.

$$GI - \frac{1}{AN + 1} = 0 \qquad EI - \frac{1}{AN^2 + AN} = 0$$

$$AR - \left(AN^2 + AN - 1\right) = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$ce := \frac{N_3}{N_1 + N_3} \qquad be := N_2 \cdot ce \qquad de := \frac{be}{N_1} \qquad fg := de \qquad gh := N_3 \cdot fg \qquad bg := N_3 - gh \qquad ar := \frac{bg}{fg}$$

Definitions.

$$be - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0 \qquad de - \frac{N_2 \cdot N_3}{N_1^2 + N_1 \cdot N_3} = 0 \qquad gh - \frac{N_2 \cdot N_3^2}{N_1^2 + N_1 \cdot N_3} = 0 \qquad bg - \frac{N_1^2 \cdot N_3 + N_1 \cdot N_3^2 - N_2 \cdot N_3^2}{N_1^2 + N_1 \cdot N_3} = 0 \qquad ar - \frac{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3}{N_2} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $N_1^{\,2} + N_1 - 1$

0, 2, 0. $-\dfrac{N_2 - 2}{N_2}$

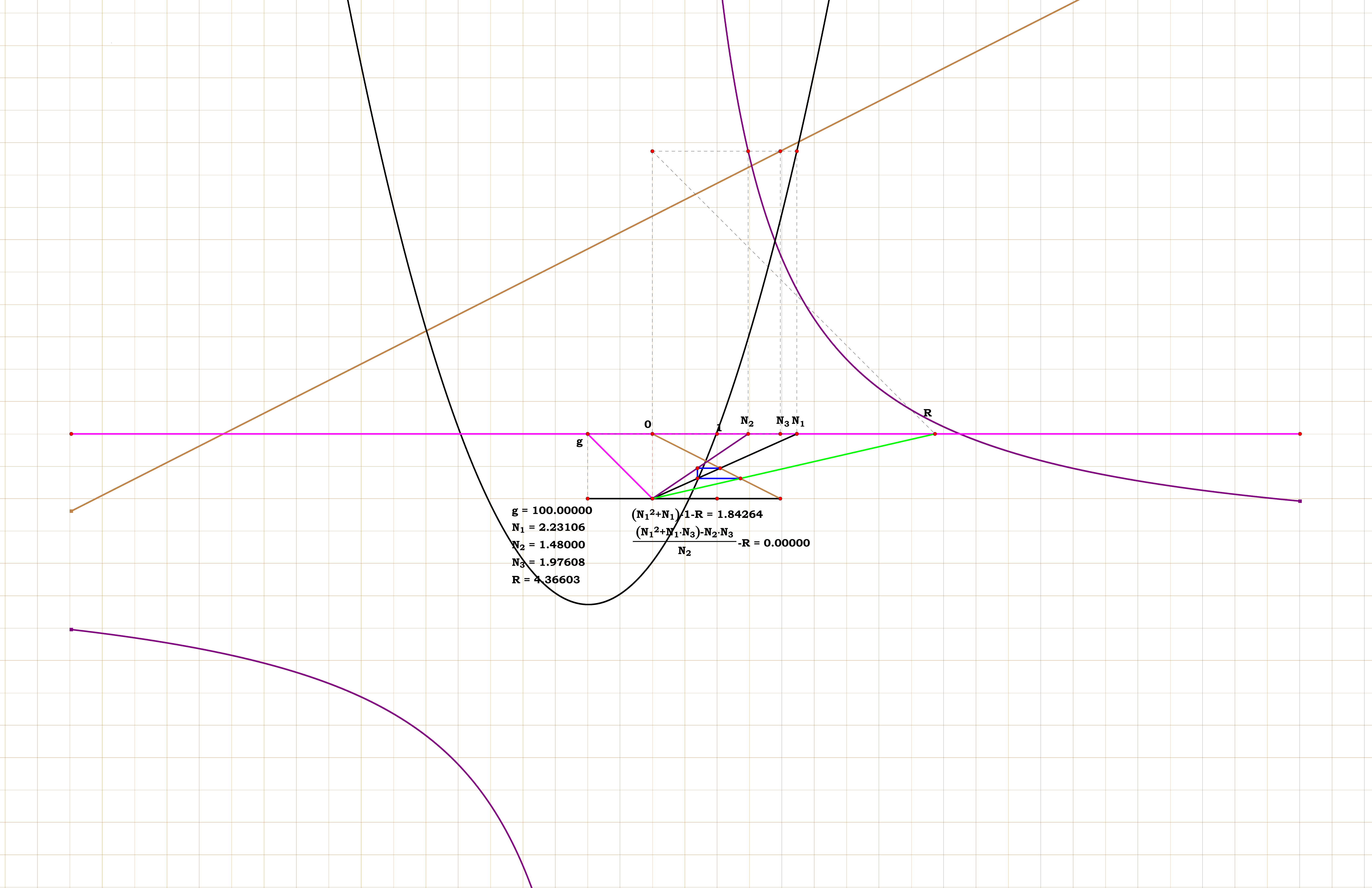
0, 0, 3. 1

1, 2, 0. $\dfrac{N_1^{\,2} + N_1 - N_2}{N_2}$

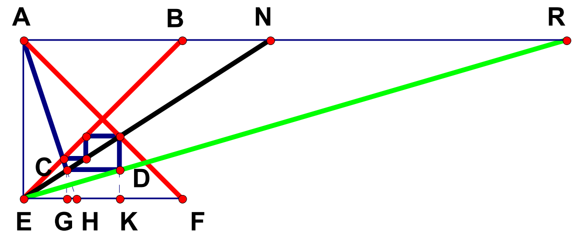
1, 0, 3. $N_1^{\,2} + N_3 \cdot N_1 - N_3$

0, 2, 3. $\dfrac{N_3 - N_2 \cdot N_3 + 1}{N_2}$

1, 2, 3. $\dfrac{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3}{N_2}$



R
0
1
N_2
N_3 N_1
g
g = 100.00000
N_1 = 2.23106
N_2 = 1.48000
N_3 = 1.97608
R = 4.36603
$(N_1^2+N_1)\cdot 1-R = 1.84264$
$\frac{(N_1^2+N_1\cdot N_3)-N_2\cdot N_3}{N_2}-R = 0.00000$



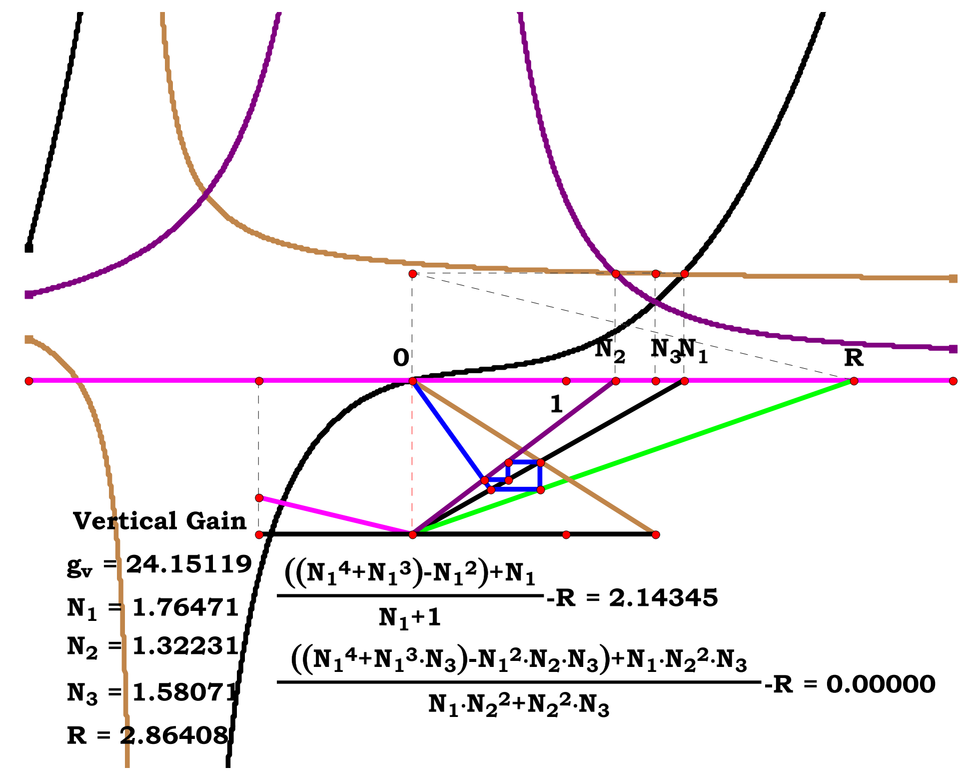
1CST5R8

Unit.

$AB := 1$

Given.

$AN := 3$

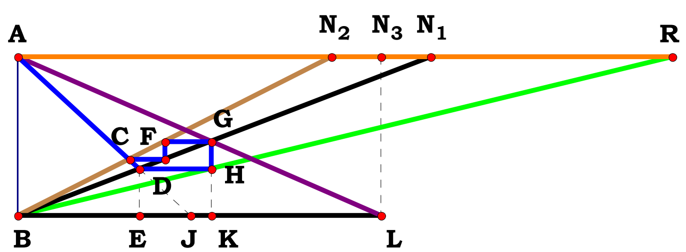


Descriptions.

$$EK := \frac{AB \cdot AN}{AB + AN} \quad EH := \frac{1}{AN^2 + AN - 1} \quad EG := \frac{EH \cdot AN}{EH + AN} \quad CG := \frac{AB \cdot EG}{AN} \quad AR := \frac{EK \cdot AB}{CG}$$

Definitions.

$$EG - \frac{AN}{AN^3 + AN^2 - AN + 1} = 0 \quad CG - \frac{1}{AN^3 + AN^2 - AN + 1} = 0 \quad AR - \frac{AN^4 + AN^3 - AN^2 + AN}{AN + 1} = 0$$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$bk := \frac{N_1 \cdot N_3}{N_1 + N_3} \quad bj := \frac{N_2^{\,2} \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} \quad be := \frac{N_1 \cdot bj}{N_1 + bj} \quad de := \frac{be}{N_1} \quad ar := \frac{bk}{de}$$

Definitions.

$$be - \frac{N_1 \cdot N_2^{\,2} \cdot N_3}{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3} = 0 \quad de - \frac{N_2^{\,2} \cdot N_3}{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3} = 0 \quad ar - \frac{N_1^{\,4} + N_1^{\,3} \cdot N_3 - N_1^{\,2} \cdot N_2 \cdot N_3 + N_1 \cdot N_2^{\,2} \cdot N_3}{N_1 \cdot N_2^{\,2} + N_2^{\,2} \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\dfrac{N_1^{\,4} + N_1^{\,3} - N_1^{\,2} + N_1}{N_1 + 1}$

0, 2, 0. $\dfrac{N_2^{\,2} - N_2 + 2}{2 \cdot N_2^{\,2}}$

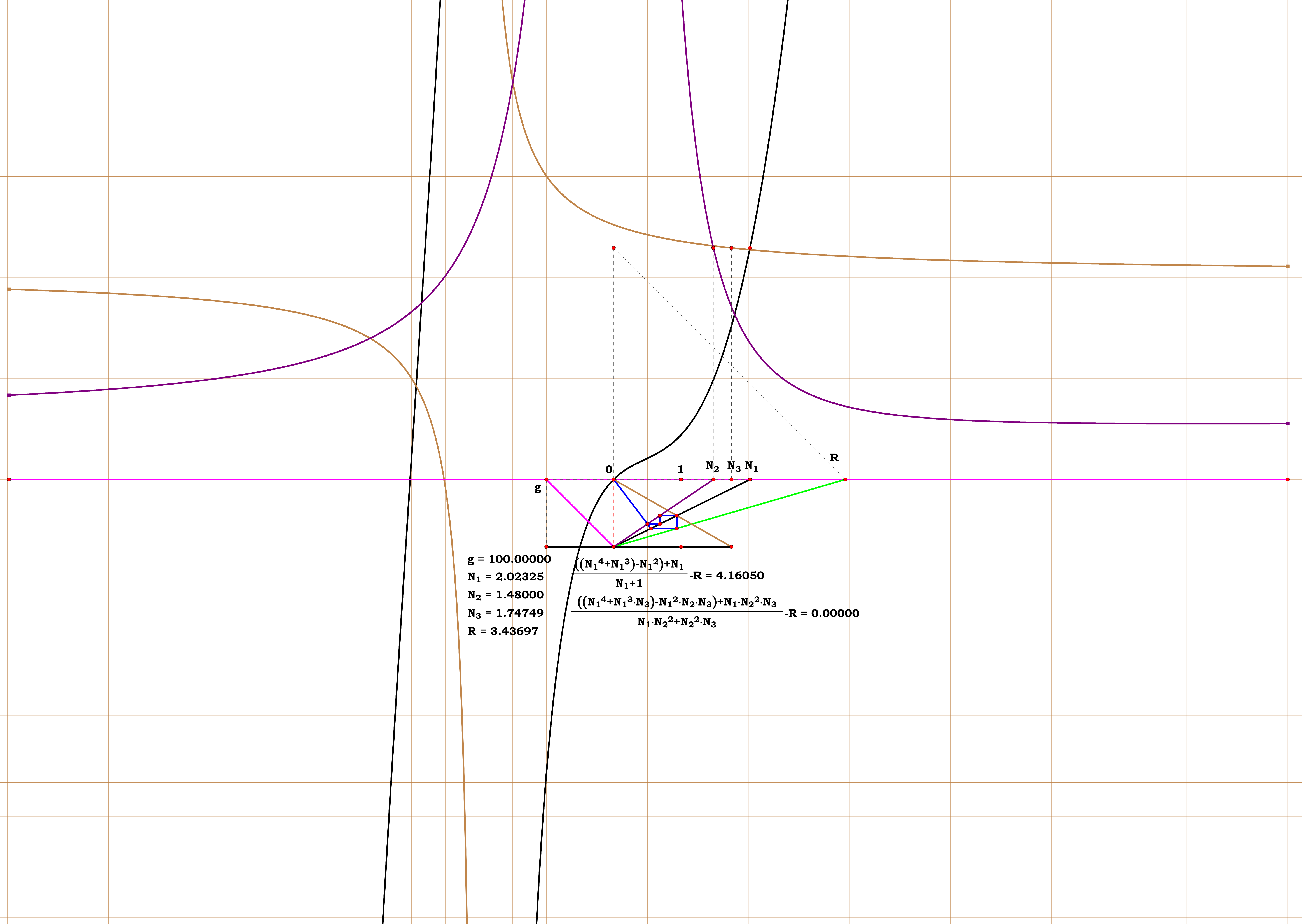
0, 0, 3. 1

1, 2, 0. $\dfrac{N_1^{\,4} + N_1^{\,3} - N_1^{\,2} \cdot N_2 + N_1 \cdot N_2^{\,2}}{N_2^{\,2} \cdot (N_1 + 1)}$

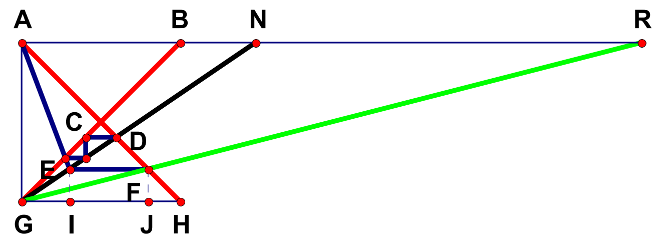
1, 0, 3. $\dfrac{N_1^{\,4} + N_3 \cdot N_1^{\,3} - N_3 \cdot N_1^{\,2} + N_3 \cdot N_1}{N_1 + N_3}$

0, 2, 3. $\dfrac{N_3 \cdot N_2^{\,2} - N_3 \cdot N_2 + N_3 + 1}{N_2^{\,2} \cdot (N_3 + 1)}$

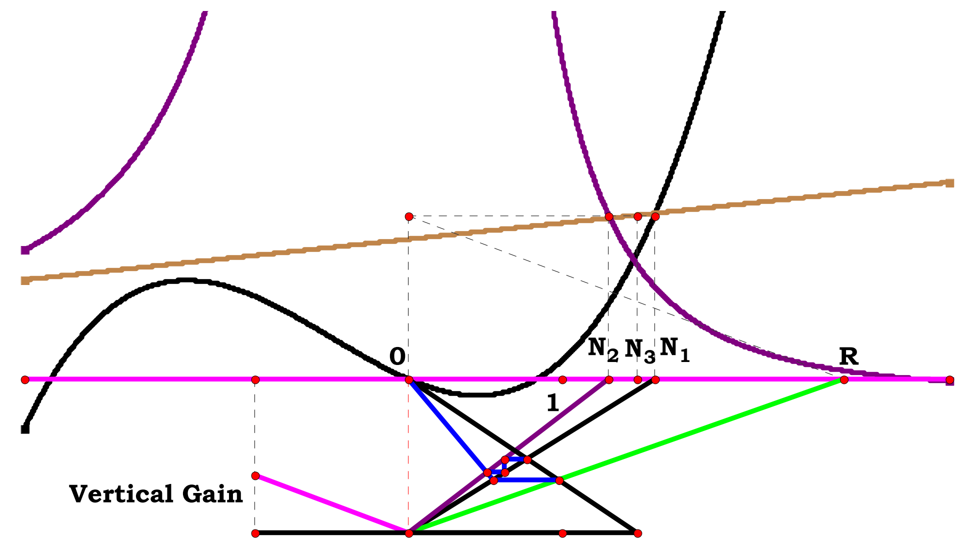
1, 2, 3. $\dfrac{N_1^{\,4} + N_1^{\,3} \cdot N_3 - N_1^{\,2} \cdot N_2 \cdot N_3 + N_1 \cdot N_2^{\,2} \cdot N_3}{N_1 \cdot N_2^{\,2} + N_2^{\,2} \cdot N_3}$



0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 2.02325
N_2 = 1.48000
N_3 = 1.74749
R = 3.43697
$\frac{((N_1^4+N_1^3)-N_1^2)+N_1}{N_1+1}$-R = 4.16050
$\frac{((N_1^4+N_1^3 \cdot N_3)-N_1^2 \cdot N_2 \cdot N_3)+N_1 \cdot N_2^2 \cdot N_3}{N_1 \cdot N_2^2+N_2^2 \cdot N_3}$-R = 0.00000



1CST5R9



$g_v = 37.65851$

$N_1 = 1.59846$

$N_2 = 1.30153$

$N_3 = 1.48720$

$R = 2.82771$

$(N_1^3+N_1^2)-N_1-R = 2.21313$

$$\frac{(N_1^3+N_1^2\cdot N_3)-N_1\cdot N_2\cdot N_3}{N_2^2}-R = 0.00000$$

Unit. $AB := 1$ **Given.** $AN := 4$

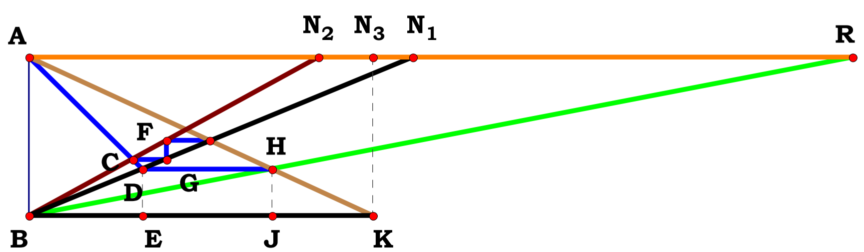
Descriptions.

$$EI := \frac{1}{AN^3 + AN^2 - AN + 1} \qquad GJ := AB - EI$$

$$FJ := EI \qquad AR := \frac{GJ \cdot AB}{FJ}$$

Definitions.

$$AR - \left(AN^3 + AN^2 - AN\right) = 0$$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$de := \frac{N_2^{\,2} \cdot N_3}{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3} \qquad jk := N_3 \cdot de \qquad bj := N_3 - jk \qquad ar := \frac{bj}{de}$$

Descriptions.

$$jk - \frac{N_2^{\,2} \cdot N_3^{\,2}}{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3} = 0 \qquad bj - \frac{N_1^{\,3} \cdot N_3 + N_1^{\,2} \cdot N_3^{\,2} - N_1 \cdot N_3^{\,2} \cdot N_2}{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3 + N_2^{\,2} \cdot N_3} = 0 \qquad ar - \frac{N_1^{\,3} + N_1^{\,2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3}{N_2^{\,2}} = 0$$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $N_1^{3} + N_1^{2} - N_1$

0, 2, 0. $-\dfrac{N_2 - 2}{N_2^{2}}$

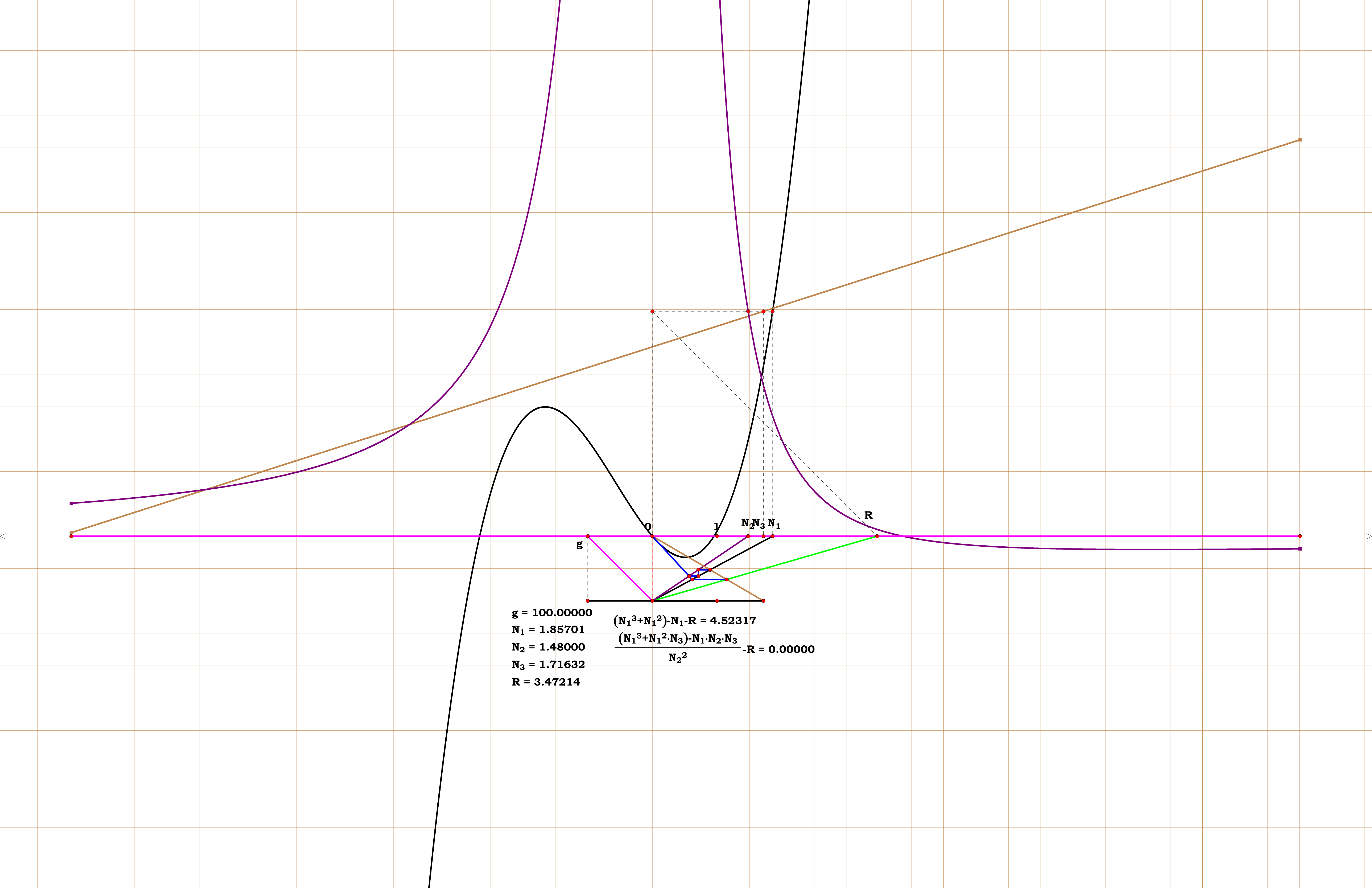
0, 0, 3. 1

1, 2, 0. $\dfrac{N_1^{3} + N_1^{2} - N_2 \cdot N_1}{N_2^{2}}$

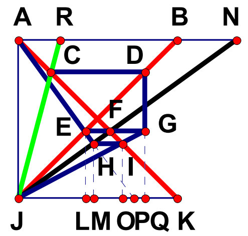
1, 0, 3. $N_1 \cdot \left(N_1^{2} + N_3 \cdot N_1 - N_3\right)$

0, 2, 3. $\dfrac{N_3 - N_2 \cdot N_3 + 1}{N_2^{2}}$

1, 2, 3. $\dfrac{N_1^{3} + N_1^{2} \cdot N_3 - N_1 \cdot N_2 \cdot N_3}{N_2^{2}}$



R
0
1
N_2 N_3 N_1
g
g = 100.00000
N_1 = 1.85701
N_2 = 1.48000
N_3 = 1.71632
R = 3.47214
$(N_1^3+N_1^2)-N_1-R = 4.52317$
$\frac{(N_1^3+N_1^2 \cdot N_3)-N_1 \cdot N_2 \cdot N_3}{N_2^2}-R = 0.00000$



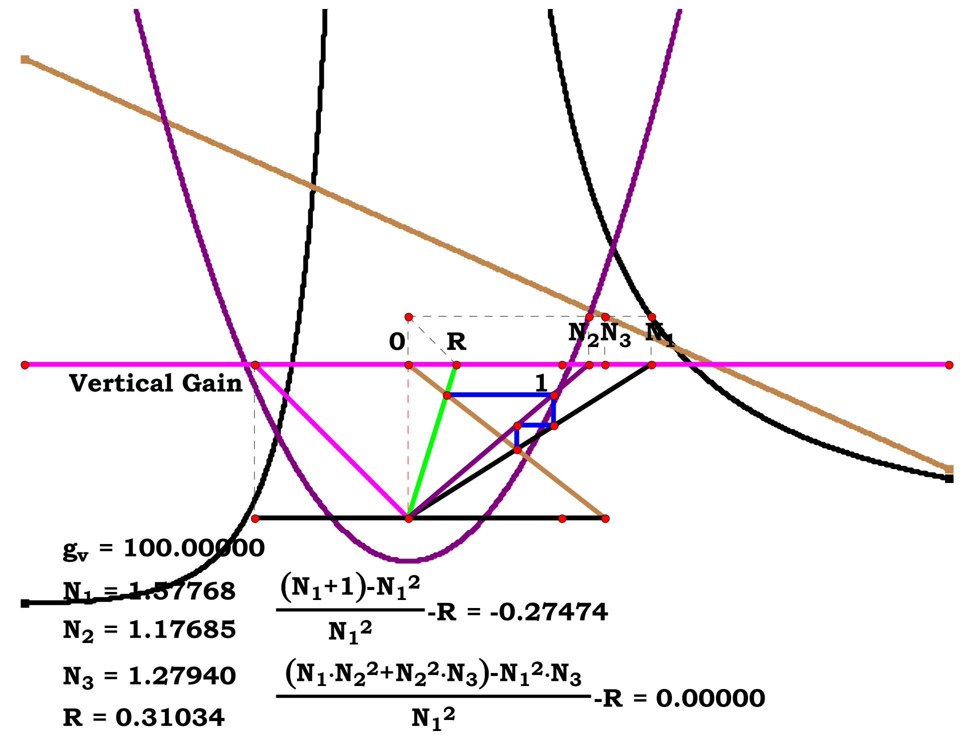
1CST5R10

Unit.

$AB := 1$

Given.

$AN := 8$



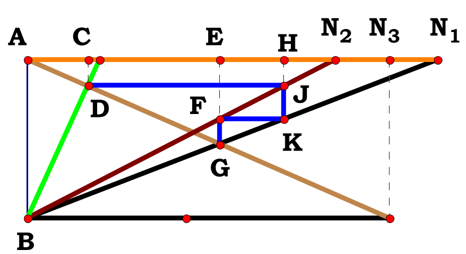
Descriptions.

$KL := \frac{AB \cdot AN}{AB + AN}$ $\quad JL := AB - KL$ $\quad JP := \frac{JL \cdot AB}{KL}$ $\quad JM := \frac{JP \cdot AN}{JP + AN}$

$HM := \frac{AB \cdot JM}{AN}$ $\quad JO := AB - HM$ $\quad JQ := \frac{JO \cdot JL}{HM}$ $\quad KQ := AB - JQ$ $\quad AR := \frac{KQ \cdot AB}{JQ}$

Definitions.

$JQ - \frac{AN^2}{AN + 1} = 0$ $\quad AR - \frac{AN - AN^2 + 1}{AN^2} = 0$



$N_1 := 2.585858$

$N_2 := 2.282828$

$N_3 := 1.939393$

$ae := \frac{N_1 \cdot N_3}{N_1 + N_3}$ $\quad ef := \frac{N_2 - ae}{N_2}$ $\quad ah := N_1 - N_1 \cdot ef$ $\quad hj := \frac{N_2 - ah}{N_2}$ $\quad ac := N_3 \cdot hj$ $\quad ar := \frac{ac}{1 - hj}$

Definitions.

$ef - \frac{N_1 \cdot N_2 + N_2 \cdot N_3 - N_1 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0$ $\quad ah - \frac{N_1{}^2 \cdot N_3}{N_1 \cdot N_2 + N_2 \cdot N_3} = 0$ $\quad hj - \frac{N_1 \cdot N_2{}^2 + N_2{}^2 \cdot N_3 - N_1{}^2 \cdot N_3}{N_1 \cdot N_2{}^2 + N_2{}^2 \cdot N_3} = 0$

$ac - \frac{N_1 \cdot N_2{}^2 \cdot N_3 + N_2{}^2 \cdot N_3{}^2 - N_1{}^2 \cdot N_3{}^2}{N_1 \cdot N_2{}^2 + N_2{}^2 \cdot N_3} = 0$ $\quad ar - \frac{N_1 \cdot N_2{}^2 + N_2{}^2 \cdot N_3 - N_1{}^2 \cdot N_3}{N_1{}^2} = 0$

Three Transforms.

0, 0, 0. 1

1, 0, 0. $\dfrac{N_1 - N_1^{\,2} + 1}{N_1^{\,2}}$

0, 2, 0. $2 \cdot N_2^{\,2} - 1$

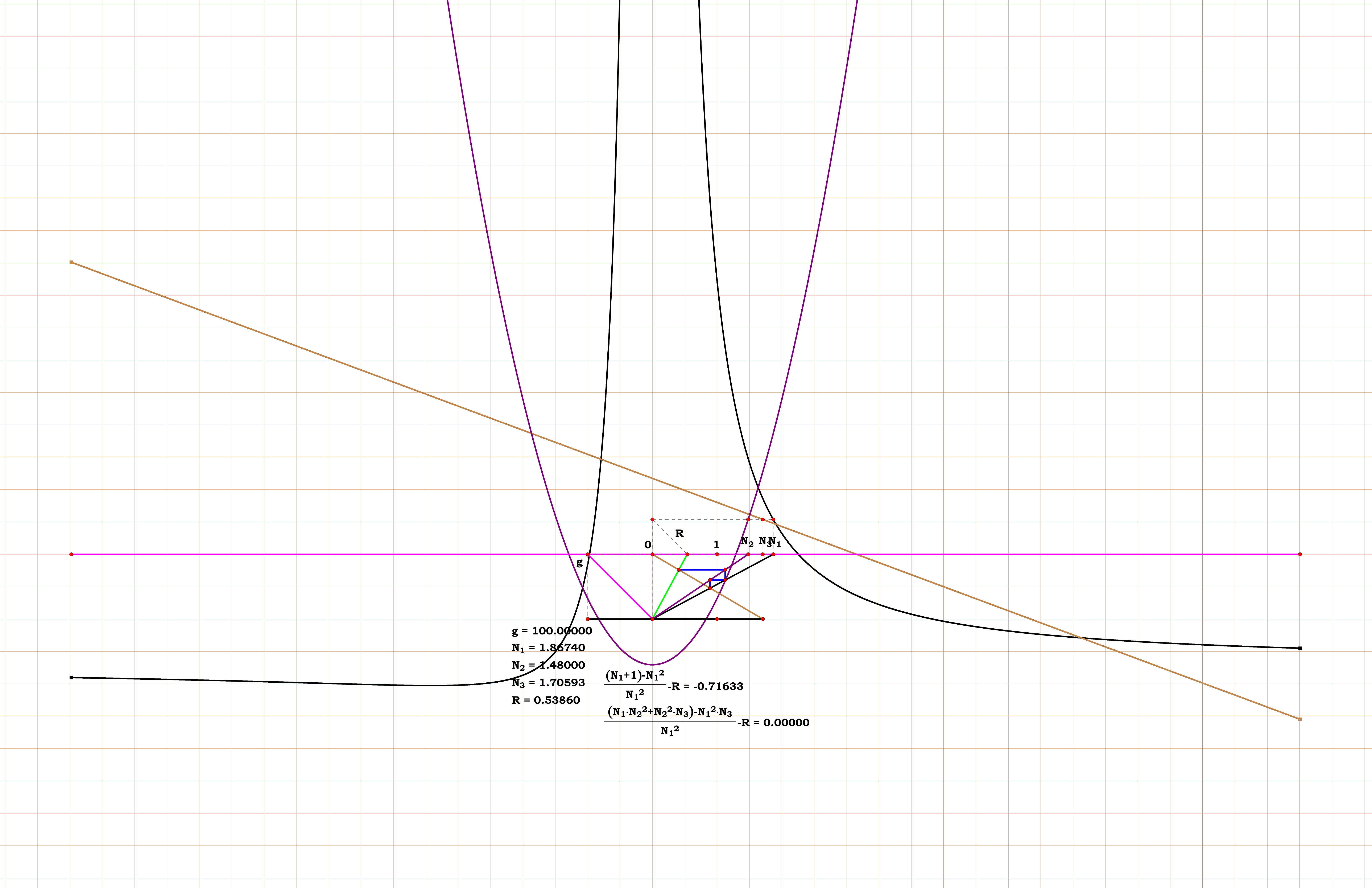
0, 0, 3. 1

1, 2, 0. $\dfrac{N_1 \cdot N_2^{\,2} - N_1^{\,2} + N_2^{\,2}}{N_1^{\,2}}$

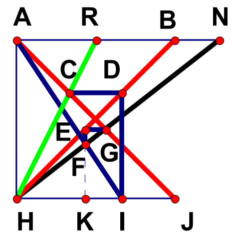
1, 0, 3. $\dfrac{N_1 - N_3 \cdot N_1^{\,2} + N_3}{N_1^{\,2}}$

0, 2, 3. $N_2^{\,2} \cdot N_3 - N_3 + N_2^{\,2}$

1, 2, 3. $\dfrac{N_1 \cdot N_2^{\,2} + N_2^{\,2} \cdot N_3 - N_1^{\,2} \cdot N_3}{N_1^{\,2}}$



R
0
1
N_2
N_3
N_1
g
g = 100.00000
N_1 = 1.86740
N_2 = 1.48000
N_3 = 1.70593
R = 0.53860
$\frac{(N_1+1)-N_1^2}{N_1^2}-R = -0.71633$
$\frac{(N_1 \cdot N_2^2+N_2^2 \cdot N_3)-N_1^2 \cdot N_3}{N_1^2}-R = 0.00000$



1CST5R11

Unit.

$AB := 1$

Given.

$AN := 3$

Descriptions.

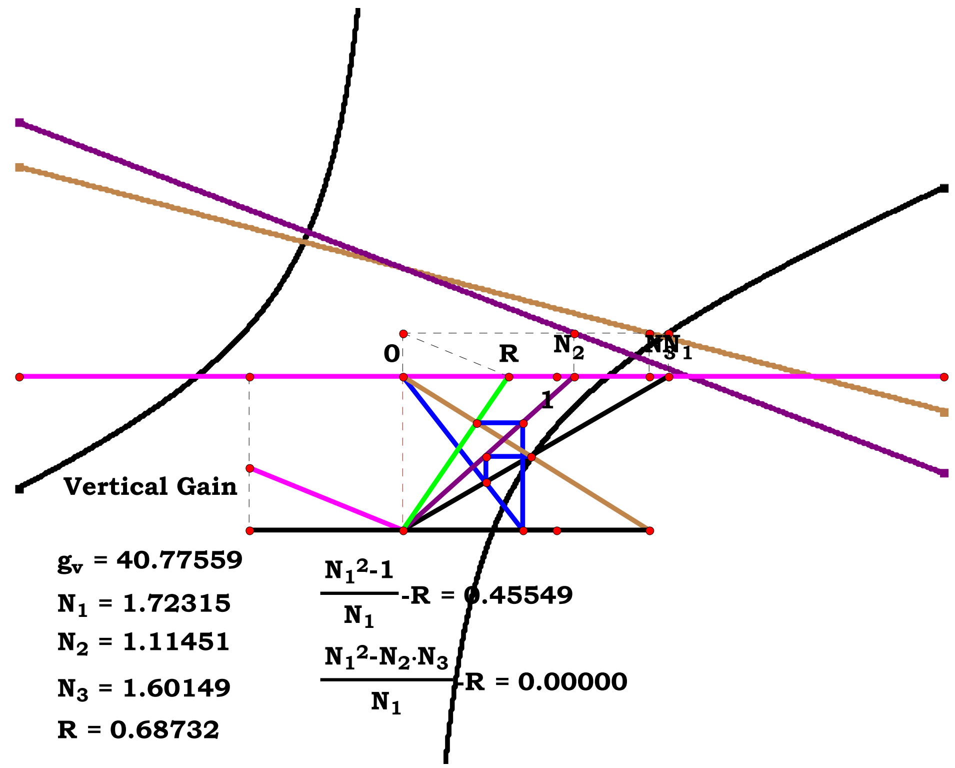
$$JK := \frac{AB \cdot AN}{AB + AN} \quad HK := AB - JK \quad FK := \frac{AB \cdot HK}{AN}$$

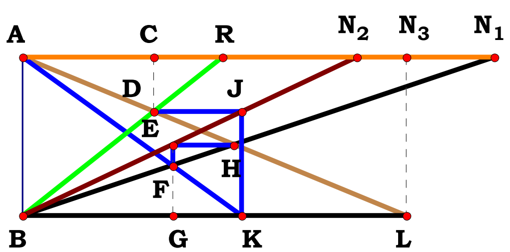
$$HI := \frac{HK \cdot AB}{AB - FK} \quad JI := AB - HI \quad AR := \frac{JI \cdot AB}{HI}$$

Definitions.

$$HK - \frac{1}{1 + AN} = 0 \qquad FK - \frac{1}{AN^2 + AN} = 0$$

$$HI - \frac{AN}{AN^2 + AN - 1} = 0 \qquad AR - \frac{AN^2 - 1}{AN} = 0$$





$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$eg := \frac{N_3}{N_1 + N_3} \quad bg := N_2 \cdot eg \quad ef := \frac{bg}{N_1} \quad bk := \frac{bg}{1 - ef} \quad hj := \frac{bk}{N_2} \quad ac := N_3 - N_3 \cdot hj \quad ar := \frac{ac}{hj}$$

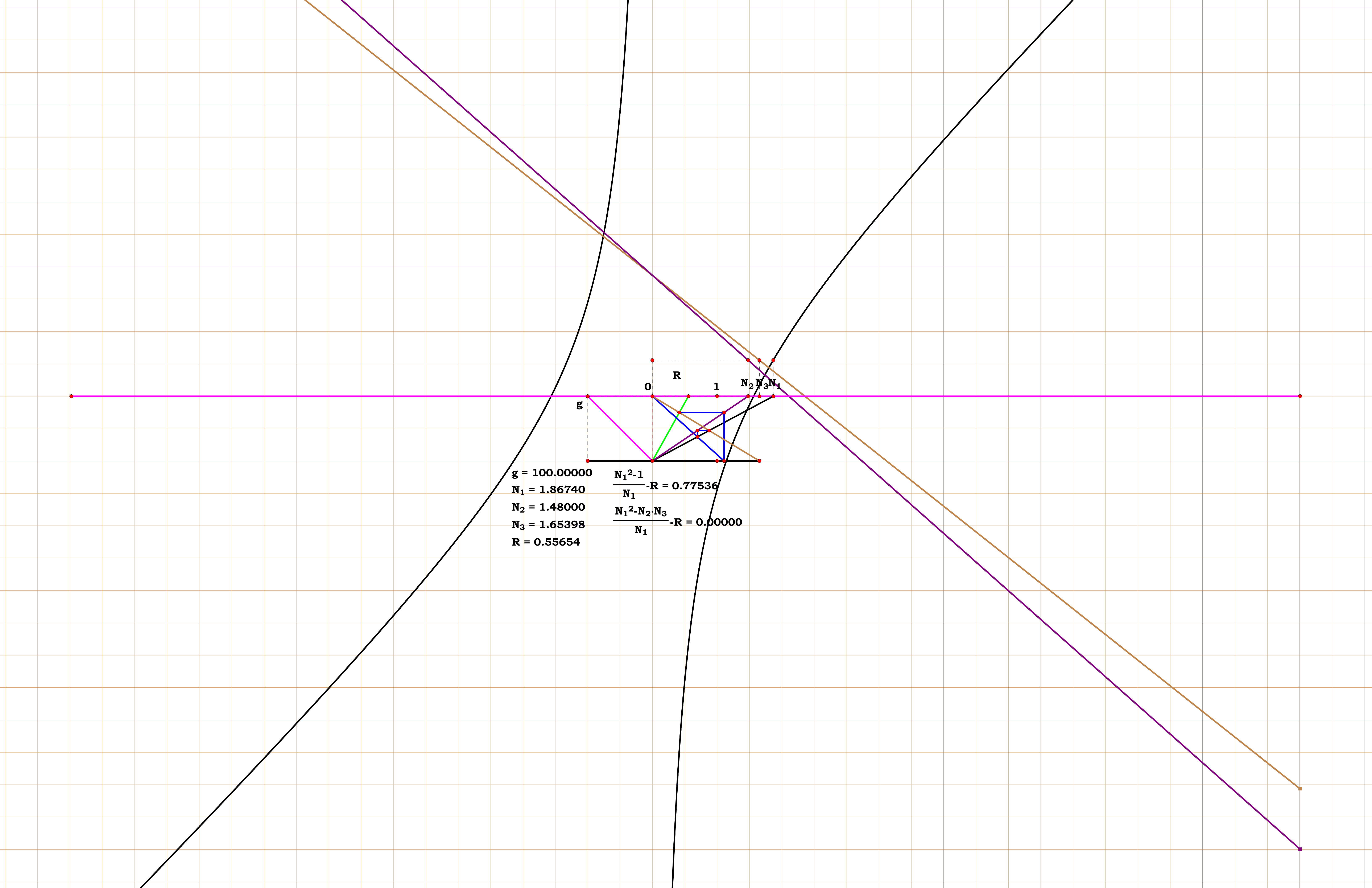
Definitions.

$$bg - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0 \quad ef - \frac{N_2 \cdot N_3}{N_1^2 + N_1 \cdot N_3} = 0 \quad bk - \frac{N_1 \cdot N_2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \quad hj - \frac{N_1 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

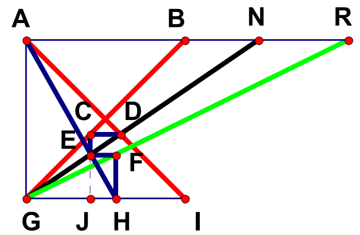
$$ac - \frac{N_1^2 \cdot N_3 - N_2 \cdot N_3^2}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1^2 - N_2 \cdot N_3}{N_1} = 0$$

Three Transforms.

0, 0, 0.	0	1, 2, 0.	$\dfrac{N_1^{\,2} - N_2}{N_1}$
1, 0, 0.	$\dfrac{N_1^{\,2} - 1}{N_1}$	1, 0, 3.	$\dfrac{N_1^{\,2} - N_3}{N_1}$
0, 2, 0.	$1 - N_2$	0, 2, 3.	$1 - N_2 \cdot N_3$
0, 0, 3.	$1 - N_3$	1, 2, 3.	$\dfrac{N_1^{\,2} - N_2 \cdot N_3}{N_1}$



R
0
1
N_2 N_3 N_1
g
g = 100.00000
N_1 = 1.86740
N_2 = 1.48000
N_3 = 1.65398
R = 0.55654
$\frac{N_1^2-1}{N_1}$-R = 0.77536
$\frac{N_1^2-N_2 \cdot N_3}{N_1}$-R = 0.00000



1CST5R12

Unit.

$AB := 1$

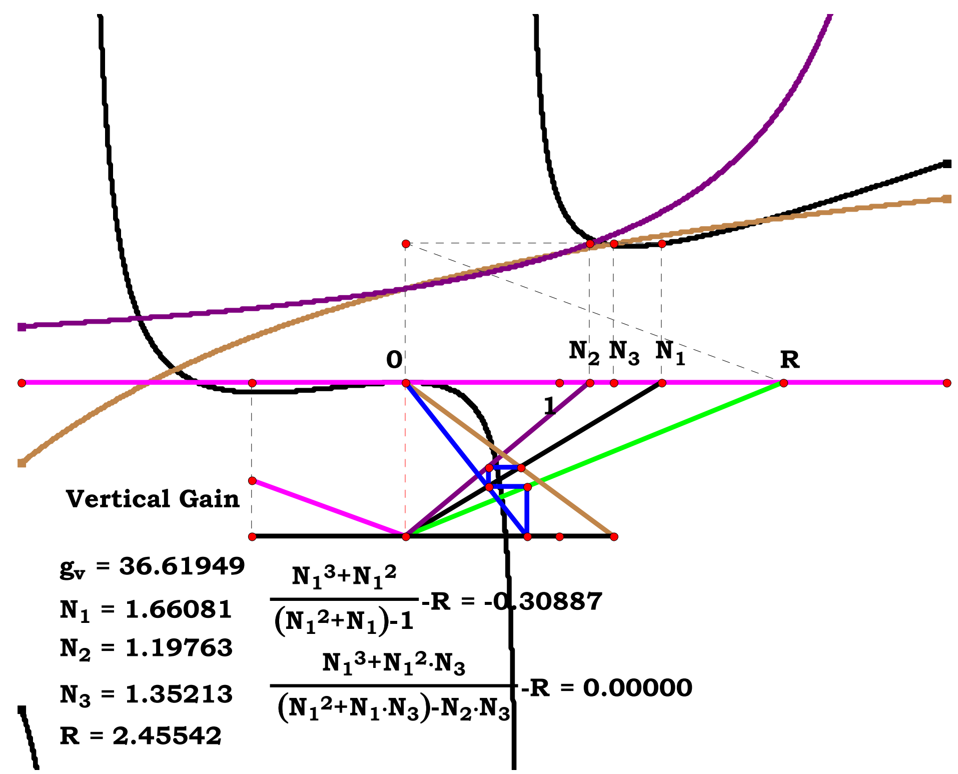
Given.

$AN := 3$

Descriptions.

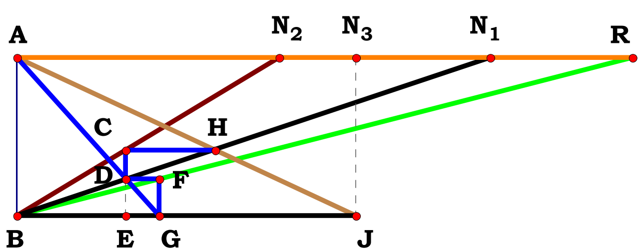
$$EJ := \frac{1}{AN^2 + AN} \qquad GH := \frac{AN}{AN^2 + AN - 1}$$

$$AR := \frac{GH \cdot AB}{EJ}$$



Definitions.

$$AR - \frac{AN^3 + AN^2}{AN^2 + AN - 1} = 0$$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$

$$bg := \frac{N_1 \cdot N_2 \cdot N_3}{N_1{}^2 + N_1 \cdot N_3 - N_2 \cdot N_3} \qquad de := \frac{bg}{bg + N_1} \qquad ar := \frac{bg}{de}$$

Definitions.

$$de - \frac{N_2 \cdot N_3}{N_1{}^2 + N_1 \cdot N_3} = 0 \qquad ar - \frac{N_1{}^3 + N_1{}^2 \cdot N_3}{N_1{}^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. 2

1, 0, 0. $\dfrac{N_1^{\,2}\cdot\left(N_1+1\right)}{N_1^{\,2}+N_1-1}$

0, 2, 0. $-\dfrac{2}{N_2-2}$

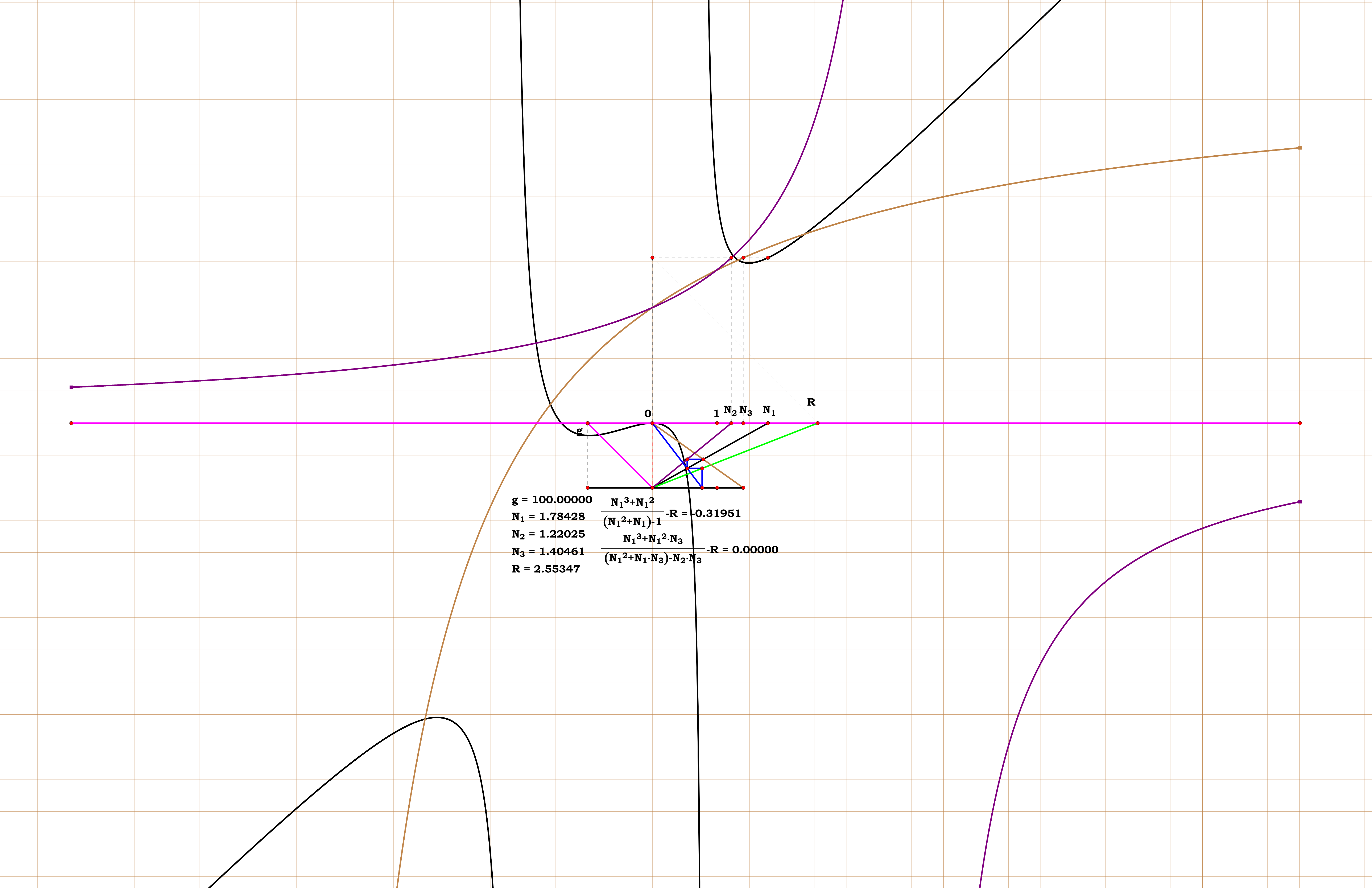
0, 0, 3. N_3+1

1, 2, 0. $\dfrac{N_1^{\,2}\cdot\left(N_1+1\right)}{N_1^{\,2}+N_1-N_2}$

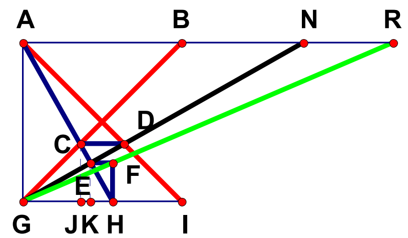
1, 0, 3. $\dfrac{N_1^{\,2}\cdot\left(N_1+N_3\right)}{N_1^{\,2}+N_3\cdot N_1-N_3}$

0, 2, 3. $\dfrac{N_3+1}{N_3-N_2\cdot N_3+1}$

1, 2, 3. $\dfrac{N_1^{\,3}+N_1^{\,2}\cdot N_3}{N_1^{\,2}+N_1\cdot N_3-N_2\cdot N_3}$



R
0
1
N_2
N_3
N_1
g
g = 100.00000
N_1 = 1.78428
N_2 = 1.22025
N_3 = 1.40461
R = 2.55347
$\frac{N_1^3+N_1^2}{(N_1^2+N_1)-1}-R = -0.31951$
$\frac{N_1^3+N_1^2 \cdot N_3}{(N_1^2+N_1 \cdot N_3)-N_2 \cdot N_3}-R = 0.00000$



1CST5R13

Unit.

$AB := 1$

Given.

$AN := 3$

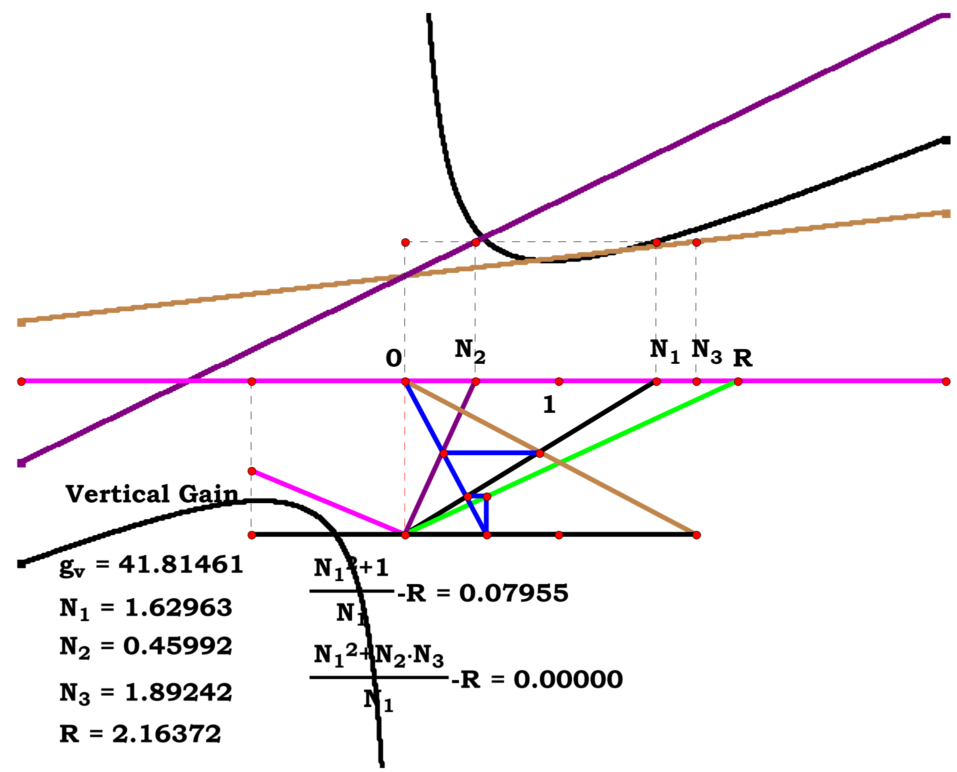
Descriptions.

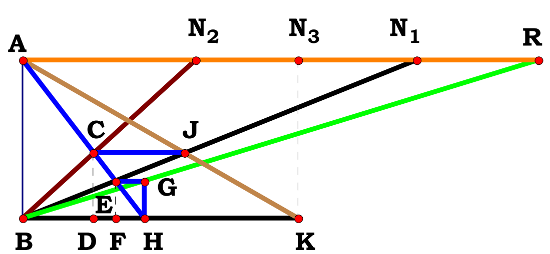
$$IJ := \frac{AB \cdot AN}{AB + AN} \quad GJ := AB - IJ \quad GH := \frac{GJ \cdot AB}{IJ}$$

$$GK := \frac{GH \cdot AN}{GH + AN} \quad EK := \frac{AB \cdot GK}{AN} \quad AR := \frac{GH \cdot AB}{EK}$$

Definitions.

$$EK - \frac{1}{AN^2 + 1} = 0 \quad AR - \frac{AN^2 + 1}{AN} = 0$$





$N_1 := 5$

$N_2 := 3$

$$N_3 := 2 \quad cd := \frac{N_3}{N_1 + N_3} \quad bd := N_2 \cdot cd \quad bh := \frac{bd}{1 - cd} \quad ef := \frac{bh}{bh + N_1} \quad ar := \frac{bh}{ef}$$

Definitions.

$$bd - \frac{N_2 \cdot N_3}{N_1 + N_3} = 0 \quad bh - \frac{N_2 \cdot N_3}{N_1} = 0 \quad ef - \frac{N_2 \cdot N_3}{N_1^{\,2} + N_2 \cdot N_3} = 0 \quad ar - \frac{N_1^{\,2} + N_2 \cdot N_3}{N_1} = 0$$

Three Transforms.

0, 0, 0. $\mathbf{2}$

1, 0, 0. $\dfrac{N_1^2 + 1}{N_1}$

0, 2, 0. $N_2 + 1$

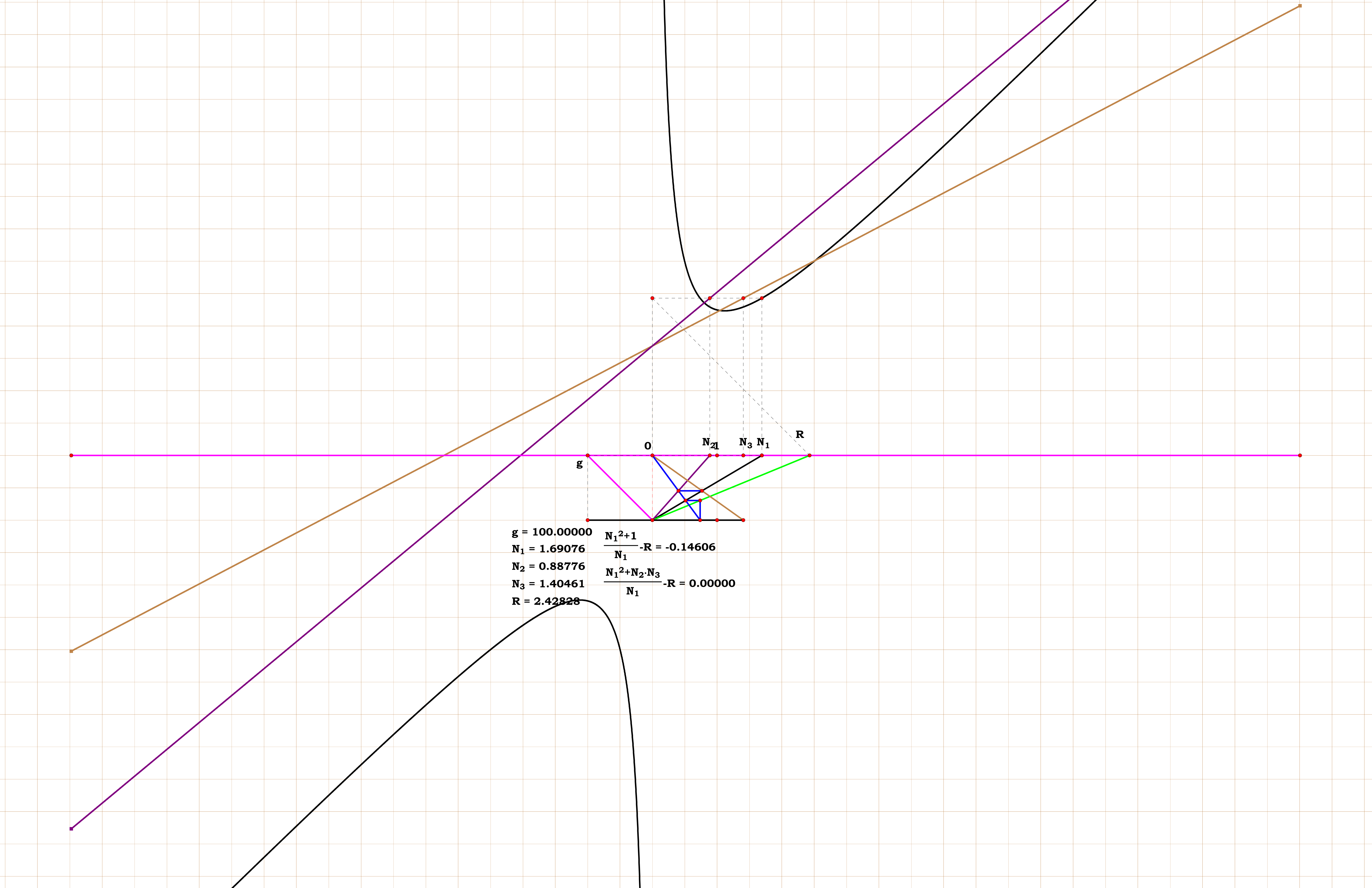
0, 0, 3. $N_3 + 1$

1, 2, 0. $\dfrac{N_1^2 + N_2}{N_1}$

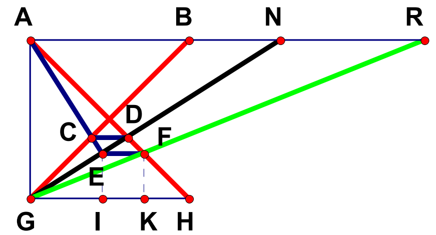
1, 0, 3. $\dfrac{N_1^2 + N_3}{N_1}$

0, 2, 3. $N_2 \cdot N_3 + 1$

1, 2, 3. $\dfrac{N_1^2 + N_2 \cdot N_3}{N_1}$



0
N_2
1
N_3
N_1
R
g
g = 100.00000
N_1 = 1.69076
N_2 = 0.88776
N_3 = 1.40461
R = 2.42828
$\frac{N_1^2+1}{N_1}$-R = -0.14606
$\frac{N_1^2+N_2 \cdot N_3}{N_1}$-R = 0.00000



1CST5R14

Unit.

$AB := 1$

Given.

$AN := 3$

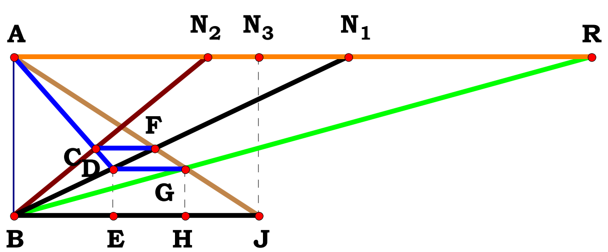
Descriptions.

$EI := \dfrac{1}{AN^2 + 1}$ $\quad HK := EI$

$GK := AB - EI$ $\quad AR := \dfrac{GK \cdot AB}{HK}$

Definitions.

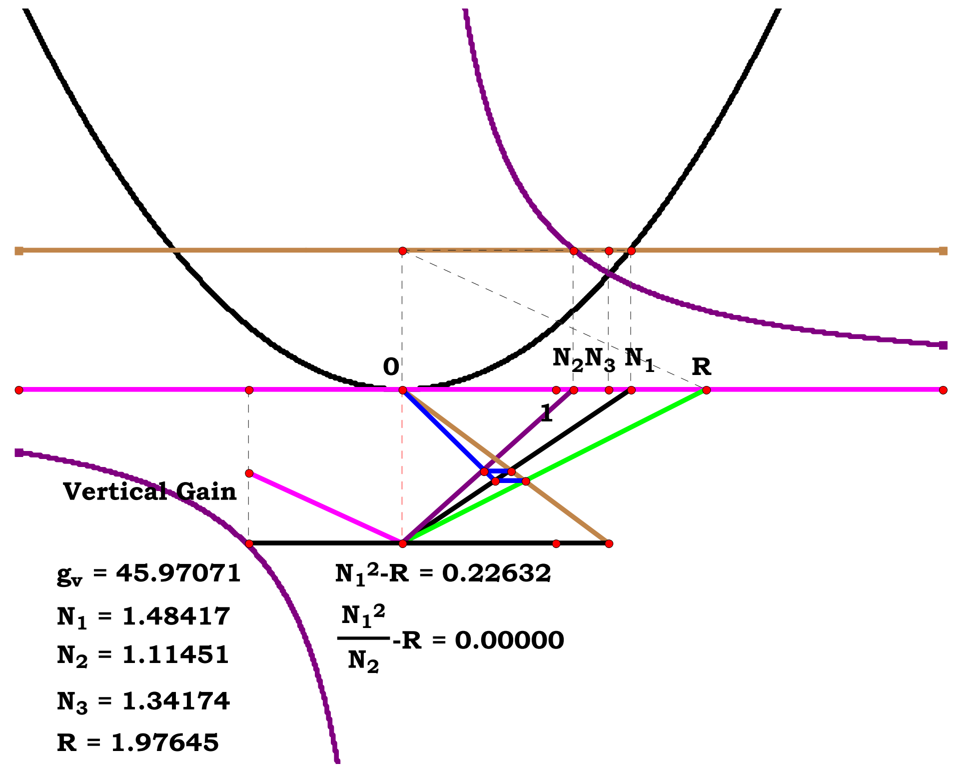
$AR - AN^2 = 0$



$N_1 := 5$

$N_2 := 3$

$N_3 := 2$



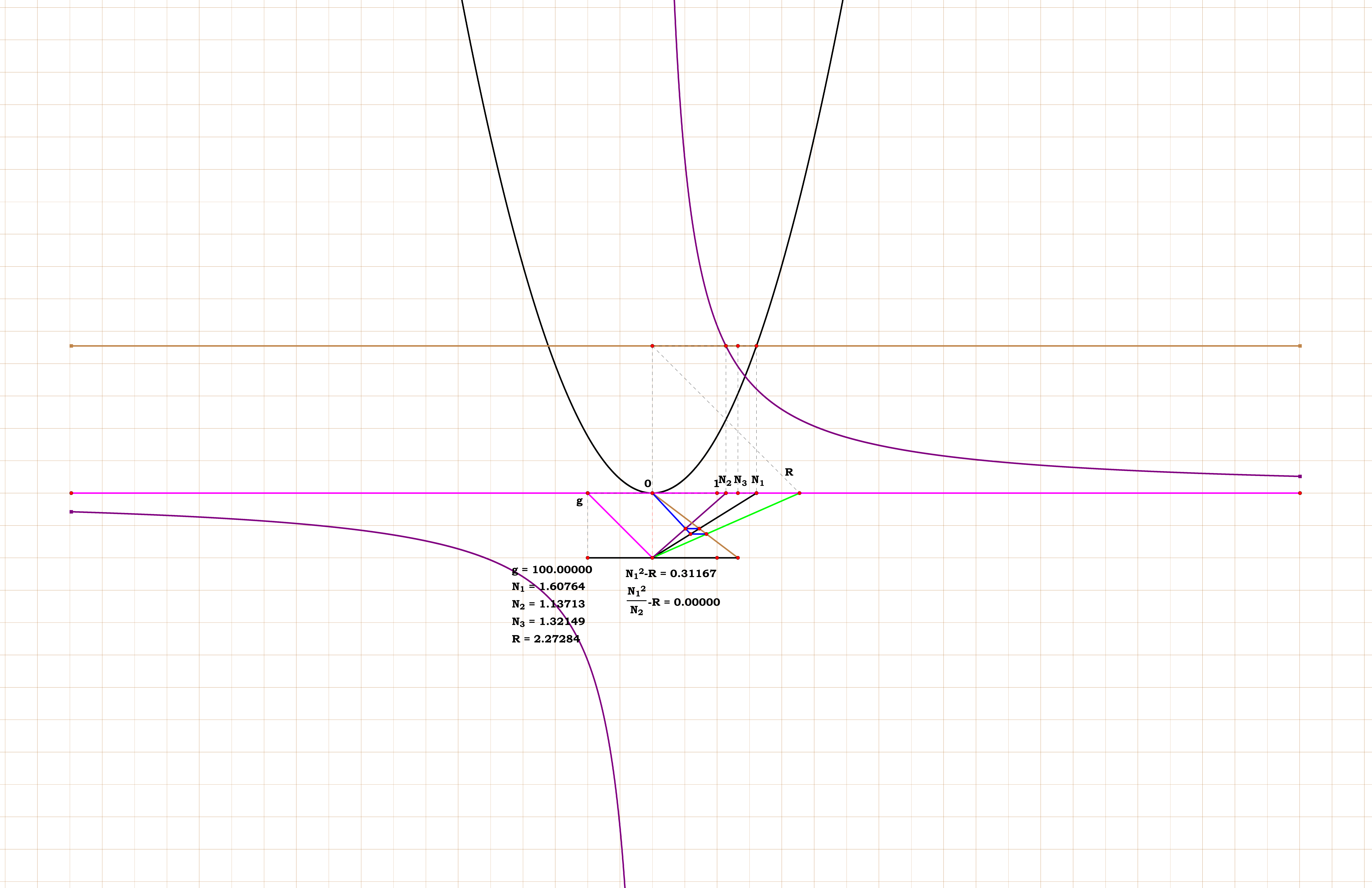
$de := \dfrac{N_2 \cdot N_3}{N_1^{\,2} + N_2 \cdot N_3}$ $\quad bh := N_3 - N_3 \cdot de$ $\quad ar := \dfrac{bh}{de}$

Descriptions.

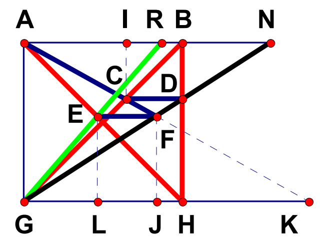
$bh - \dfrac{N_1^{\,2} \cdot N_3}{N_1^{\,2} + N_2 \cdot N_3} = 0$ $\quad ar - \dfrac{N_1^{\,2}}{N_2} = 0$

Three Transforms.

0, 0, 0.	1	1, 2, 0.	$\frac{N_1^{\,2}}{N_2}$
1, 0, 0.	$N_1^{\,2}$	1, 0, 3.	$N_1^{\,2}$
0, 2, 0.	$\frac{1}{N_2}$	0, 2, 3.	$\frac{1}{N_2}$
0, 0, 3.	1	1, 2, 3.	$\frac{N_1^{\,2}}{N_2}$



0
1
N_2
N_3
N_1
R
g
g = 100.00000
N_1 = 1.60764
N_2 = 1.13713
N_3 = 1.32149
R = 2.27284
N_1^2-R = 0.31167
$\frac{N_1^2}{N_2}$-R = 0.00000



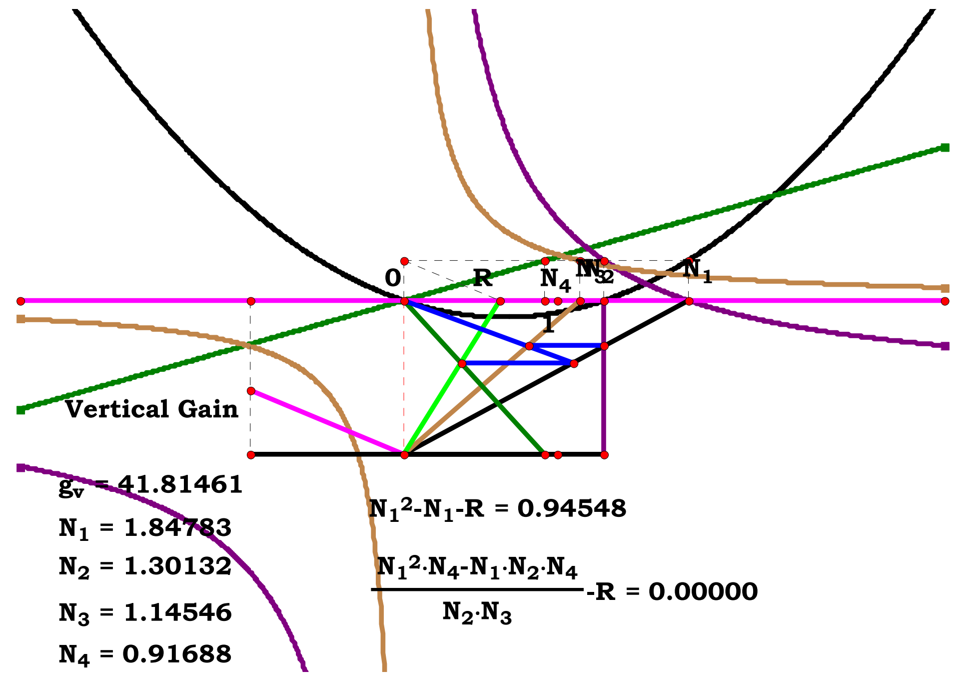
1CST6R0

Unit.

$AB := 1$

Given.

$AN := 3$



Descriptions.

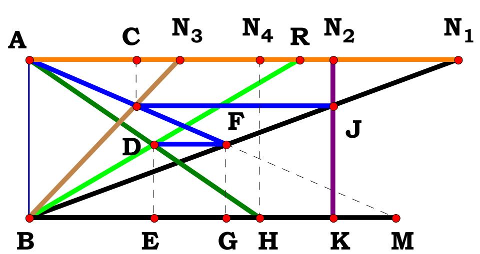
$$AI := \frac{1}{AN} \quad CI := \frac{AN-1}{AN} \quad GK := \frac{AI \cdot AB}{CI}$$

$$GJ := \frac{GK \cdot AN}{GK + AN} \quad FJ := \frac{AB \cdot GJ}{AN}$$

$$GL := AB - FJ \quad AR := \frac{GL \cdot AB}{FJ}$$

Definitions.

$$AR - \left(AN^2 - AN\right) = 0$$



$$N_1 := 5 \quad N_2 := 4 \quad N_3 := 3 \quad N_4 := 2$$

$$jn2 := \frac{N_1 - N_2}{N_1} \quad ac := N_3 - N_3 \cdot jn2 \quad bm := \frac{ac}{jn2} \quad fg := \frac{bm}{bm + N_1} \quad be := N_4 - N_4 \cdot fg \quad ar := \frac{be}{fg}$$

$$ac - \frac{N_2 \cdot N_3}{N_1} = 0 \quad bm - \frac{N_2 \cdot N_3}{N_1 - N_2} = 0 \quad fg - \frac{N_2 \cdot N_3}{N_1^2 - N_1 \cdot N_2 + N_2 \cdot N_3} = 0 \quad be - \frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_1^2 - N_1 \cdot N_2 + N_2 \cdot N_3} = 0 \quad ar - \frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2 \cdot N_3} = 0$$

Four Transforms.

0, 0, 0, 0. 0

1, 0, 0, 0. $N_1 \cdot (N_1 - 1)$

0, 2, 0, 0. $-\dfrac{N_2 - 1}{N_2}$

0, 0, 3, 0. 0

0, 0, 0, 4. 0

1, 2, 0, 0. $\dfrac{N_1^{\,2} - N_1 \cdot N_2}{N_2}$

1, 0, 3, 0. $\dfrac{N_1 \cdot (N_1 - 1)}{N_3}$

1, 0, 0, 4. $N_1^{\,2} \cdot N_4 - N_1 \cdot N_4$

0, 2, 3, 0. $-\dfrac{N_2 - 1}{N_2 \cdot N_3}$

0, 2, 0, 4. $-\dfrac{N_4 \cdot (N_2 - 1)}{N_2}$

0, 0, 3, 4. 0

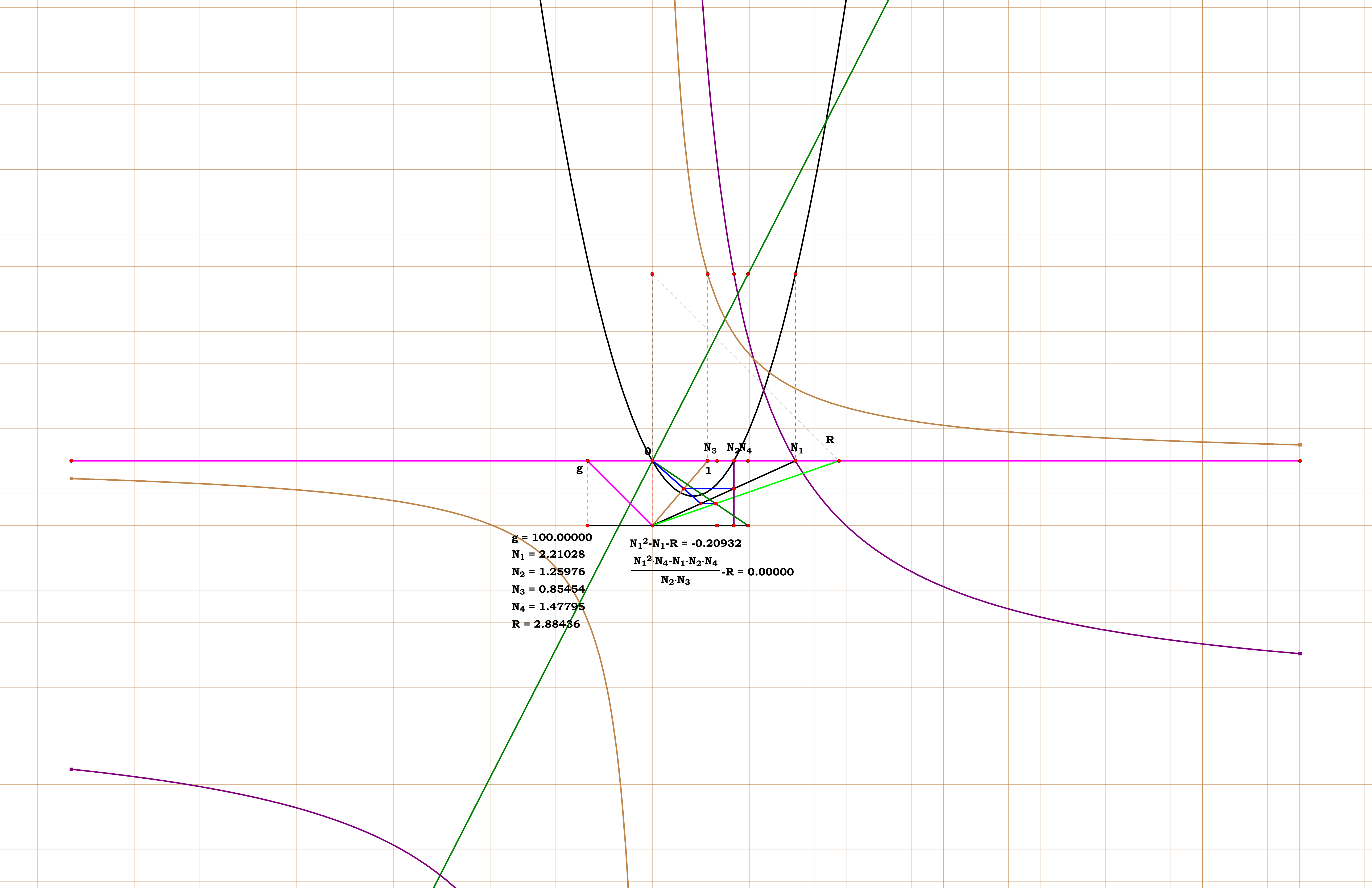
1, 2, 3, 0. $\dfrac{N_1^{\,2} - N_1 \cdot N_2}{N_2 \cdot N_3}$

1, 0, 3, 4. $\dfrac{N_1 \cdot N_4 \cdot (N_1 - 1)}{N_3}$

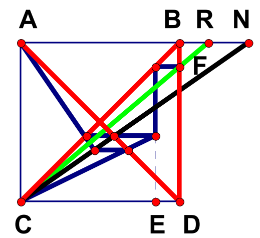
0, 2, 3, 4. $-\dfrac{N_4 \cdot (N_2 - 1)}{N_2 \cdot N_3}$

1, 2, 0, 4. $\dfrac{N_1 \cdot N_4 \cdot (N_1 - N_2)}{N_2}$

1, 2, 3, 4. $\dfrac{N_1^{\,2} \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2 \cdot N_3}$



O
g
1
N_3
N_2
N_4
N_1
R
g = 100.00000
N_1 = 2.21028
N_2 = 1.25976
N_3 = 0.85454
N_4 = 1.47795
R = 2.88436
$N_1^2 - N_1 - R = -0.20932$
$\frac{N_1^2 \cdot N_4 - N_1 \cdot N_2 \cdot N_4}{N_2 \cdot N_3} - R = 0.00000$



1CST6R1

Unit.

$AB := 1$

Given.

$AN := 8$

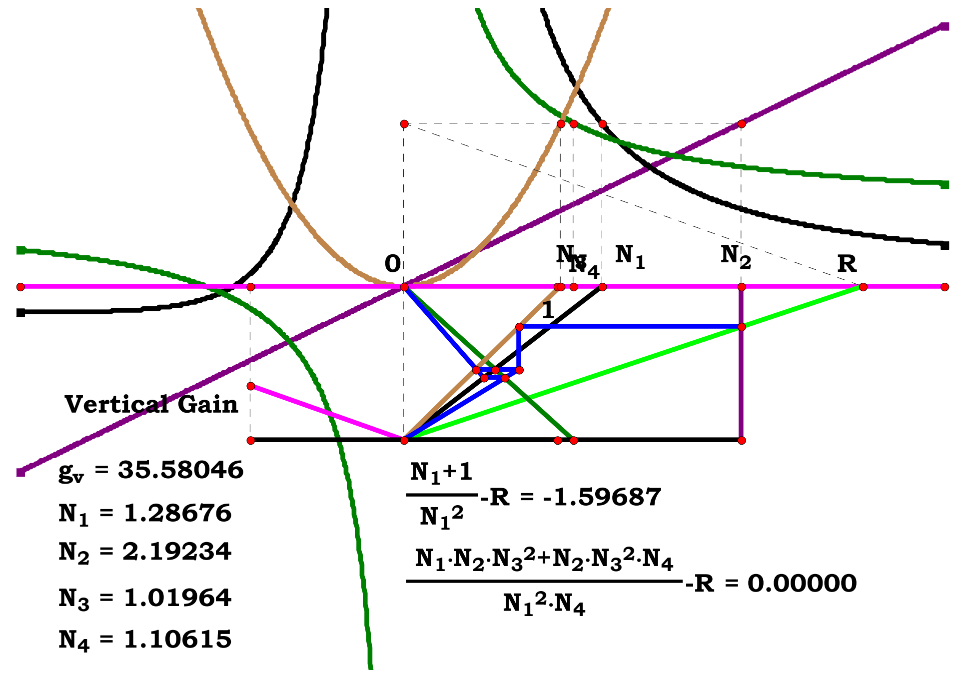
Descriptions.

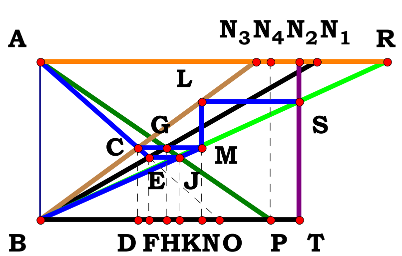
$$CE := \frac{AN^2}{AN+1} \qquad DE := AB - CE$$

$$BF := DE \qquad AR := \frac{AB}{CE}$$

Definitions.

$$AR - \frac{AN+1}{AN^2} = 0$$





$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

$$gh := \frac{N_4}{N_1 + N_4} \qquad bd := N_3 \cdot gh \qquad bo := \frac{bd}{1 - gh} \qquad ef := \frac{bo}{bo + N_1}$$

$$bk := N_4 - N_4 \cdot ef \qquad bn := \frac{bk \cdot gh}{ef} \qquad ln := \frac{bn}{N_3} \qquad ar := \frac{N_2}{ln}$$

Definitions.

$$bd - \frac{N_3 \cdot N_4}{N_1 + N_4} = 0 \qquad bo - \frac{N_3 \cdot N_4}{N_1} = 0 \qquad ef - \frac{N_3 \cdot N_4}{N_1^2 + N_3 \cdot N_4} = 0 \qquad bk - \frac{N_1^2 \cdot N_4}{N_1^2 + N_3 \cdot N_4} = 0$$

$$bn - \frac{N_1^2 \cdot N_4}{N_1 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad ln - \frac{N_1^2 \cdot N_4}{N_1 \cdot N_3^2 + N_3^2 \cdot N_4} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_3^2 + N_2 \cdot N_3^2 \cdot N_4}{N_1^2 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. 2

1, 0, 0, 0. $\dfrac{N_1 + 1}{N_1^{\,2}}$

0, 2, 0, 0. $2 \cdot N_2$

0, 0, 3, 0. $2 \cdot N_3^{\,2}$

0, 0, 0, 4. $\dfrac{N_4 + 1}{N_4}$

1, 2, 0, 0. $\dfrac{N_2 \cdot (N_1 + 1)}{N_1^{\,2}}$

1, 0, 3, 0. $\dfrac{N_1 \cdot N_3^{\,2} + N_3^{\,2}}{N_1^{\,2}}$

1, 0, 0, 4. $\dfrac{N_1 + N_4}{N_1^{\,2} \cdot N_4}$

0, 2, 3, 0. $2 \cdot N_2 \cdot N_3^{\,2}$

0, 2, 0, 4. $\dfrac{N_2 \cdot (N_4 + 1)}{N_4}$

0, 0, 3, 4. $\dfrac{N_3^{\,2} \cdot N_4 + N_3^{\,2}}{N_4}$

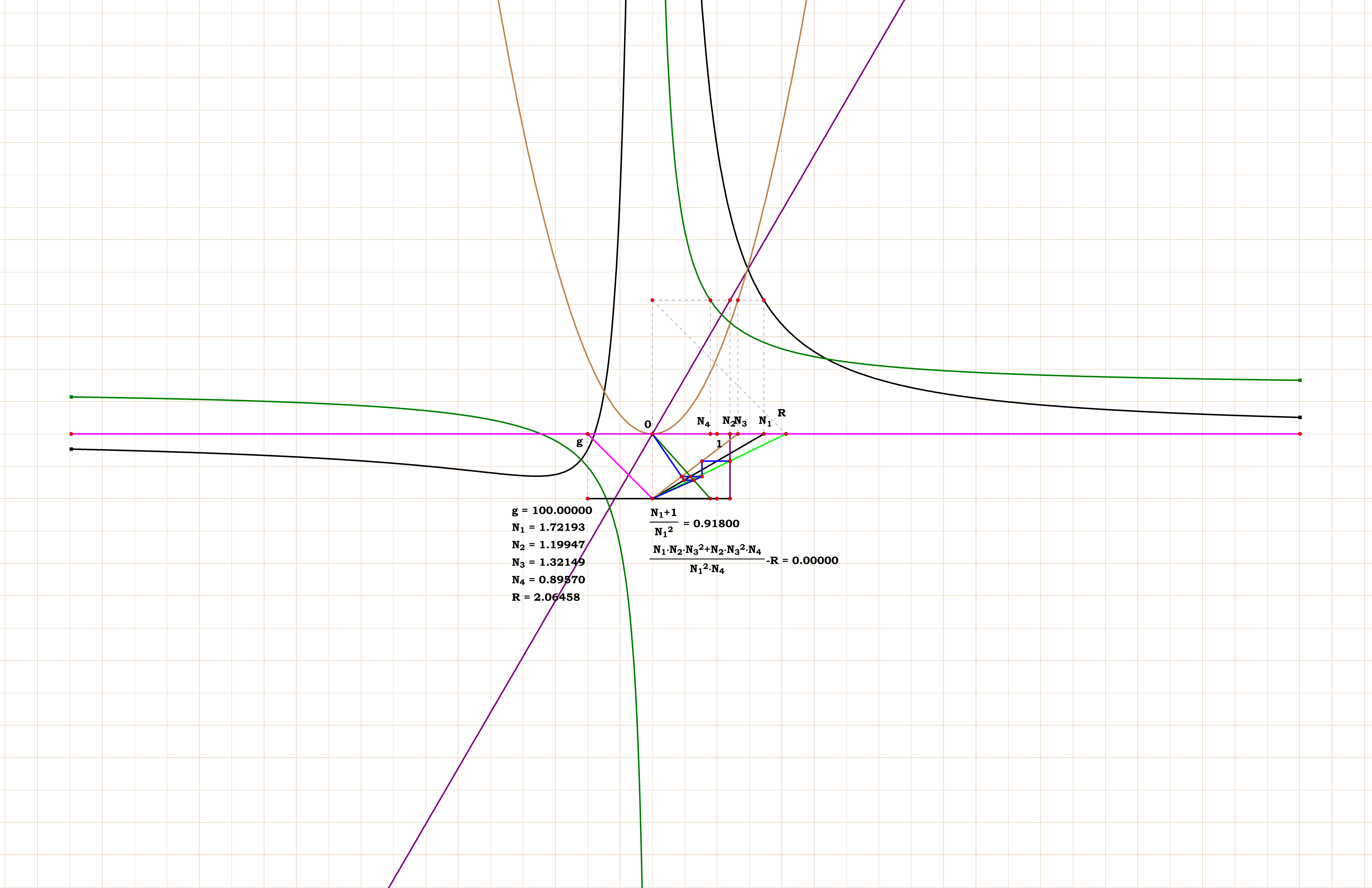
1, 2, 3, 0. $\dfrac{N_2 \cdot N_3^{\,2} \cdot (N_1 + 1)}{N_1^{\,2}}$

1, 0, 3, 4. $\dfrac{N_3^{\,2} \cdot (N_1 + N_4)}{N_1^{\,2} \cdot N_4}$

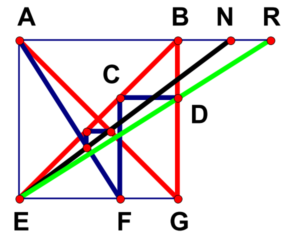
0, 2, 3, 4. $\dfrac{N_2 \cdot N_3^{\,2} \cdot (N_4 + 1)}{N_4}$

1, 2, 0, 4. $\dfrac{N_2 \cdot (N_1 + N_4)}{N_1^{\,2} \cdot N_4}$

1, 2, 3, 4. $\dfrac{N_1 \cdot N_2 \cdot N_3^{\,2} + N_2 \cdot N_3^{\,2} \cdot N_4}{N_1^{\,2} \cdot N_4}$



R
0
N4
N2
N3
N1
g
1
g = 100.00000
N1 = 1.72193
N2 = 1.19947
N3 = 1.32149
N4 = 0.89570
R = 2.06458
(N1+1)/N1^2 = 0.91800
(N1·N2·N3^2+N2·N3^2·N4)/(N1^2·N4)-R = 0.00000



1CST6R2

Unit.

$AB := 1$

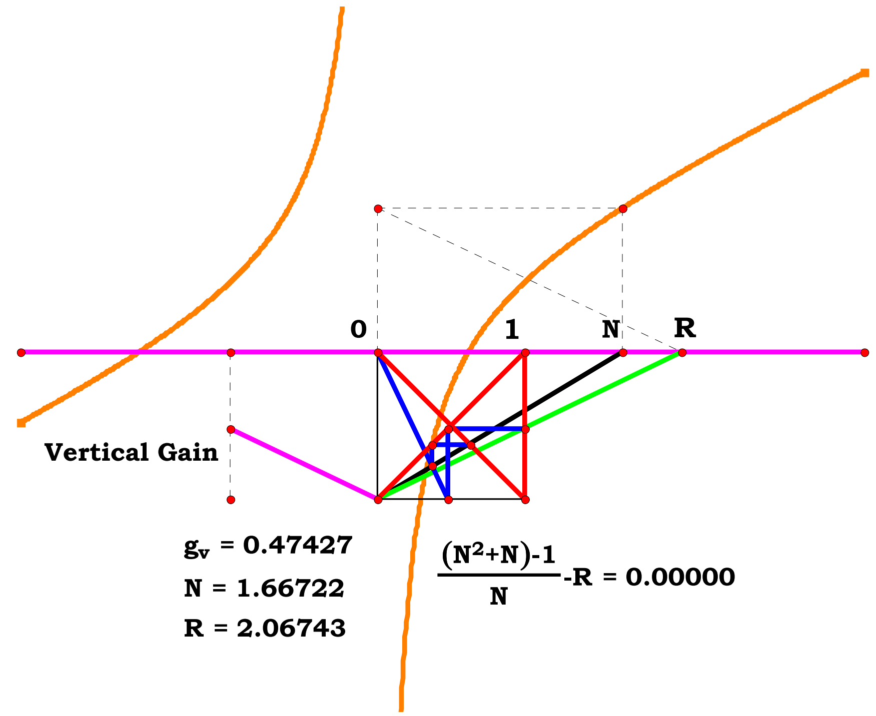
Given.

$AN := 3$

Descriptions.

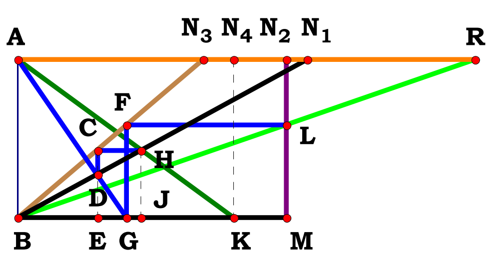
$$EF := \frac{AN}{AN^2 + AN - 1} \qquad BD := AB - EF$$

$$AR := \frac{AB^2}{EF}$$



Definitions.

$$AR - \frac{AN^2 + AN - 1}{AN} = 0$$



$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

$$hj := \frac{N_4}{N_1 + N_4} \quad be := N_3 \cdot hj \quad de := \frac{be}{N_1} \quad bg := \frac{be}{1 - de} \quad fg := \frac{bg}{N_3} \quad ar := \frac{N_2}{fg}$$

Definitions.

$$be - \frac{N_3 \cdot N_4}{N_1 + N_4} = 0 \qquad de - \frac{N_3 \cdot N_4}{N_1^2 + N_4 \cdot N_1} = 0 \qquad bg - \frac{N_1 \cdot N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad fg - \frac{N_1 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. 1

1, 0, 0, 0. $\dfrac{N_1^{\,2}+N_1-1}{N_1}$

0, 2, 0, 0. N_2

0, 0, 3, 0. $2-N_3$

0, 0, 0, 4. $\dfrac{1}{N_4}$

1, 2, 0, 0. $\dfrac{N_2\cdot\left(N_1^{\,2}+N_1-1\right)}{N_1}$

1, 0, 3, 0. $\dfrac{N_1^{\,2}+N_1-N_3}{N_1}$

1, 0, 0, 4. $\dfrac{N_1^{\,2}+N_4\cdot N_1-N_4}{N_1\cdot N_4}$

0, 2, 3, 0. $2\cdot N_2-N_2\cdot N_3$

0, 2, 0, 4. $\dfrac{N_2}{N_4}$

0, 0, 3, 4. $\dfrac{N_4-N_3\cdot N_4+1}{N_4}$

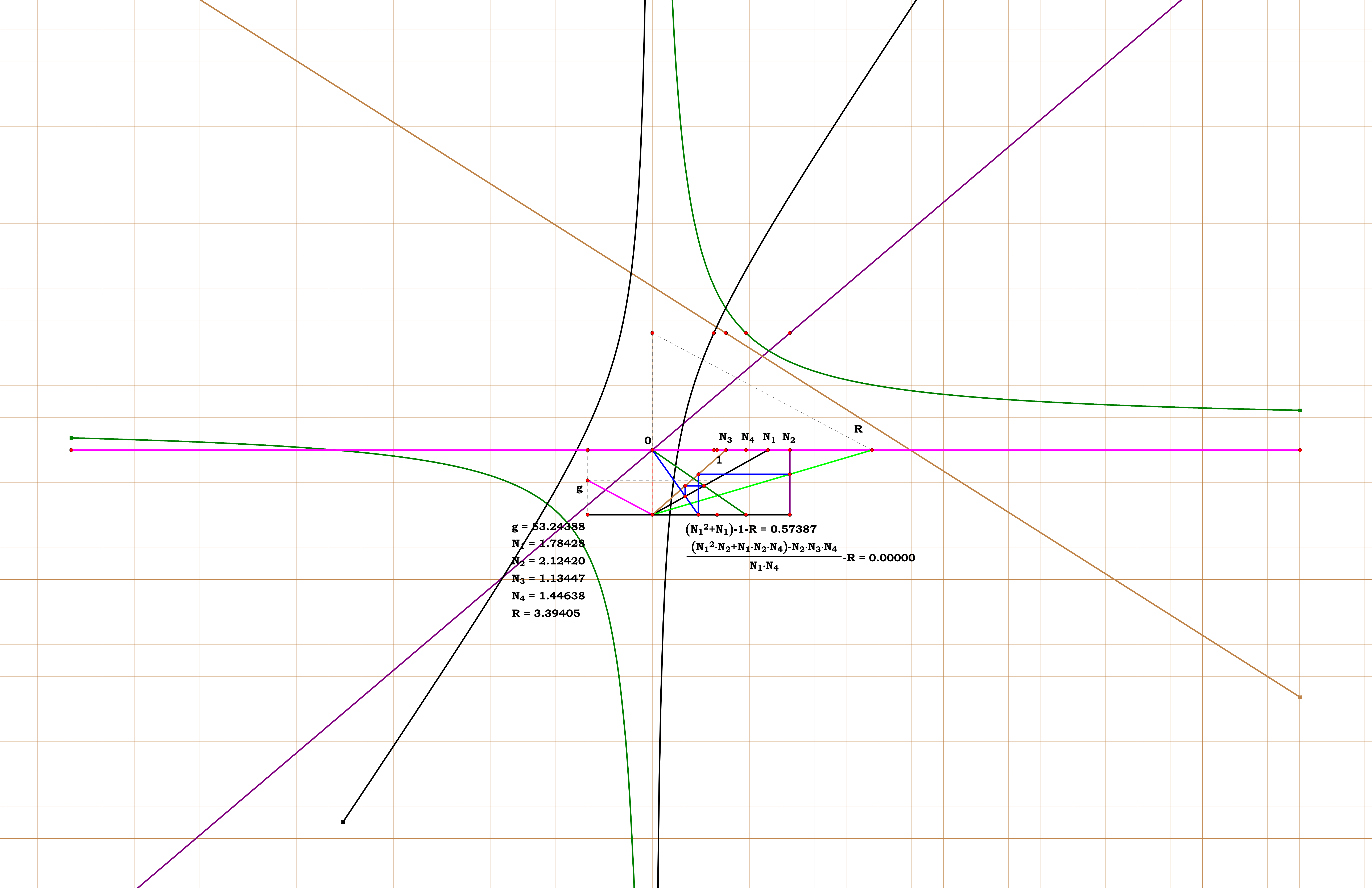
1, 2, 3, 0. $\dfrac{N_2\cdot\left(N_1^{\,2}+N_1-N_3\right)}{N_1}$

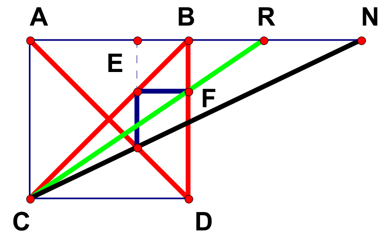
1, 0, 3, 4. $\dfrac{N_1^{\,2}+N_4\cdot N_1-N_3\cdot N_4}{N_1\cdot N_4}$

0, 2, 3, 4. $\dfrac{N_2+N_2\cdot N_4-N_2\cdot N_3\cdot N_4}{N_4}$

1, 2, 0, 4. $\dfrac{N_2\cdot\left(N_1^{\,2}+N_4\cdot N_1-N_4\right)}{N_1\cdot N_4}$

1, 2, 3, 4. $\dfrac{N_1^{\,2}\cdot N_2+N_1\cdot N_2\cdot N_4-N_2\cdot N_3\cdot N_4}{N_1\cdot N_4}$


0
N3
N4
N1
N2
R
1
g
g = 53.24388
N1 = 1.78428
N2 = 2.12420
N3 = 1.13447
N4 = 1.44638
R = 3.39405
(N1²+N1)-1-R = 0.57387
((N1²·N2+N1·N2·N4)-N2·N3·N4) / (N1·N4) -R = 0.00000



1CST6R3

Unit.

$AB := 1$

Given.

$AN := 3$

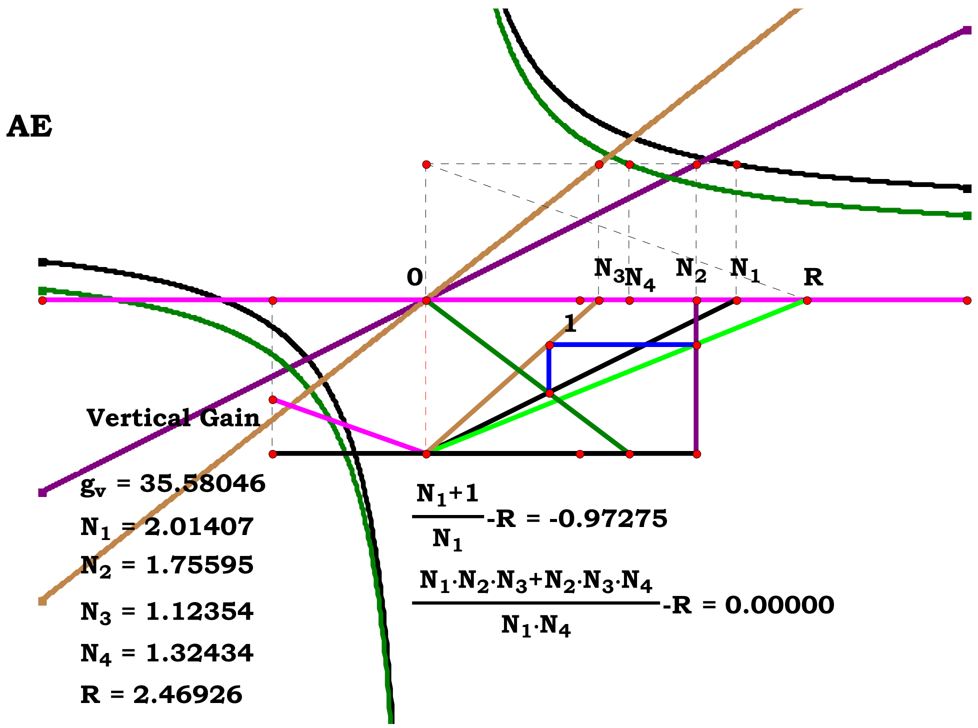
Descriptions.

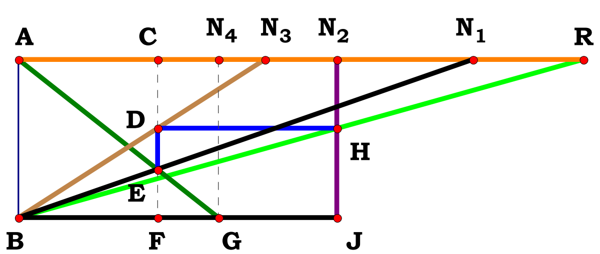
$$AE := \frac{AB \cdot AN}{AB + AN} \qquad BF := AB - AE$$

$$AR := \frac{AB^2}{AE}$$

Definitions.

$$AR - \frac{AN + 1}{AN} = 0$$





$N_1 := 5 \quad N_2 := 4 \quad N_3 := 3 \quad N_4 := 2$

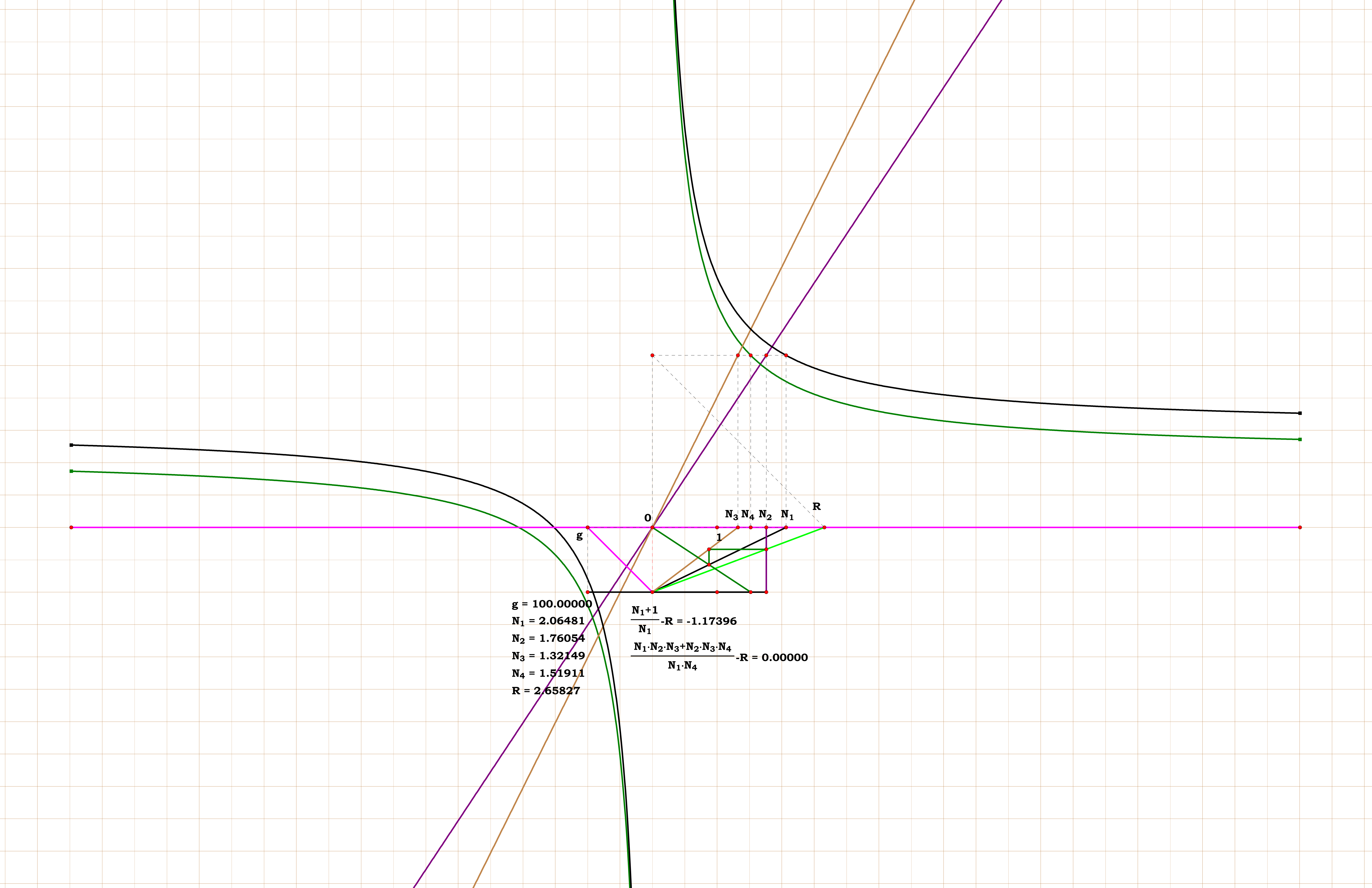
$$bf := \frac{N_1 \cdot N_4}{N_1 + N_4} \qquad cd := \frac{N_3 - bf}{N_3} \qquad ar := \frac{N_2}{1 - cd}$$

Definitions.

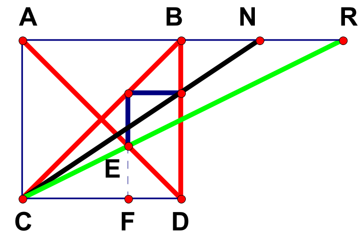
$$cd - \frac{N_1 \cdot N_3 + N_3 \cdot N_4 - N_1 \cdot N_4}{N_1 \cdot N_3 + N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0.	2	0, 2, 3, 0.	$2 \cdot N_2 \cdot N_3$
1, 0, 0, 0.	$\frac{N_1 + 1}{N_1}$	0, 2, 0, 4.	$\frac{N_2 + N_2 \cdot N_4}{N_4}$
0, 2, 0, 0.	$2 \cdot N_2$	0, 0, 3, 4.	$\frac{N_3 + N_3 \cdot N_4}{N_4}$
0, 0, 3, 0.	$2 \cdot N_3$	1, 2, 3, 0.	$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3}{N_1}$
0, 0, 0, 4.	$\frac{1 + N_4}{N_4}$	1, 0, 3, 4.	$\frac{N_1 \cdot N_3 + N_3 \cdot N_4}{N_1 \cdot N_4}$
1, 2, 0, 0.	$\frac{N_1 \cdot N_2 + N_2}{N_1}$	0, 2, 3, 4.	$\frac{N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_4}$
1, 0, 3, 0.	$\frac{N_1 \cdot N_3 + N_3}{N_1}$	1, 2, 0, 4.	$\frac{N_1 \cdot N_2 + N_2 \cdot N_4}{N_1 \cdot N_4}$
1, 0, 0, 4.	$\frac{N_1 + N_4}{N_1 \cdot N_4}$	1, 2, 3, 4.	$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4}$



0
g
1
N_3 N_4 N_2 N_1
R
g = 100.00000
N_1 = 2.06481
N_2 = 1.76054
N_3 = 1.32149
N_4 = 1.51911
R = 2.65827
$\frac{N_1+1}{N_1}$-R = -1.17396
$\frac{N_1 \cdot N_2 \cdot N_3 + N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4}$-R = 0.00000



1CST6R4

Unit.

$AB := 1$

Given.

$AN := 3$

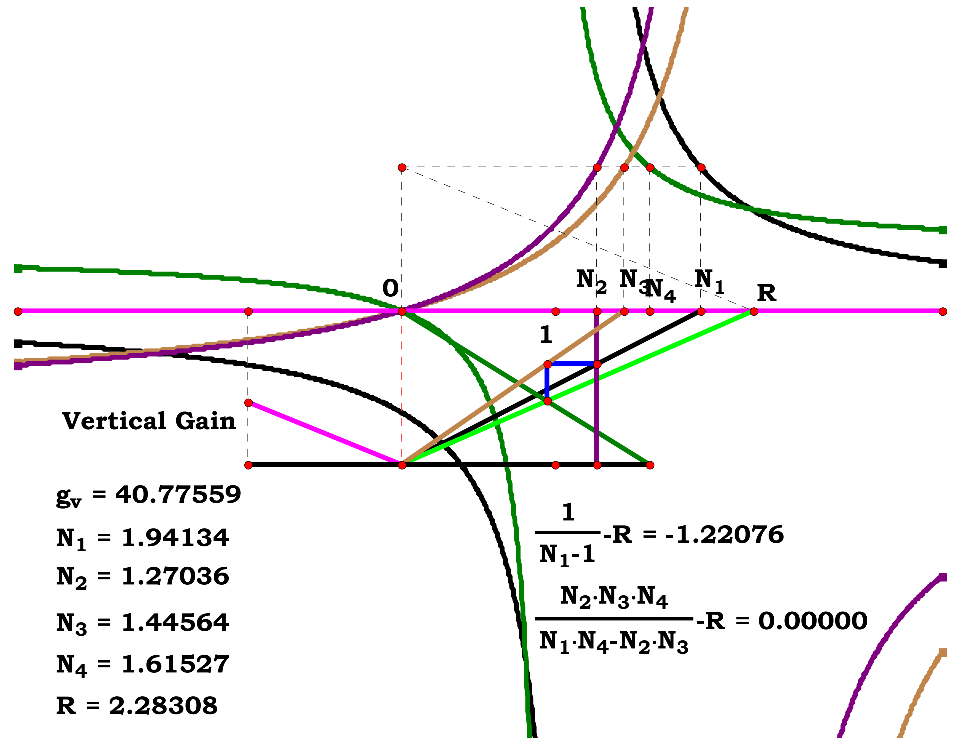
Descriptions.

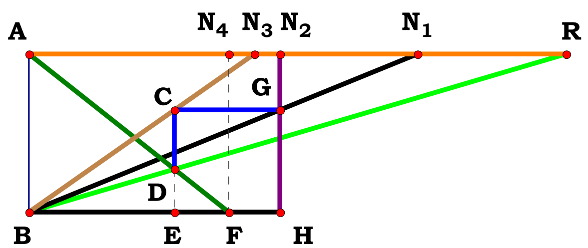
$$CF := \frac{1}{AN} \qquad EF := AB - CF$$

$$AR := \frac{CF \cdot AB}{EF}$$

Definitions.

$$AR - \frac{1}{AN - 1} = 0$$





$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

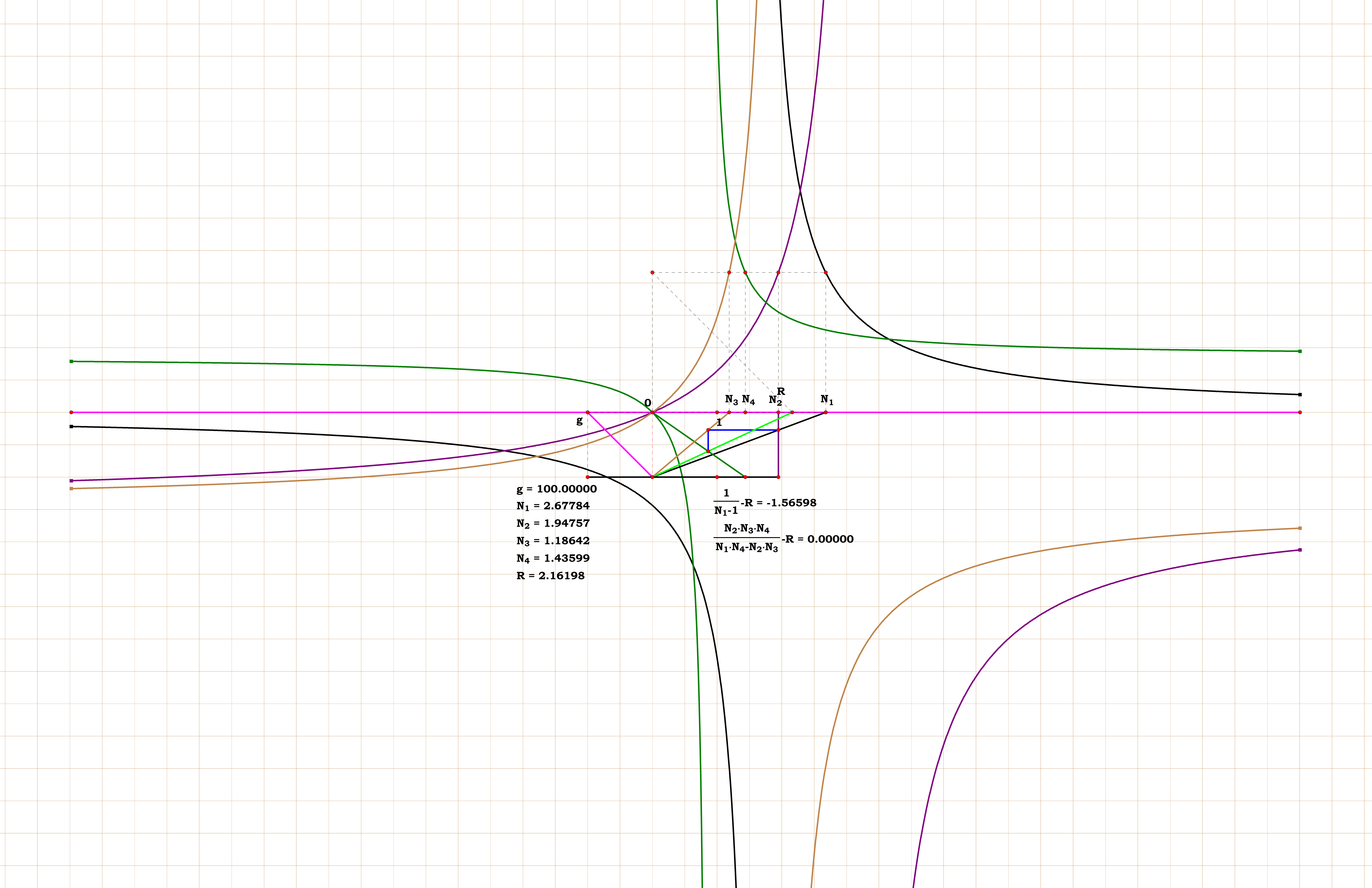
$$gh := \frac{N_2}{N_1} \qquad be := N_3 \cdot gh \qquad de := \frac{N_4 - be}{N_4} \qquad ar := \frac{be}{de}$$

Definitions.

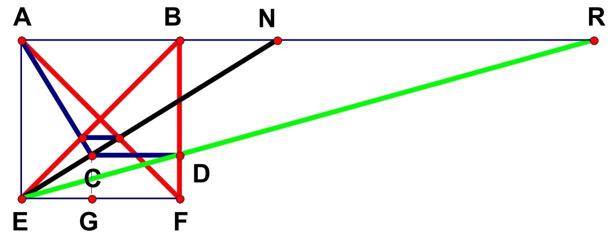
$$be - \frac{N_2 \cdot N_3}{N_1} = 0 \qquad de - \frac{N_1 \cdot N_4 - N_2 \cdot N_3}{N_1 \cdot N_4} = 0 \qquad ar - \frac{N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0$$

Four Transforms.

0, 0, 0, 0.	0	0, 2, 3, 0.	$N_2 \cdot \frac{N_3}{1 - N_2 \cdot N_3}$
1, 0, 0, 0.	$\frac{1}{N_1 - 1}$	0, 2, 0, 4.	$N_2 \cdot \frac{N_4}{N_4 - N_2}$
0, 2, 0, 0.	$\frac{N_2}{1 - N_2}$	0, 0, 3, 4.	$N_3 \cdot \frac{N_4}{N_4 - N_3}$
0, 0, 3, 0.	$\frac{N_3}{1 - N_3}$	1, 2, 3, 0.	$N_2 \cdot \frac{N_3}{N_1 - N_2 \cdot N_3}$
0, 0, 0, 4.	$\frac{N_4}{N_4 - 1}$	1, 0, 3, 4.	$N_3 \cdot \frac{N_4}{N_1 \cdot N_4 - N_3}$
1, 2, 0, 0.	$\frac{N_2}{N_1 - N_2}$	0, 2, 3, 4.	$N_2 \cdot N_3 \cdot \frac{N_4}{N_4 - N_2 \cdot N_3}$
1, 0, 3, 0.	$\frac{N_3}{N_1 - N_3}$	1, 2, 0, 4.	$N_2 \cdot \frac{N_4}{N_1 \cdot N_4 - N_2}$
1, 0, 0, 4.	$\frac{N_4}{N_1 \cdot N_4 - 1}$	1, 2, 3, 4.	$\frac{N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3}$



0
g
1
N3 N4 N2 N1
R
g = 100.00000
N1 = 2.67784
N2 = 1.94757
N3 = 1.18642
N4 = 1.43599
R = 2.16198
$\frac{1}{N_1-1}-R = -1.56598$
$\frac{N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3}-R = 0.00000$



1CST6R5

Unit.

$AB := 1$

Given.

$AN := 3$

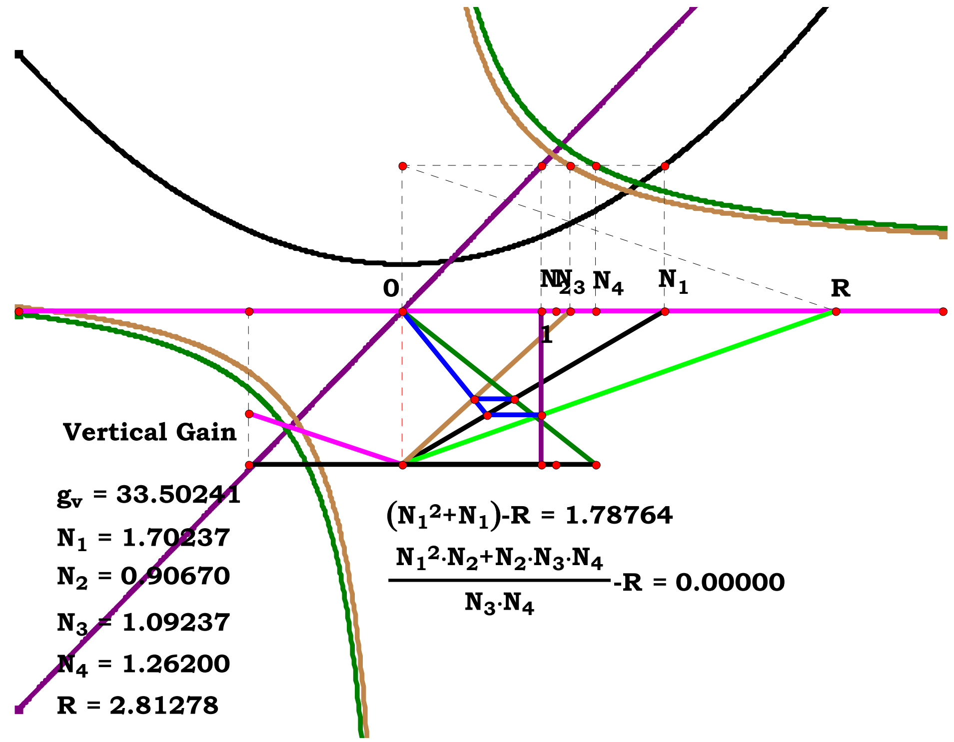
Descriptions.

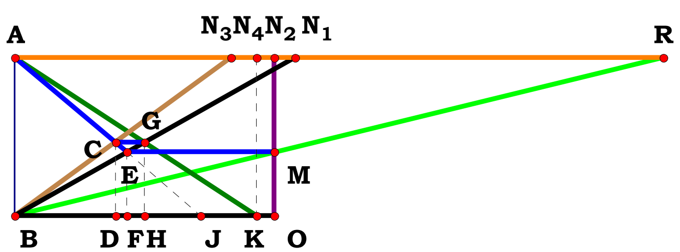
$$CG := \frac{1}{AN^2 + 1} \qquad DF := CG$$

$$AR := \frac{AB^2}{DF}$$

Definitions.

$$AR - \left(AN^2 + 1\right) = 0$$





$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

$$gh := \frac{N_3}{N_1 + N_3} \qquad bd := N_4 \cdot gh \qquad bj := \frac{bd}{1 - gh} \qquad ef := \frac{bj}{N_1 + bj} \qquad ar := \frac{N_2}{ef}$$

Definitions.

$$bd - \frac{N_3 \cdot N_4}{N_1 + N_3} = 0 \qquad bj - \frac{N_3 \cdot N_4}{N_1} = 0 \qquad ef - \frac{N_3 \cdot N_4}{N_1^2 + N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1^2 \cdot N_2 + N_2 \cdot N_3 \cdot N_4}{N_3 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. 2

1, 0, 0, 0. N_1^2+1

0, 2, 0, 0. $2\cdot N_2$

0, 0, 3, 0. $\dfrac{1+N_3}{N_3}$

0, 0, 0, 4. $\dfrac{1+N_4}{N_4}$

1, 2, 0, 0. $N_1^2\cdot N_2+N_2$

1, 0, 3, 0. $\dfrac{N_1^2+N_3}{N_3}$

1, 0, 0, 4. $\dfrac{N_1^2+N_4}{N_4}$

0, 2, 3, 0. $\dfrac{N_2+N_2\cdot N_3}{N_3}$

0, 2, 0, 4. $\dfrac{N_2+N_2\cdot N_4}{N_4}$

0, 0, 3, 4. $\dfrac{1+N_3\cdot N_4}{N_3\cdot N_4}$

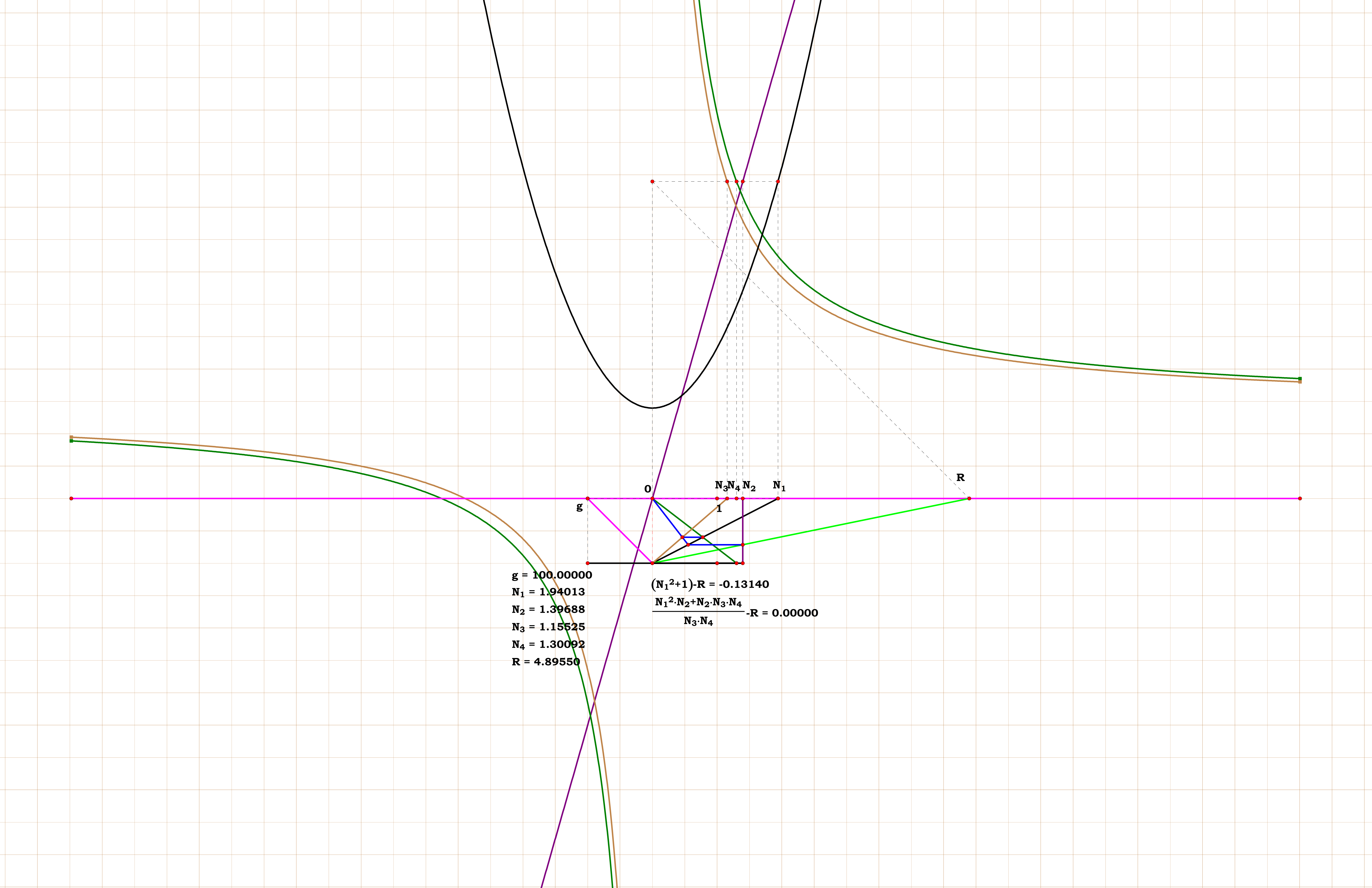
1, 2, 3, 0. $\dfrac{N_1^2\cdot N_2+N_2\cdot N_3}{N_3}$

1, 0, 3, 4. $\dfrac{N_1^2+N_3\cdot N_4}{N_3\cdot N_4}$

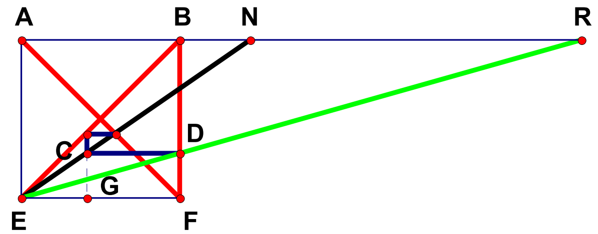
0, 2, 3, 4. $\dfrac{N_2+N_2\cdot N_3\cdot N_4}{N_3\cdot N_4}$

1, 2, 0, 4. $\dfrac{N_1^2\cdot N_2+N_2\cdot N_4}{N_4}$

1, 2, 3, 4. $\dfrac{N_1^2\cdot N_2+N_2\cdot N_3\cdot N_4}{N_3\cdot N_4}$



0
N3 N4 N2
N1
R
g
1
g = 100.00000
N1 = 1.94013
N2 = 1.39688
N3 = 1.15525
N4 = 1.30092
R = 4.89550
(N1²+1)-R = -0.13140
(N1²·N2+N2·N3·N4)/(N3·N4)-R = 0.00000



1CST6R6

Unit. $AB := 1$ **Given.** $AN := 3$

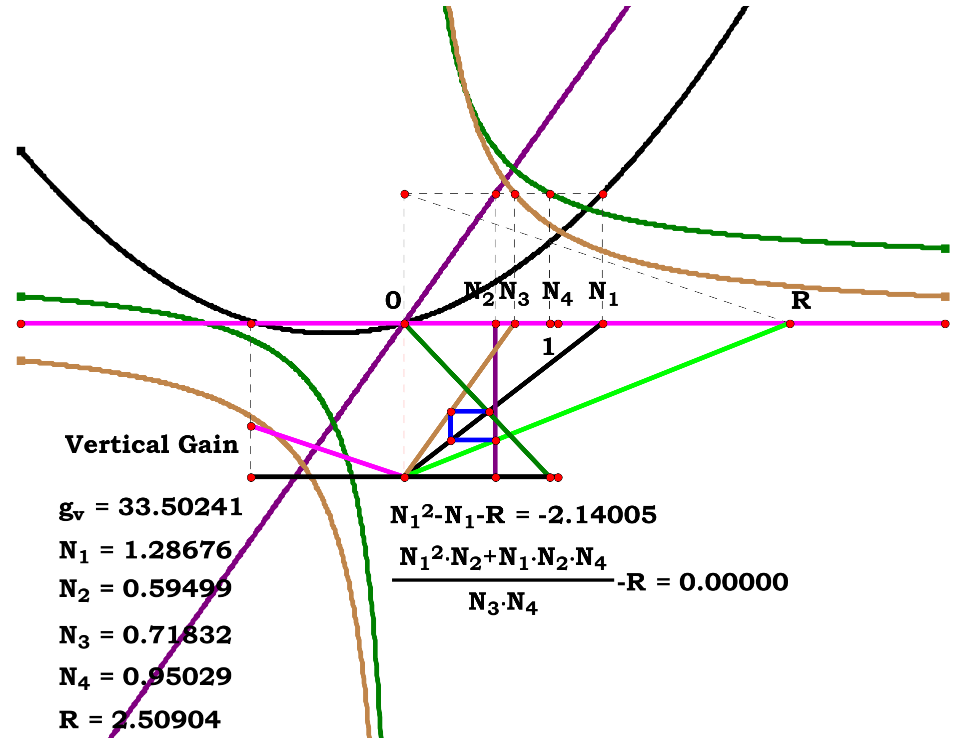
Descriptions.

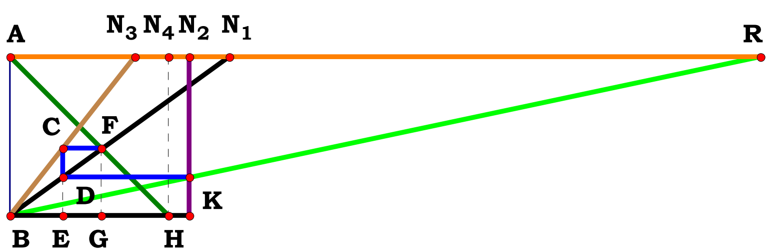
$$CG := \frac{1}{AN^2 + AN} \qquad DF := CG$$

$$AR := \frac{AB^2}{DF}$$

Definitions.

$$AR - \left(AN^2 + AN\right) = 0$$





$N_1 := 5 \quad N_2 := 4 \quad N_3 := 3 \quad N_4 := 2$

$$fg := \frac{N_4}{N_1 + N_4} \qquad be := N_3 \cdot fg \qquad de := \frac{be}{N_1} \qquad ar := \frac{N_2}{de}$$

Definitions.

$$be - \frac{N_3 \cdot N_4}{N_1 + N_4} = 0 \qquad de - \frac{N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4} = 0 \qquad ar - \frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4}{N_3 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. 2

1, 0, 0, 0. $N_1^{2} + N_1$

0, 2, 0, 0. $2 \cdot N_2$

0, 0, 3, 0. $\dfrac{2}{N_3}$

0, 0, 0, 4. $\dfrac{N_4 + 1}{N_4}$

1, 2, 0, 0. $N_1 \cdot N_2 \cdot \left(N_1 + 1\right)$

1, 0, 3, 0. $\dfrac{N_1 \cdot \left(N_1 + 1\right)}{N_3}$

1, 0, 0, 4. $\dfrac{N_1 \cdot \left(N_1 + N_4\right)}{N_4}$

0, 2, 3, 0. $\dfrac{2 \cdot N_2}{N_3}$

0, 2, 0, 4. $\dfrac{N_2 \cdot \left(N_4 + 1\right)}{N_4}$

0, 0, 3, 4. $\dfrac{N_4 + 1}{N_3 \cdot N_4}$

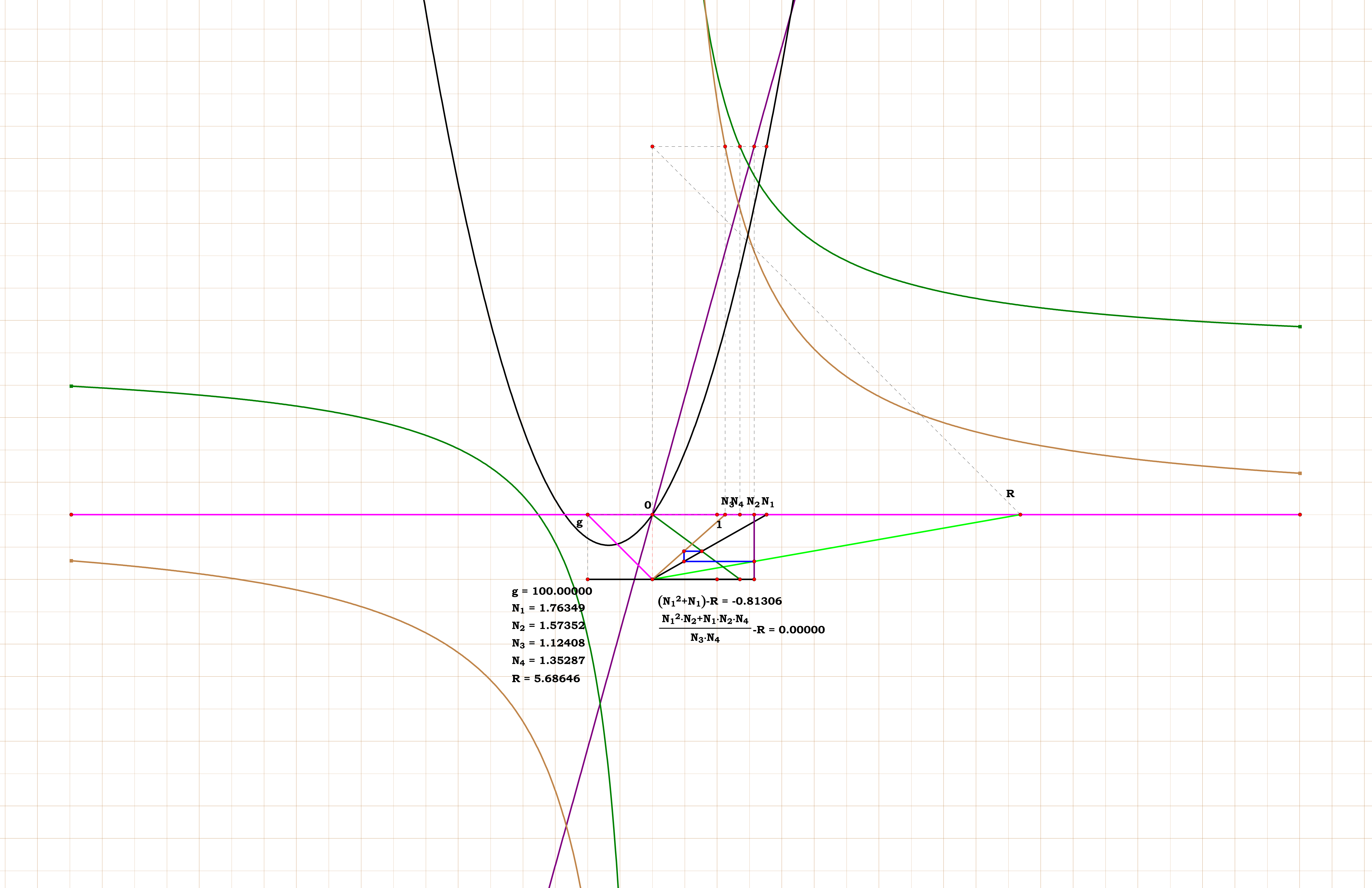
1, 2, 3, 0. $\dfrac{\left(N_2 \cdot N_1^{2} + N_2 \cdot N_1\right)}{N_3}$

1, 0, 3, 4. $\dfrac{N_1^{2} + N_4 \cdot N_1}{N_3 \cdot N_4}$

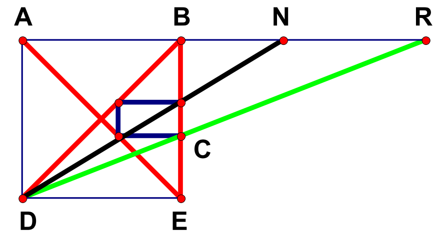
0, 2, 3, 4. $\dfrac{N_2 \cdot \left(N_4 + 1\right)}{N_3 \cdot N_4}$

1, 2, 0, 4. $\dfrac{N_1 \cdot N_2 \cdot \left(N_1 + N_4\right)}{N_4}$

1, 2, 3, 4. $\dfrac{N_1^{2} \cdot N_2 + N_1 \cdot N_2 \cdot N_4}{N_3 \cdot N_4}$



0
N3 N4 N2 N1
R
g
1
g = 100.00000
N1 = 1.76349
N2 = 1.57352
N3 = 1.12408
N4 = 1.35287
R = 5.68646
(N1²+N1)-R = -0.81306
(N1²·N2+N1·N2·N4)/(N3·N4)-R = 0.00000



1CST6R7

Unit.

$AB := 1$

Given.

$AN := 3$

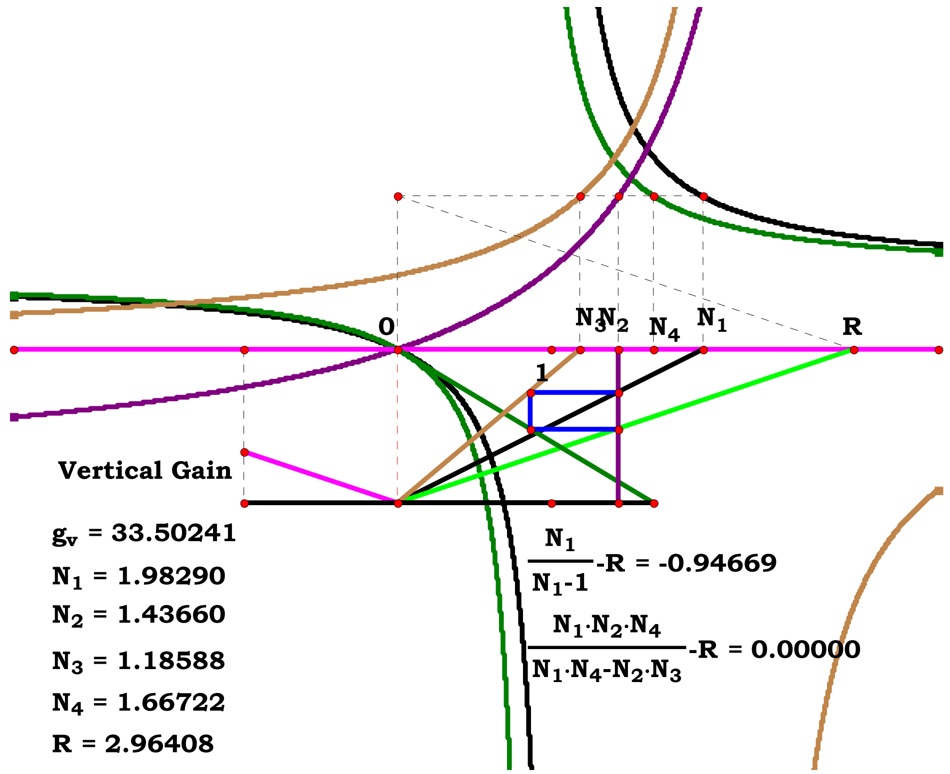
Descriptions.

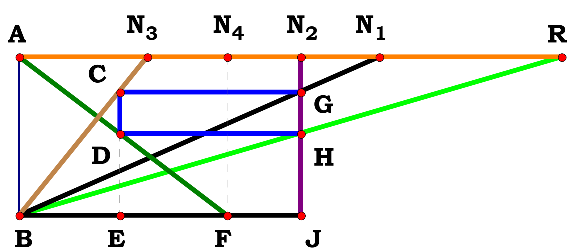
$$BC := \frac{1}{AN} \qquad CE := AB - BC$$

$$AR := \frac{AB^2}{CE}$$

Definitions.

$$AR - \frac{AN}{AN - 1} = 0$$





$$N_1 := 5 \quad N_2 := 4 \quad N_3 := 3 \quad N_4 := 2$$

$$gj := \frac{N_2}{N_1} \quad be := N_3 \cdot gj \quad de := \frac{N_4 - be}{N_4} \quad ar := \frac{N_2}{de}$$

Definitions.

$$be - \frac{N_2 \cdot N_3}{N_1} = 0 \quad de - \frac{N_1 \cdot N_4 - N_2 \cdot N_3}{N_1 \cdot N_4} = 0 \quad ar - \frac{N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3} = 0$$

Four Transforms.

0, 0, 0, 0. 0

1, 0, 0, 0. $\frac{N_1}{N_1 - 1}$

0, 2, 0, 0. $-\frac{N_2}{N_2 - 1}$

0, 0, 3, 0. $-\frac{1}{N_3 - 1}$

0, 0, 0, 4. $\frac{N_4}{N_4 - 1}$

1, 2, 0, 0. $\frac{N_1 \cdot N_2}{N_1 - N_2}$

1, 0, 3, 0. $\frac{N_1}{N_1 - N_3}$

1, 0, 0, 4. $\frac{N_1 \cdot N_4}{N_1 \cdot N_4 - 1}$

0, 2, 3, 0. $-\frac{N_2}{N_2 \cdot N_3 - 1}$

0, 2, 0, 4. $-\frac{N_2 \cdot N_4}{N_2 - N_4}$

0, 0, 3, 4. $-\frac{N_4}{N_3 - N_4}$

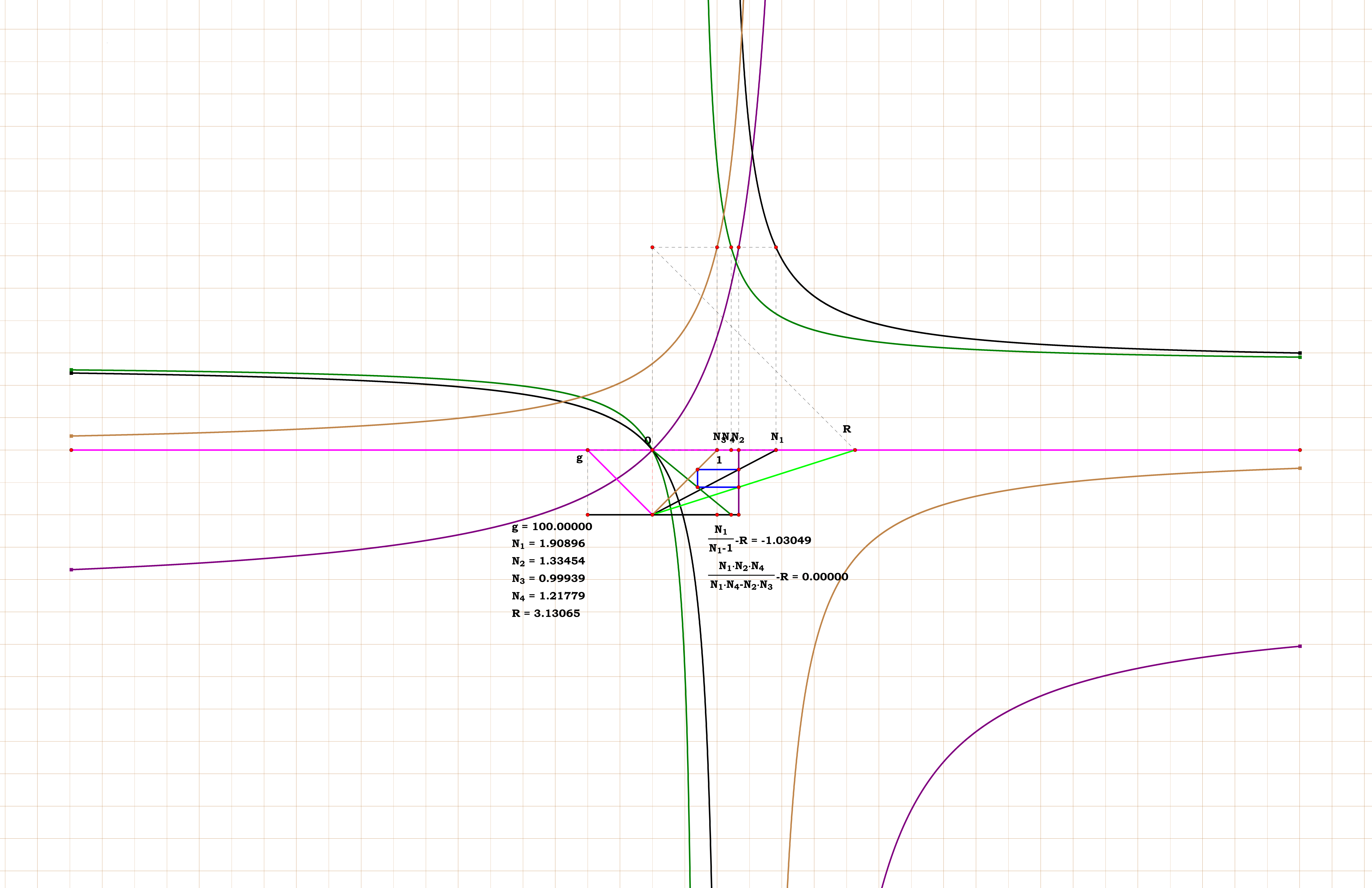
1, 2, 3, 0. $\frac{N_1 \cdot N_2}{N_1 - N_2 \cdot N_3}$

1, 0, 3, 4. $-\frac{N_1 \cdot N_4}{N_3 - N_1 \cdot N_4}$

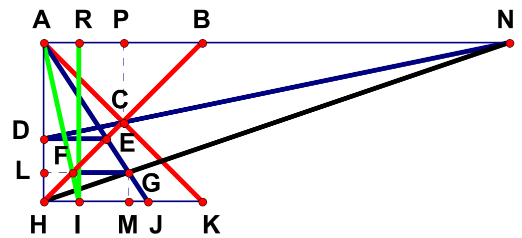
0, 2, 3, 4. $\frac{N_2 \cdot N_4}{N_4 - N_2 \cdot N_3}$

1, 2, 0, 4. $-\frac{N_1 \cdot N_2 \cdot N_4}{N_2 - N_1 \cdot N_4}$

1, 2, 3, 4. $\frac{N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3}$



O
g
N3
N4
N2
N1
R
1
g = 100.00000
N1 = 1.90896
N2 = 1.33454
N3 = 0.99939
N4 = 1.21779
R = 3.13065
$\frac{N_1}{N_1-1}$-R = -1.03049
$\frac{N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_2 \cdot N_3}$-R = 0.00000



Unit.

$AB := 1$

Given.

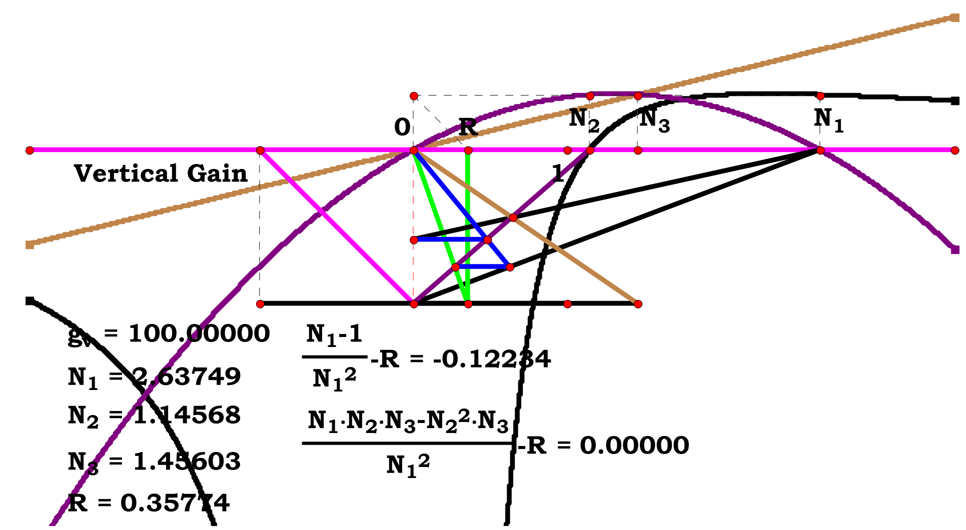
$AN := 3$

1CST7R0

Descriptions.

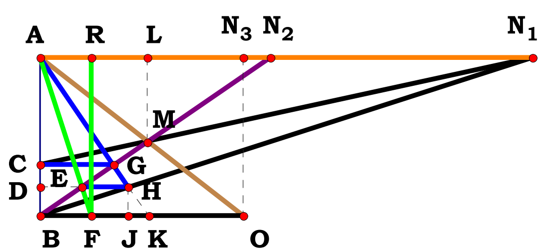
$AP := \frac{AB}{2}$ $\quad NP := AN - AP$ $\quad CP := AP$ $\quad AD := \frac{CP \cdot AN}{NP}$ $\quad DH := AB - AD$ $\quad DE := DH$ $\quad HJ := \frac{DE \cdot AB}{AD}$

$HM := \frac{HJ \cdot AN}{HJ + AN}$ $\quad GM := \frac{AB \cdot HM}{AN}$ $\quad AL := AB - GM$ $\quad FL := GM$ $\quad HI := \frac{FL \cdot AB}{AL}$ $\quad AR := HI$



Definitions.

$AD - \frac{AN}{2 \cdot AN - 1} = 0$ $\quad HJ - \frac{AN - 1}{AN} = 0$ $\quad DH - \frac{AN - 1}{2 \cdot AN - 1} = 0$ $\quad HM - \frac{AN^2 - AN}{AN^2 + AN - 1} = 0$ $\quad AR - \frac{AN - 1}{AN^2} = 0$



$N_1 := 4$ $\quad N_2 := 3$ $\quad N_3 := 2$ $\quad lm := \frac{N_2}{N_2 + N_3}$

$al := N_3 \cdot lm$ $\quad ac := \frac{lm \cdot N_1}{N_1 - al}$ $\quad bc := 1 - ac$ $\quad cg := N_2 \cdot bc$

$bk := \frac{cg}{ac}$ $\quad gj := \frac{bk}{N_1 + bk}$ $\quad ad := 1 - gj$ $\quad de := N_2 \cdot gj$ $\quad ar := \frac{de}{ad}$

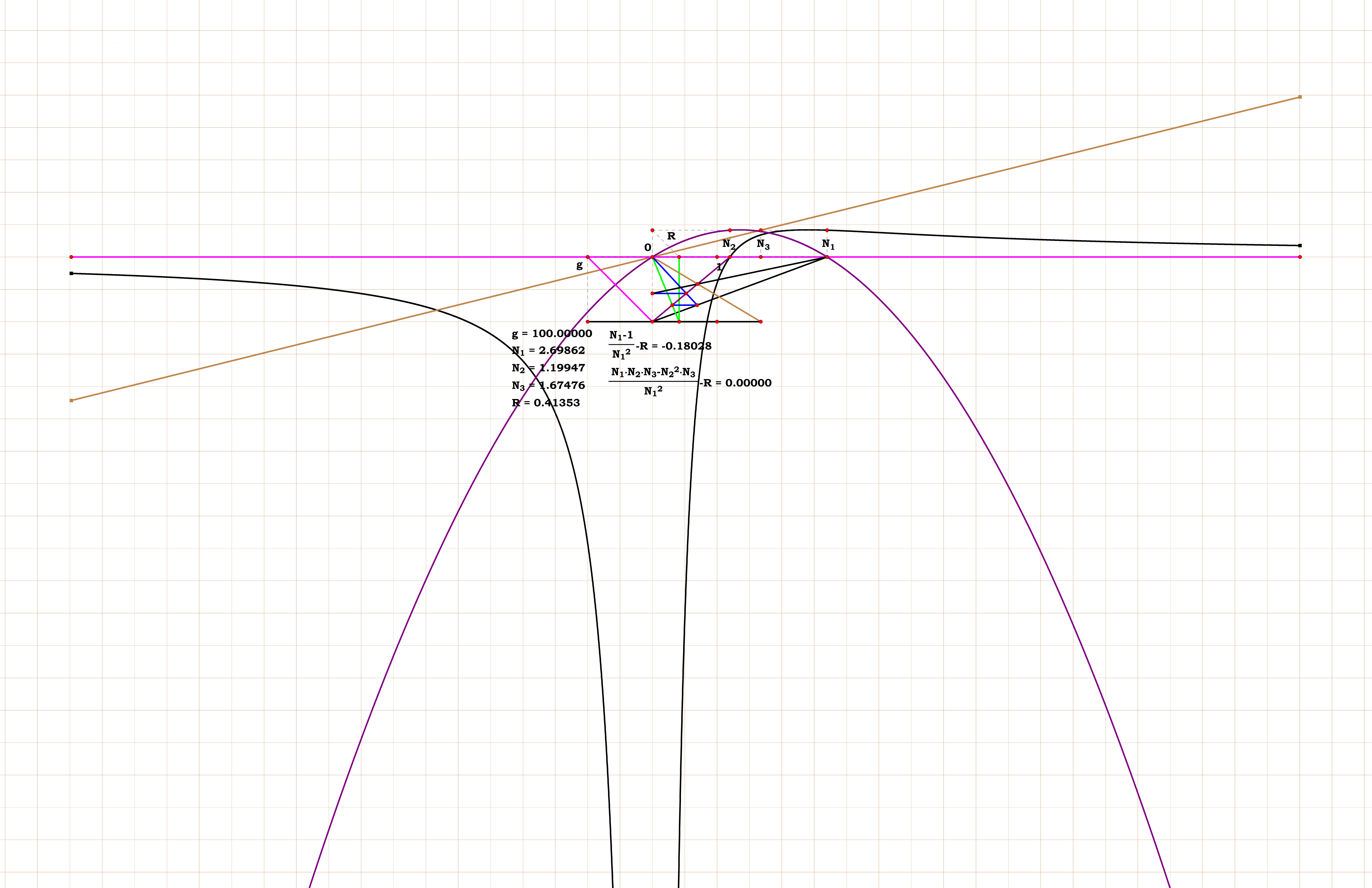
Definitions.

$al - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0$ $\quad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad bc - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad cg - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad bk - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} = 0$

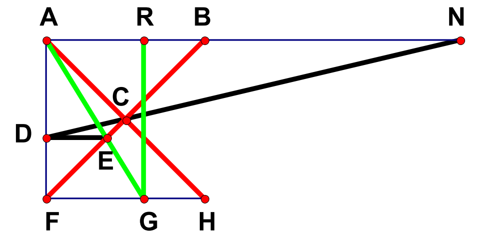
$gj - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad ad - \frac{N_1^{\,2}}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad de - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$ $\quad ar - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1^{\,2}} = 0$

Three Transforms.

0, 0, 0.	0	1, 2, 0.	$\dfrac{N_1 \cdot N_2 - N_2^{\,2}}{N_1^{\,2}}$
1, 0, 0.	$\dfrac{N_1 - 1}{N_1^{\,2}}$	1, 0, 3.	$\dfrac{N_1 \cdot N_3 - N_3}{N_1^{\,2}}$
0, 2, 0.	$N_2 - N_2^{\,2}$	0, 2, 3.	$N_2 \cdot N_3 - N_2^{\,2} \cdot N_3$
0, 0, 3.	0	1, 2, 3.	$\dfrac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1^{\,2}}$



R
0
N_2
N_3
N_1
g
1
g = 100.00000
N_1 = 2.69862
N_2 = 1.19947
N_3 = 1.67476
R = 0.41353
$\frac{N_1-1}{N_1^2}-R = -0.18028$
$\frac{N_1 \cdot N_2 \cdot N_3 - N_2^2 \cdot N_3}{N_1^2}-R = 0.00000$



1CST7R1

Unit.

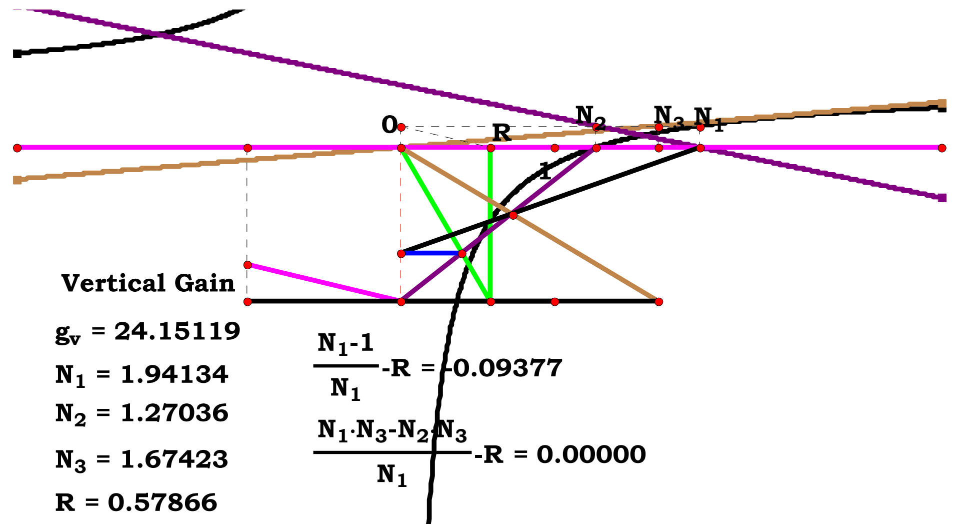
$AB := 1$

Given.

$AN := 3$

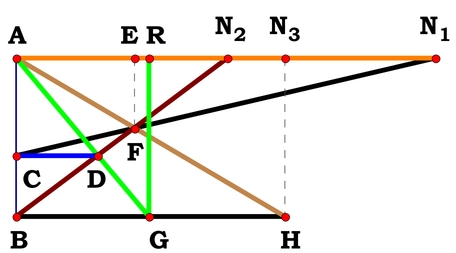
Descriptions.

$$FG := \frac{AN-1}{AN} \quad AR := FG$$



Definitions.

$$AR - \frac{AN-1}{AN} = 0$$



$N_1 := 4$

$N_2 := 3$

$N_3 := 2$

$$ef := \frac{N_2}{N_2+N_3} \quad ae := N_3 \cdot ef \quad ac := \frac{ef \cdot N_1}{N_1 - ae} \quad cd := N_2 - ac \cdot N_2 \quad ar := \frac{cd}{ac}$$

Definitions.

$$ae - \frac{N_2 \cdot N_3}{N_2+N_3} = 0 \quad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_3 + N_1 \cdot N_2 - N_2 \cdot N_3} = 0 \quad cd - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \quad ar - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} = 0$$

Three Transforms.

0, 0, 0. 0

1, 0, 0. $\dfrac{N_1 - 1}{N_1}$

0, 2, 0. $1 - N_2$

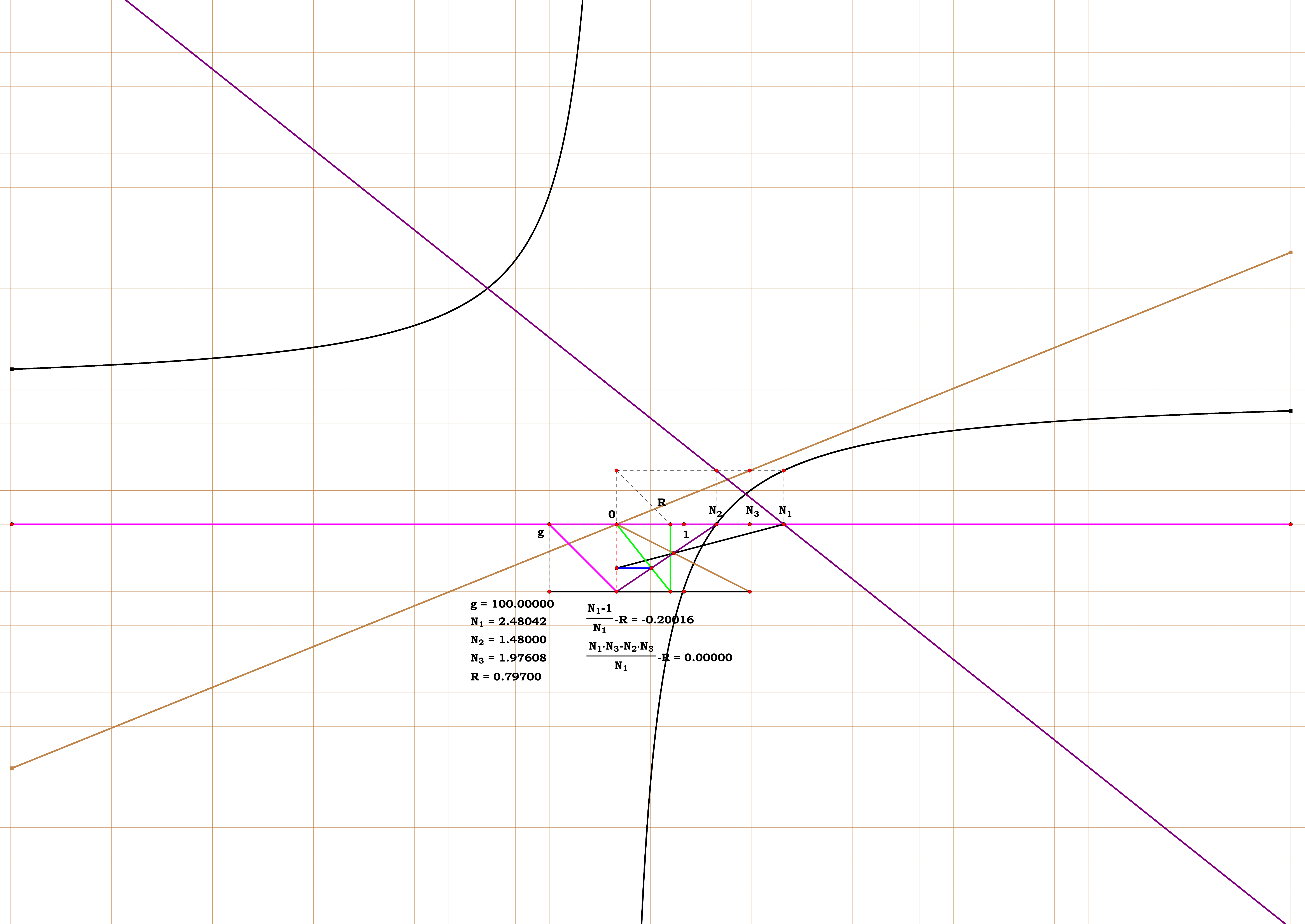
0, 0, 3. 0

1, 2, 0. $\dfrac{N_1 - N_2}{N_1}$

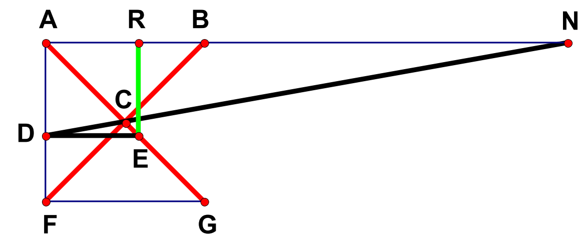
1, 0, 3. $\dfrac{N_1 \cdot N_3 - N_3}{N_1}$

0, 2, 3. $N_3 - N_2 \cdot N_3$

1, 2, 3. $\dfrac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1}$



R
0
N2
N3
N1
g
1
g = 100.00000
N1 = 2.48042
N2 = 1.48000
N3 = 1.97608
R = 0.79700
$\frac{N_1-1}{N_1}-R = -0.20016$
$\frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1}-R = 0.00000$



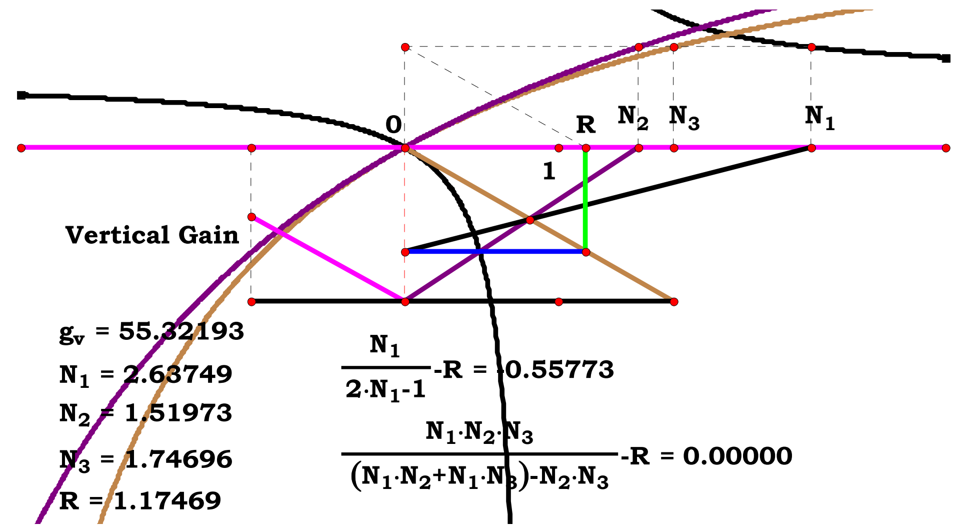
1CST7R2

Unit.

$AB := 1$

Given.

$AN := 4$

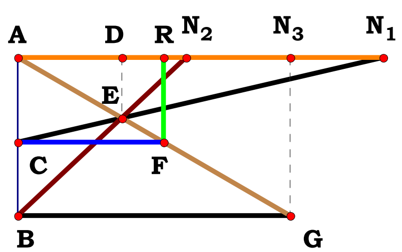


Descriptions.

$$AD := \frac{AN}{2 \cdot AN - 1} \qquad AR := AD$$

Definitions.

$$AR - \frac{AN}{2 \cdot AN - 1} = 0$$



$N_1 := 4$

$N_2 := 3$

$N_3 := 2$

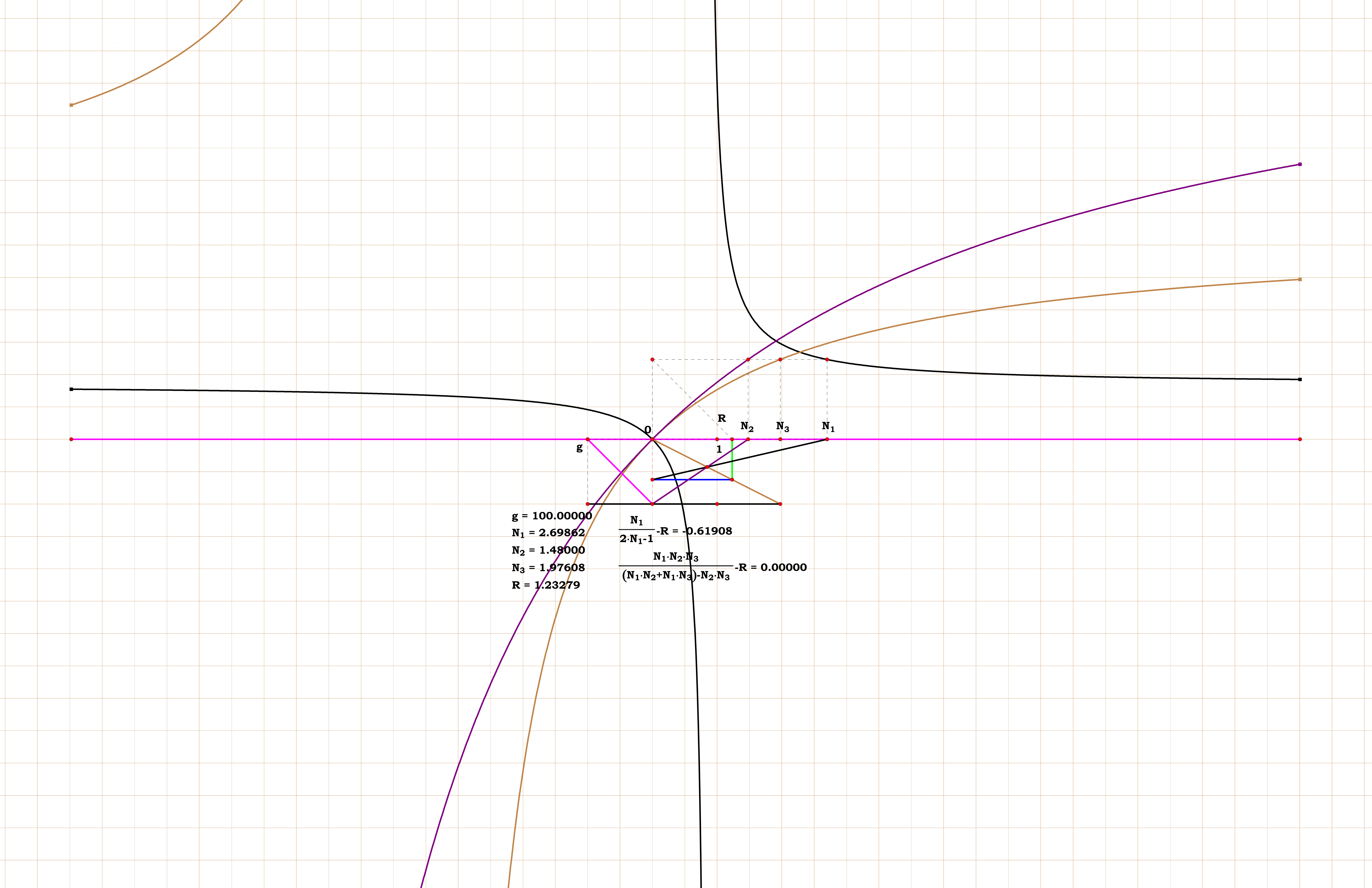
$$de := \frac{N_2}{N_2 + N_3} \qquad ad := N_3 \cdot de \qquad ac := \frac{de \cdot N_1}{N_1 - ad} \qquad ar := N_3 \cdot ac$$

Definitions.

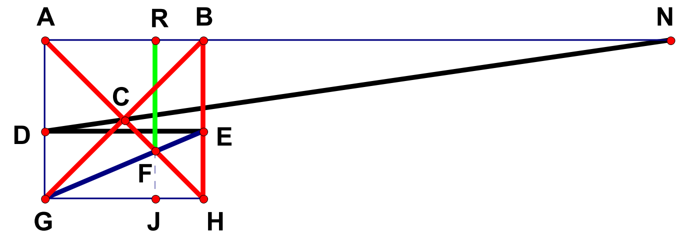
$$ad - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0 \qquad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0.	1	1, 2, 0.	$\dfrac{N_1 \cdot N_2}{N_1 - N_2 + N_1 \cdot N_2}$
1, 0, 0.	$\dfrac{N_1}{2 \cdot N_1 - 1}$	1, 0, 3.	$N_1 \cdot \dfrac{N_3}{N_1 + N_1 \cdot N_3 - N_3}$
0, 2, 0.	N_2	0, 2, 3.	$N_2 \cdot \dfrac{N_3}{N_2 + N_3 - N_2 \cdot N_3}$
0, 0, 3.	N_3	1, 2, 3.	$\dfrac{N_1 \cdot N_2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3}$



R
0
N_2
N_3
N_1
g
1
g = 100.00000
N_1 = 2.69862
N_2 = 1.48000
N_3 = 1.97608
R = 1.23279
$\frac{N_1}{2 \cdot N_1 - 1} - R = -0.61908$
$\frac{N_1 \cdot N_2 \cdot N_3}{(N_1 \cdot N_2 + N_1 \cdot N_3) - N_2 \cdot N_3} - R = 0.00000$



1CST7R3

Unit. $AB := 1$ **Given.** $AN := 4$

Descriptions.

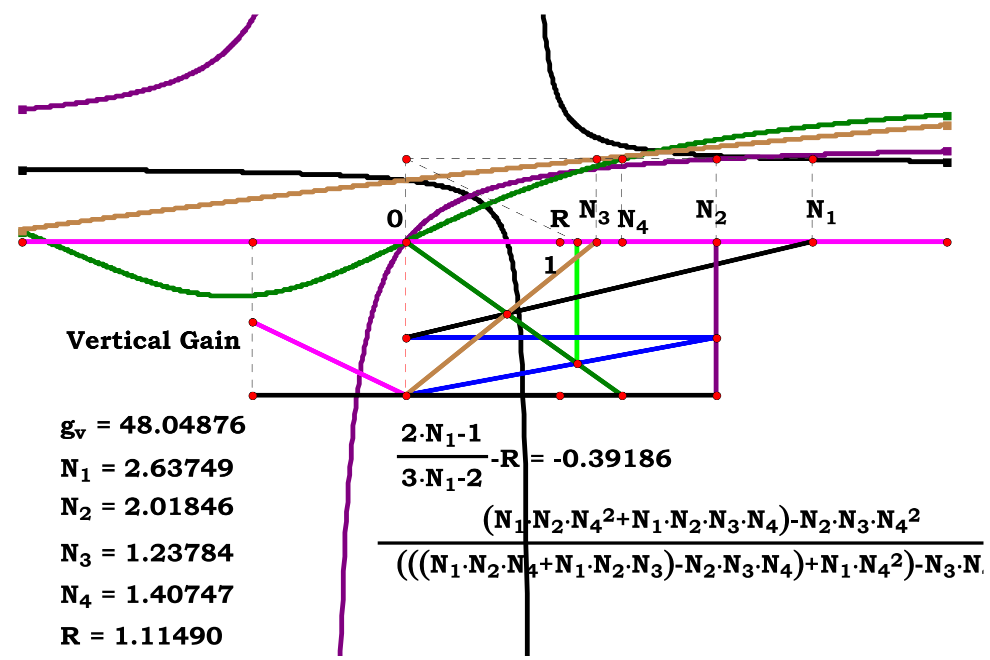
$$DG := \frac{AN - 1}{2 \cdot AN - 1} \qquad EH := DG$$

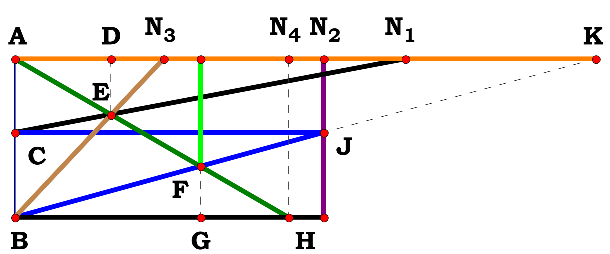
$$FJ := \frac{AB \cdot EH}{AB + EH}$$

$$GJ := AB - FJ \qquad AR := GJ$$

Definitions.

$$AR - \frac{2AN - 1}{3AN - 2} = 0$$





$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

$$de := \frac{N_3}{N_3 + N_4} \qquad ad := N_4 \cdot de \qquad ac := \frac{de \cdot N_1}{N_1 - ad} \qquad bc := 1 - ac \qquad ak := \frac{N_2}{bc} \qquad ar := \frac{ak \cdot N_4}{ak + N_4}$$

Definitions.

$$ad - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \qquad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_4 + N_1 \cdot N_3 - N_3 \cdot N_4} = 0 \qquad bc - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0$$

$$ak - \frac{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_4^2 + N_1 \cdot N_2 \cdot N_3 \cdot N_4 - N_2 \cdot N_3 \cdot N_4^2}{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4 + N_1 \cdot N_4^2 - N_3 \cdot N_4^2} = 0$$

Four Transforms.

0, 0, 0, 0. $$1$$

1, 0, 0, 0. $$\frac{2 \cdot N_1 - 1}{3 \cdot N_1 - 2}$$

0, 2, 0, 0. $$1$$

0, 0, 3, 0. $$-\frac{1}{N_3 - 2}$$

0, 0, 0, 4. $$N_4$$

1, 2, 0, 0. $$-\frac{N_2 - 2 \cdot N_1 \cdot N_2}{N_1 - N_2 + 2 \cdot N_1 \cdot N_2 - 1}$$

1, 0, 3, 0. $$\frac{N_1 - N_3 + N_1 \cdot N_3}{2 \cdot N_1 - 2 \cdot N_3 + N_1 \cdot N_3}$$

1, 0, 0, 4. $$\frac{N_1 \cdot N_4^2 - N_4^2 + N_1 \cdot N_4}{N_1 - N_4 + N_1 \cdot N_4^2 - N_4^2 + N_1 \cdot N_4}$$

0, 2, 3, 0. $$\frac{N_2}{N_2 - N_3 + 1}$$

0, 2, 0, 4. $$N_4$$

0, 0, 3, 4. $$\frac{N_4^2 - N_3 \cdot N_4^2 + N_3 \cdot N_4}{N_3 + N_4 - N_3 \cdot N_4^2 + N_4^2 - N_3 \cdot N_4}$$

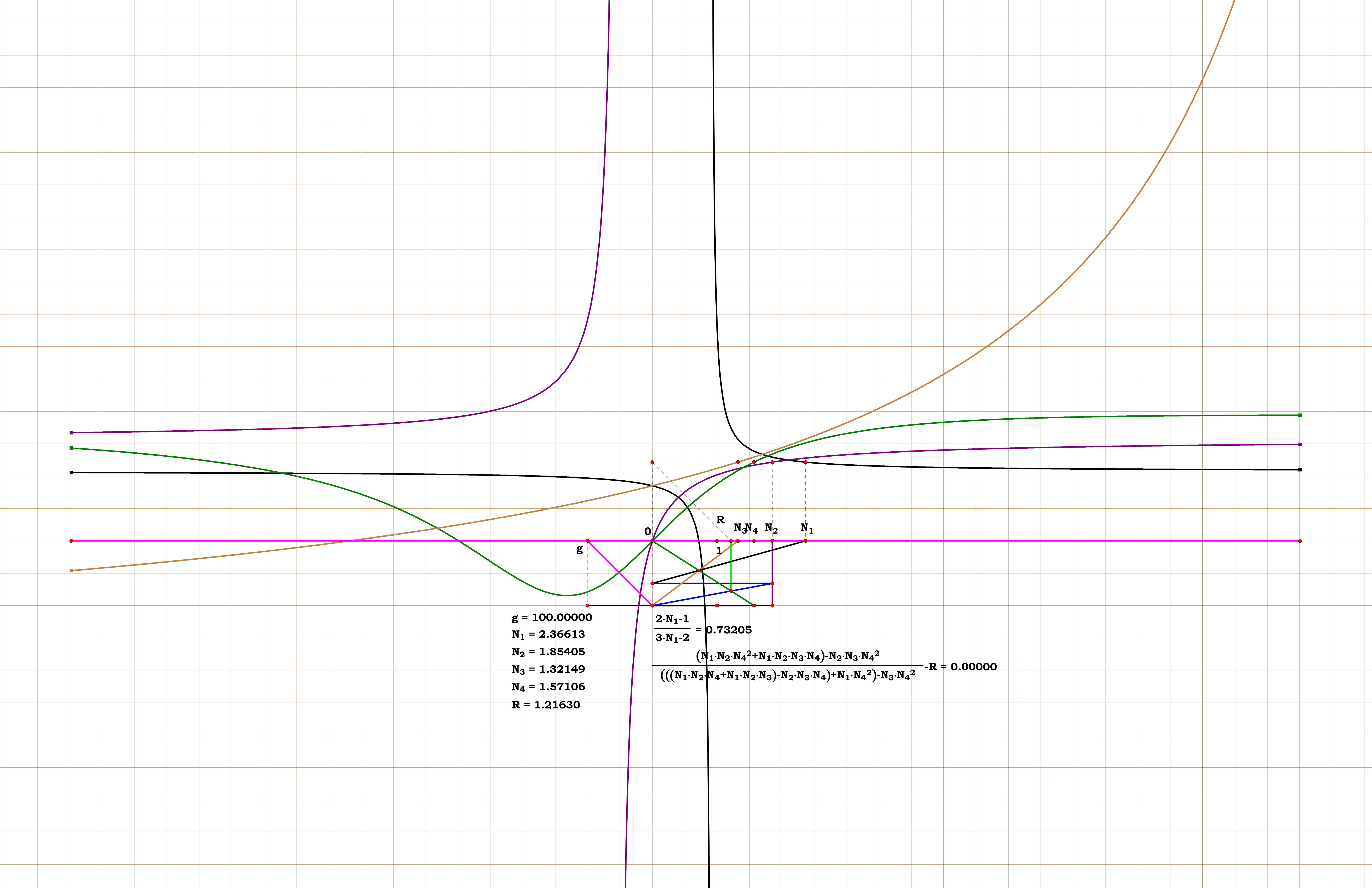
1, 2, 3, 0. $$\frac{N_1 \cdot N_2 - N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_3}{N_1 - N_3 + N_1 \cdot N_2 - N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_3}$$

1, 0, 3, 4. $$\frac{N_1 \cdot N_4^2 - N_3 \cdot N_4^2 + N_1 \cdot N_3 \cdot N_4}{N_1 \cdot N_4^2 - N_3 \cdot N_4^2 + N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4}$$

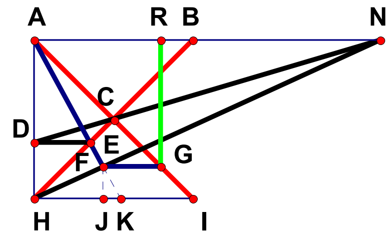
0, 2, 3, 4. $$\frac{N_2 \cdot N_4 \cdot \left(N_3 + N_4 - N_3 \cdot N_4\right)}{N_4^2 - N_3 \cdot N_4^2 + N_2 \cdot N_3 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}$$

1, 2, 0, 4. $$\frac{N_2 \cdot N_4 \cdot \left(N_1 - N_4 + N_1 \cdot N_4\right)}{N_1 \cdot N_4^2 - N_4^2 + N_1 \cdot N_2 - N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4}$$

1, 2, 3, 4. $$\frac{N_1 \cdot N_2 \cdot N_4^2 + N_1 \cdot N_2 \cdot N_3 \cdot N_4 - N_2 \cdot N_3 \cdot N_4^2}{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4 + N_1 \cdot N_4^2 - N_3 \cdot N_4^2}$$



R
0
N3
N4
N2
N1
g
1
g = 100.00000
N1 = 2.36613
N2 = 1.85405
N3 = 1.32149
N4 = 1.57106
R = 1.21630
$\frac{2 \cdot N_1 - 1}{3 \cdot N_1 - 2} = 0.73205$
$\frac{(N_1 \cdot N_2 \cdot N_4^2 + N_1 \cdot N_2 \cdot N_3 \cdot N_4) - N_2 \cdot N_3 \cdot N_4^2}{(((N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4) + N_1 \cdot N_4^2) - N_3 \cdot N_4^2} - R = 0.00000$



1CST7R4

Unit.

$AB := 1$

Given.

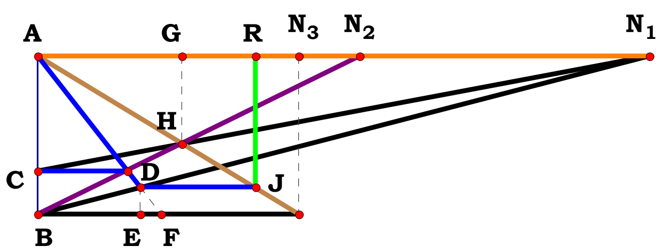
$AN := 3$

Descriptions.

$$HJ := \frac{AN^2 - AN}{AN^2 + AN - 1} \qquad FJ := \frac{AB \cdot HJ}{AN} \qquad BR := FJ \qquad AR := AB - BR$$

Definitions.

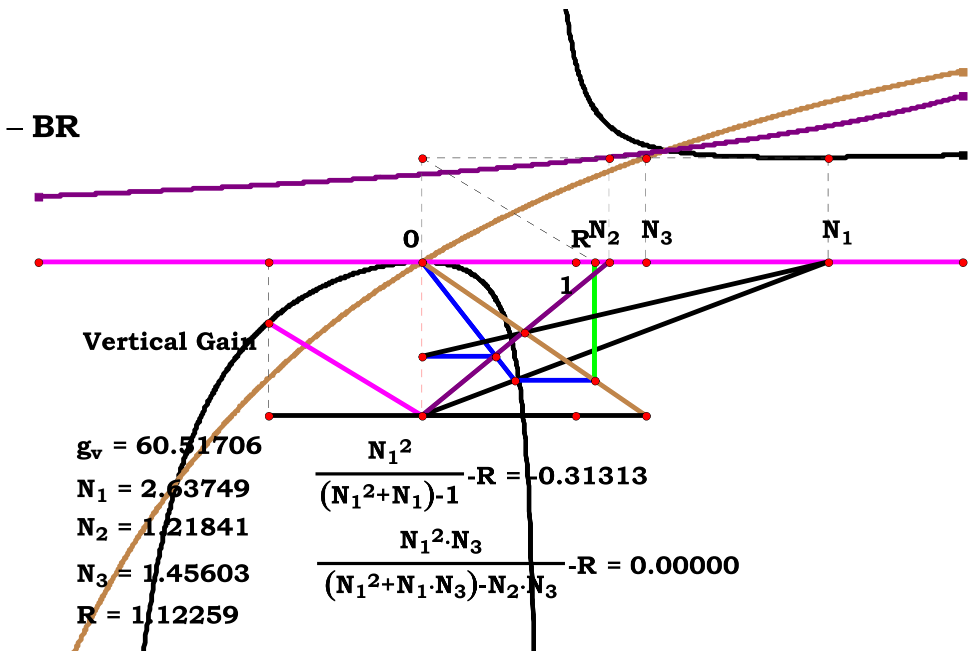
$$FJ - \frac{AN - 1}{AN^2 + AN - 1} = 0 \qquad AR - \frac{AN^2}{AN^2 + AN - 1} = 0$$



$N_1 := 4$

$N_2 := 3$

$N_3 := 2$



$$gh := \frac{N_2}{N_2 + N_3} \qquad ag := N_3 \cdot gh \qquad ac := \frac{gh \cdot N_1}{N_1 - ag} \qquad cd := N_2 - N_2 \cdot ac \qquad bf := \frac{cd}{ac}$$

$$de := \frac{bf}{N_1 + bf} \qquad ar := N_3 - N_3 \cdot de$$

Definitions.

$$ag - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0 \qquad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad cd - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^{\,2} \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

$$bf - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} = 0 \qquad de - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1^{\,2} \cdot N_3}{N_1^{\,2} + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

Three Transforms.

0, 0, 0. $$1$$

1, 0, 0. $$\frac{N_1^2}{N_1^2 + N_1 - 1}$$

0, 2, 0. $$\frac{1}{2 - N_2}$$

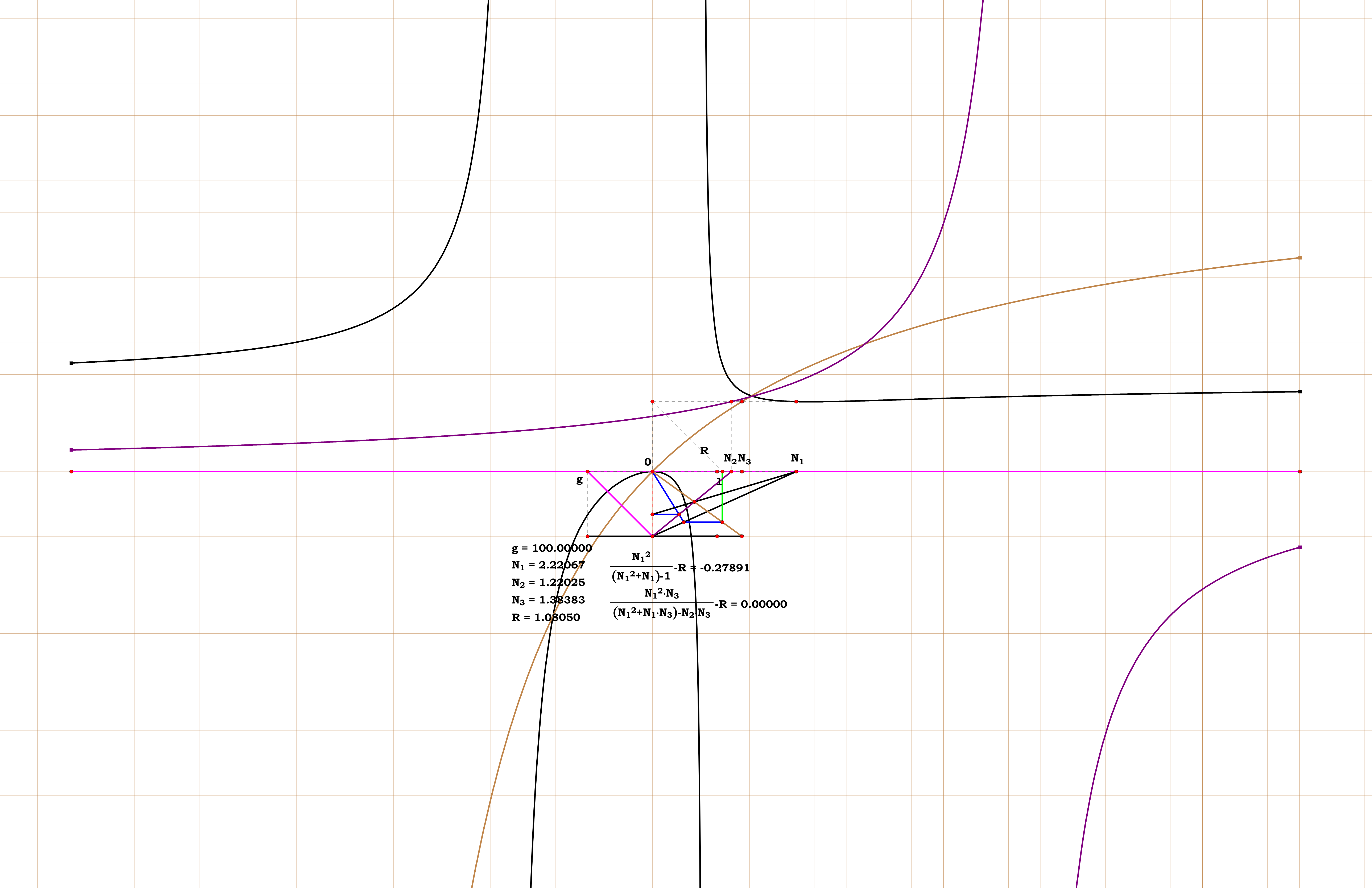
0, 0, 3. $$N_3$$

1, 2, 0. $$\frac{N_1^2}{N_1^2 + N_1 - N_2}$$

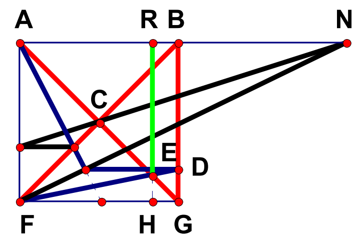
1, 0, 3. $$N_1^2 \cdot \frac{N_3}{N_1^2 + N_1 \cdot N_3 - N_3}$$

0, 2, 3. $$\frac{N_3}{1 + N_3 - N_2 \cdot N_3}$$

1, 2, 3. $$\frac{N_1^2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3}$$



0
R
N_2 N_3
N_1
g
1
g = 100.00000
N_1 = 2.22067
N_2 = 1.22025
N_3 = 1.38383
R = 1.08050
$\frac{N_1^2}{(N_1^2+N_1)-1}$-R = -0.27891
$\frac{N_1^2 \cdot N_3}{(N_1^2+N_1 \cdot N_3)-N_2 N_3}$-R = 0.00000



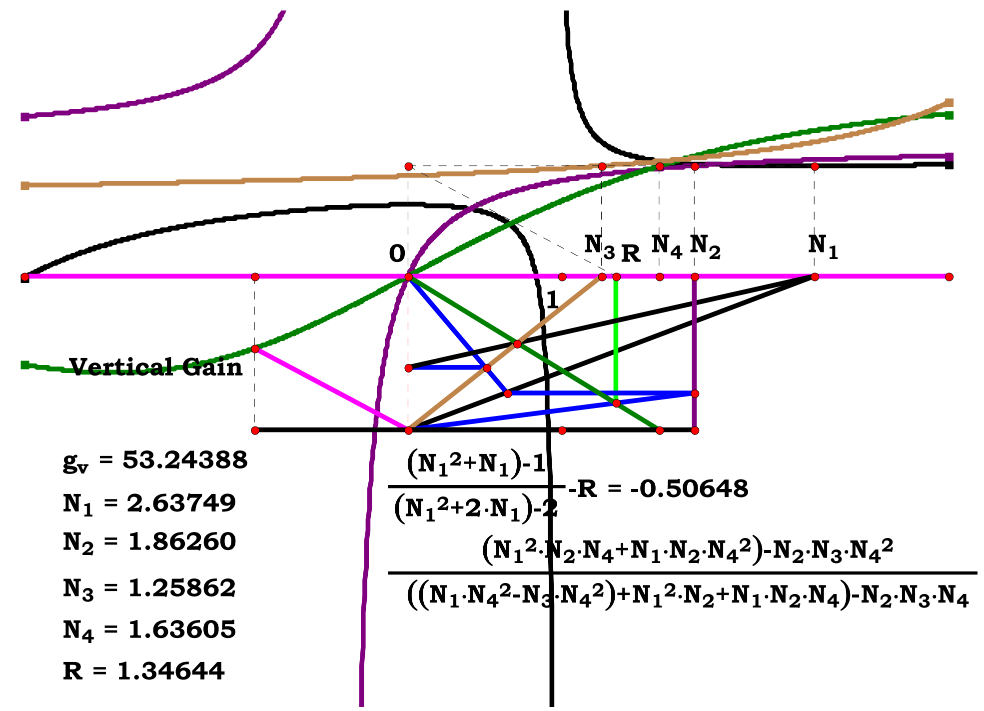
1CST7R5

Unit.

$AB := 1$

Given.

$AN := 3$



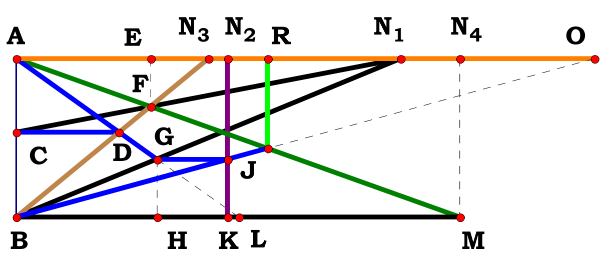
Descriptions.

$$DG := \frac{AN - 1}{AN^2 + AN - 1} \qquad EH := \frac{AB \cdot DG}{AB + DG}$$

$$AR := AB - EH$$

Definitions.

$$AR - \frac{AN^2 + AN - 1}{AN^2 + 2AN - 2} = 0$$



$$N_1 := 5 \qquad N_2 := 4 \qquad N_3 := 3 \qquad N_4 := 2$$

$$ef := \frac{N_3}{N_3 + N_4} \qquad ae := N_4 \cdot ef \qquad ac := \frac{ef \cdot N_1}{N_1 - ae}$$

$$cd := N_3 - N_3 \cdot ac \qquad bl := \frac{cd}{ac} \qquad gh := \frac{bl}{N_1 + bl} \qquad ao := \frac{N_2}{gh} \qquad ar := \frac{N_4 \cdot ao}{N_4 + ao}$$

Definitions.

$$ae - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \qquad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad cd - \frac{N_1 \cdot N_3 \cdot N_4 - N_3^2 \cdot N_4}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad bl - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1} = 0$$

$$gh - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ao - \frac{N_2 \cdot N_1^2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1^2 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4^2 - N_2 \cdot N_3 \cdot N_4^2}{N_1 \cdot N_4^2 - N_3 \cdot N_4^2 + N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. 1

1, 0, 0, 0. $$\frac{N_1{}^2 + N_1 - 1}{N_1{}^2 + 2 \cdot N_1 - 2}$$

0, 2, 0, 0. 1

0, 0, 3, 0. $$\frac{N_3 - 2}{2 \cdot N_3 - 3}$$

0, 0, 0, 4. N_4

1, 2, 0, 0. $$\frac{N_2 \cdot N_1{}^2 + N_2 \cdot N_1 - N_2}{N_1 - N_2 + N_1{}^2 \cdot N_2 + N_1 \cdot N_2 - 1}$$

1, 0, 3, 0. $$\frac{N_1{}^2 + N_1 - N_3}{N_1{}^2 + 2 \cdot N_1 - 2 \cdot N_3}$$

1, 0, 0, 4. $$\frac{N_1{}^2 \cdot N_4 + N_1 \cdot N_4{}^2 - N_4{}^2}{N_1{}^2 + N_1 \cdot N_4{}^2 + N_1 \cdot N_4 - N_4{}^2 - N_4}$$

0, 2, 3, 0. $$\frac{N_2 \cdot N_3 - 2 \cdot N_2}{N_3 - 2 \cdot N_2 + N_2 \cdot N_3 - 1}$$

0, 2, 0, 4. N_4

0, 0, 3, 4. $$\frac{N_4 - N_3 \cdot N_4{}^2 + N_4{}^2}{N_4 - N_3 \cdot N_4{}^2 + N_4{}^2 - N_3 \cdot N_4 + 1}$$

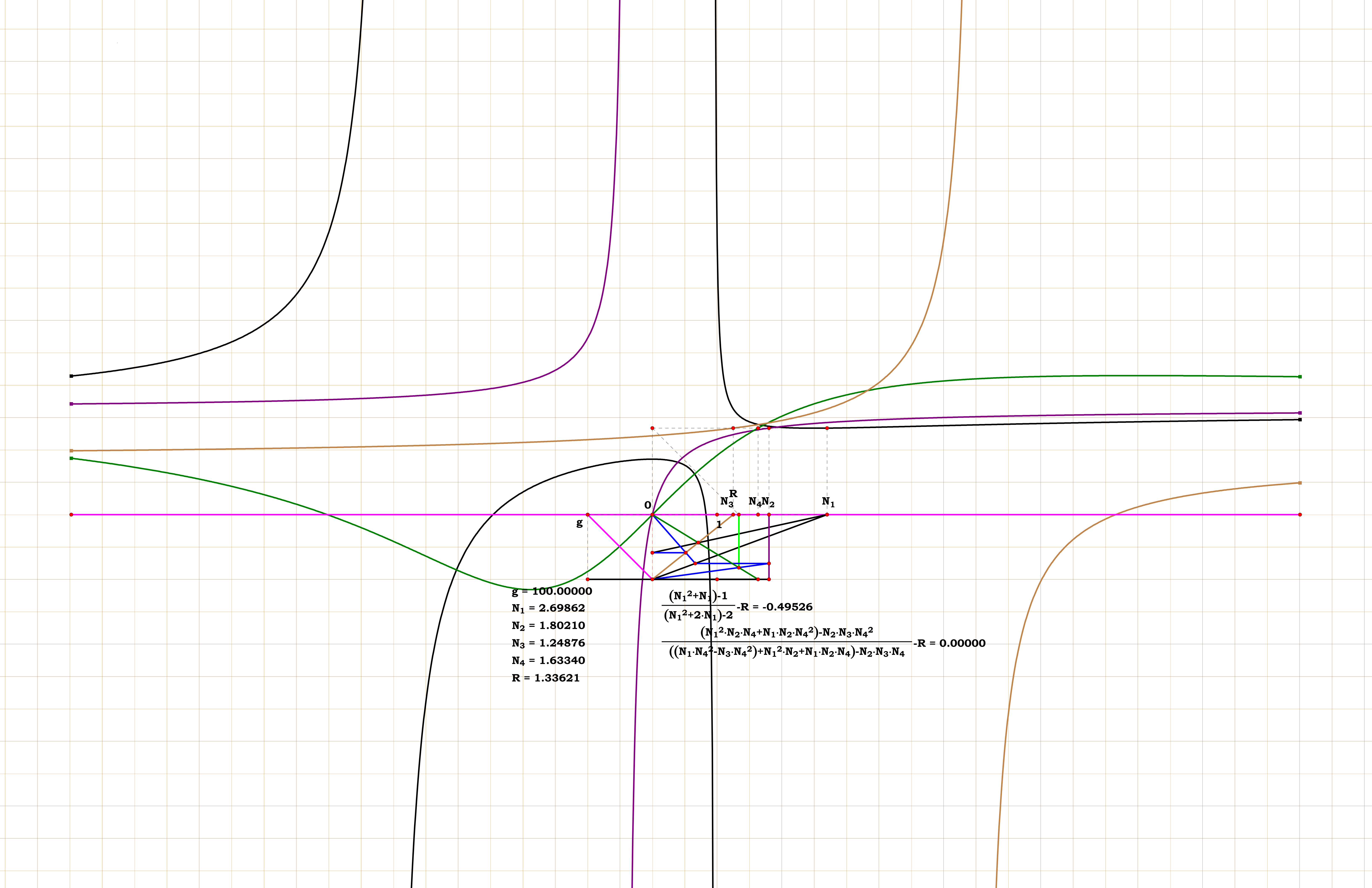
1, 2, 3, 0. $$\frac{N_2 \cdot N_1{}^2 + N_2 \cdot N_1 - N_2 \cdot N_3}{N_1 - N_3 + N_1{}^2 \cdot N_2 + N_1 \cdot N_2 - N_2 \cdot N_3}$$

1, 0, 3, 4. $$\frac{N_1{}^2 \cdot N_4 + N_1 \cdot N_4{}^2 - N_3 \cdot N_4{}^2}{N_1{}^2 + N_1 \cdot N_4{}^2 + N_1 \cdot N_4 - N_3 \cdot N_4{}^2 - N_3 \cdot N_4}$$

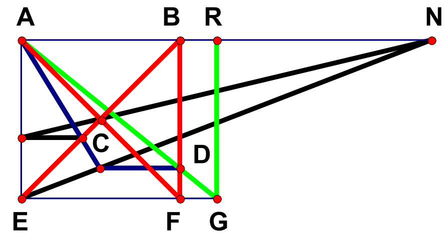
0, 2, 3, 4. $$\frac{N_2 \cdot N_4 \cdot \left(N_4 - N_3 \cdot N_4 + 1\right)}{N_2 - N_3 \cdot N_4{}^2 + N_4{}^2 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}$$

1, 2, 0, 4. $$\frac{N_2 \cdot N_4 \cdot \left(N_1{}^2 + N_4 \cdot N_1 - N_4\right)}{N_2 \cdot N_1{}^2 + N_1 \cdot N_4{}^2 + N_2 \cdot N_1 \cdot N_4 - N_4{}^2 - N_2 \cdot N_4}$$

1, 2, 3, 4. $$\frac{N_1{}^2 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4{}^2 - N_2 \cdot N_3 \cdot N_4{}^2}{N_1 \cdot N_4{}^2 - N_3 \cdot N_4{}^2 + N_1{}^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}$$



0
g
1
N3
R
N4
N2
N1
g = 100.00000
N1 = 2.69862
N2 = 1.80210
N3 = 1.24876
N4 = 1.63340
R = 1.33621
((N1²+N1)-1)/((N1²+2·N1)-2) -R = -0.49526
((N1²·N2·N4+N1·N2·N4²)-N2·N3·N4²)/(((N1·N4²-N3·N4²)+N1²·N2+N1·N2·N4)-N2·N3·N4) -R = 0.00000



1CST7R6

Unit.

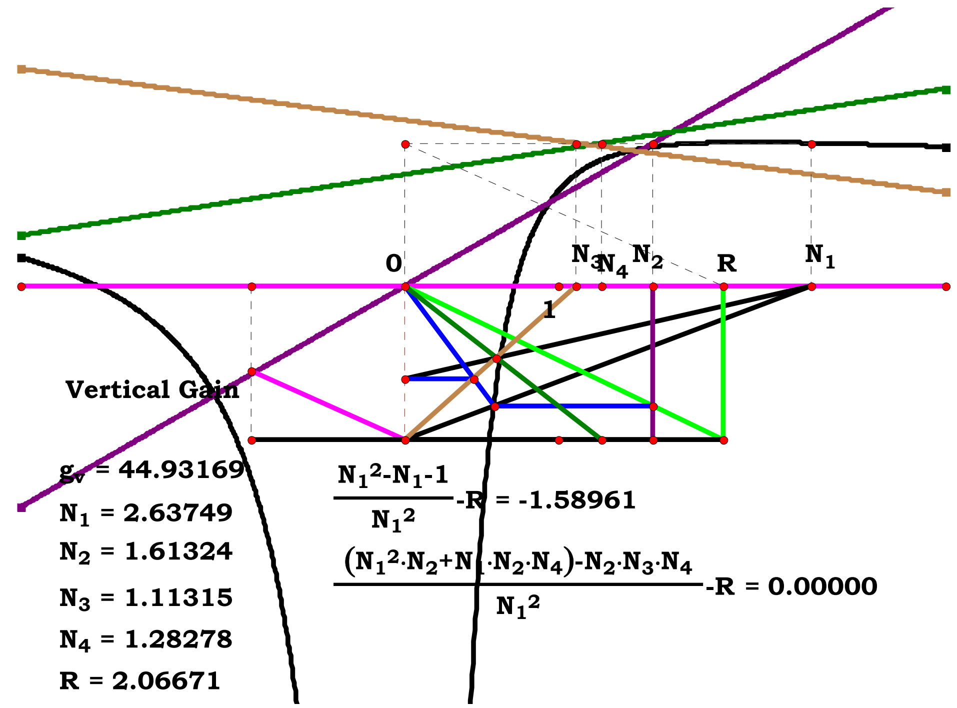
$AB := 1$

Given.

$AN := 3$

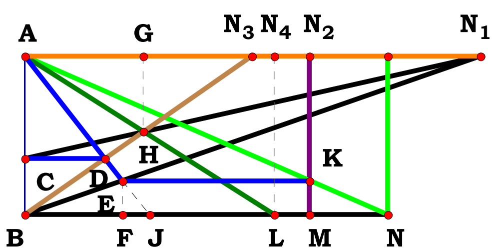
Descriptions.

$$DF := \frac{AN - 1}{AN^2 + AN - 1} \quad BD := AB - DF \quad EG := \frac{AB^2}{BD} \quad AR := EG$$



Definitions.

$$AR - \frac{AN^2 + AN - 1}{AN^2} = 0$$



$N_1 := 5$

$N_2 := 4$

$N_3 := 3$

$N_4 := 2$

$$gh := \frac{N_3}{N_3 + N_4} \quad ag := N_4 \cdot gh \quad ac := \frac{gh \cdot N_1}{N_1 - ag} \quad cd := N_3 - N_3 \cdot ac \quad bj := \frac{cd}{ac} \quad de := \frac{bj}{N_1 + bj} \quad ar := \frac{N_2}{1 - de}$$

Definitions.

$$ag - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \quad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad cd - \frac{N_1 \cdot N_3 \cdot N_4 - N_3^2 \cdot N_4}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0$$

$$bj - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1} = 0 \quad de - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad ar - \frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1^2} = 0$$

Four Transforms.

0, 0, 0, 0. 1

1, 0, 0, 0. $\frac{N_1^2 + N_1 - 1}{N_1^2}$

0, 2, 0, 0. N_2

0, 0, 3, 0. $2 - N_3$

0, 0, 0, 4. 1

1, 2, 0, 0. $\frac{N_2 \cdot \left(N_1^2 + N_1 - 1\right)}{N_1^2}$

1, 0, 3, 0. $\frac{N_1^2 + N_1 - N_3}{N_1^2}$

1, 0, 0, 4. $\frac{N_1^2 + N_4 \cdot N_1 - N_4}{N_1^2}$

0, 2, 3, 0. $2 \cdot N_2 - N_2 \cdot N_3$

0, 2, 0, 4. N_2

0, 0, 3, 4. $\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1^2}$

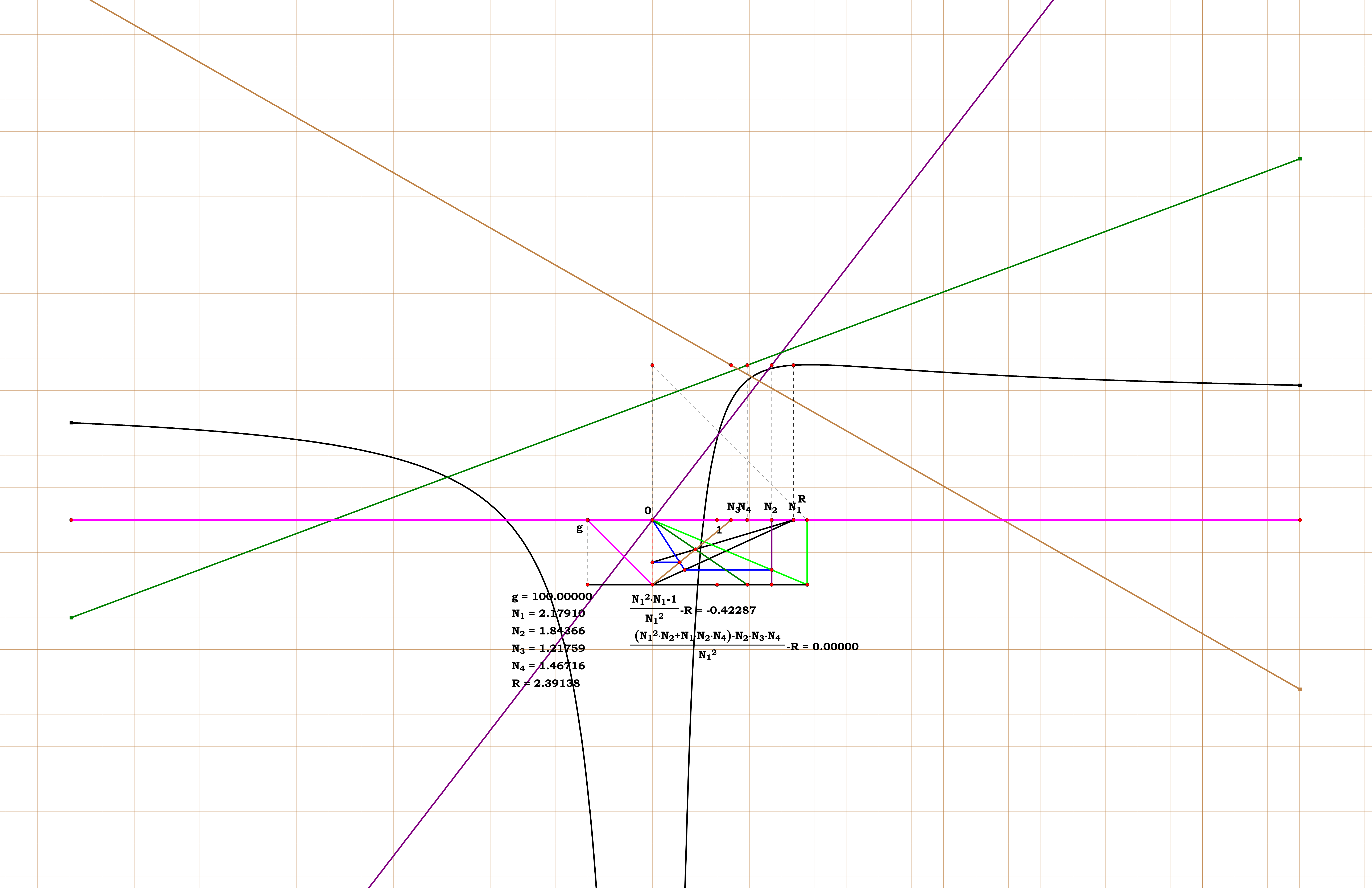
1, 2, 3, 0. $\frac{N_2 \cdot \left(N_1^2 + N_1 - N_3\right)}{N_1^2}$

1, 0, 3, 4. $\frac{N_1^2 + N_4 \cdot N_1 - N_3 \cdot N_4}{N_1^2}$

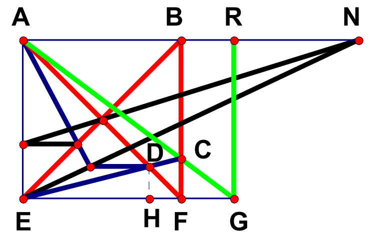
0, 2, 3, 4. $N_2 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4$

1, 2, 0, 4. $\frac{N_2 \cdot \left(N_1^2 + N_4 \cdot N_1 - N_4\right)}{N_1^2}$

1, 2, 3, 4. $\frac{N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1^2}$



0
g
1
N_3 N_4
N_2
N_1
R
g = 100.00000
N_1 = 2.17910
N_2 = 1.84366
N_3 = 1.21759
N_4 = 1.46716
R = 2.39138
$\frac{N_1^2 \cdot N_1 - 1}{N_1^2} - R = -0.42287$
$\frac{(N_1^2 \cdot N_2 + N_1 \cdot N_2 \cdot N_4) - N_2 \cdot N_3 \cdot N_4}{N_1^2} - R = 0.00000$



1CST7R7

Unit.

$AB := 1$

Given.

$AN := -3$

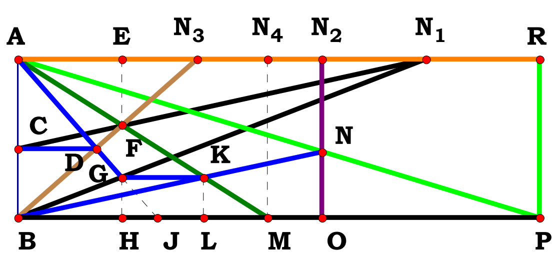
Descriptions.

$$DH := \frac{AN - 1}{AN^2 + AN - 1} \quad EH := AB - DH \quad CF := \frac{DH \cdot AB}{EH} \quad BC := AB - CF$$

$$EG := \frac{AB^2}{BC} \quad AR := EG$$

Definitions.

$$AR - \frac{AN^2}{AN^2 - AN + 1} = 0$$

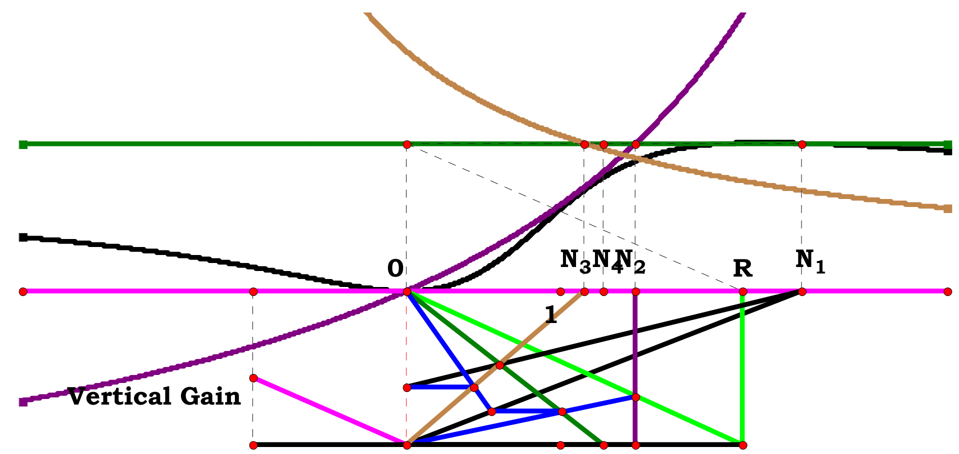


$N_1 := 5$

$N_2 := 4$

$N_3 := 3$

$N_4 := 2$



$g_v = 43.89266$

$N_1 = 2.56476$

$N_2 = 1.48856$

$N_3 = 1.15471$

$N_4 = 1.28278$

$R = 2.18611$

$$\frac{N_1^2}{(N_1^2 - N_1) + 1} - R = -0.87398$$

$$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_2) + N_2 \cdot N_3} - R = 0.00000$$

$$ef := \frac{N_3}{N_3 + N_4} \quad ae := N_4 \cdot ef \quad ac := \frac{ef \cdot N_1}{N_1 - ae} \quad cd := N_3 - N_3 \cdot ac \quad bj := \frac{cd}{ac} \quad gh := \frac{bj}{N_1 + bj} \quad bl := N_4 - N_4 \cdot gh \quad no := \frac{gh \cdot N_2}{bl} \quad ar := \frac{N_2}{1 - no}$$

Definitions.

$$ae - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \quad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad cd - \frac{N_1 \cdot N_3 \cdot N_4 - N_3^2 \cdot N_4}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad bj - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1} = 0$$

$$gh - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad bl - \frac{N_1^2 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad no - \frac{N_1 \cdot N_2 - N_2 \cdot N_3}{N_1^2} = 0 \quad ar - \frac{N_1^2 \cdot N_2}{N_1^2 - N_1 \cdot N_2 + N_2 \cdot N_3} = 0$$

Four Transforms.

0, 0, 0, 0. 1

1, 0, 0, 0. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_1 + 1}$

0, 2, 0, 0. N_2

0, 0, 3, 0. $\dfrac{1}{N_3}$

0, 0, 0, 4. 1

1, 2, 0, 0. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_2 \cdot N_1 + N_2}$

1, 0, 3, 0. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_1 + N_3}$

1, 0, 0, 4. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_1 + 1}$

0, 2, 3, 0. $\dfrac{N_2}{N_2 \cdot N_3 - N_2 + 1}$

0, 2, 0, 4. N_2

0, 0, 3, 4. $\dfrac{1}{N_3}$

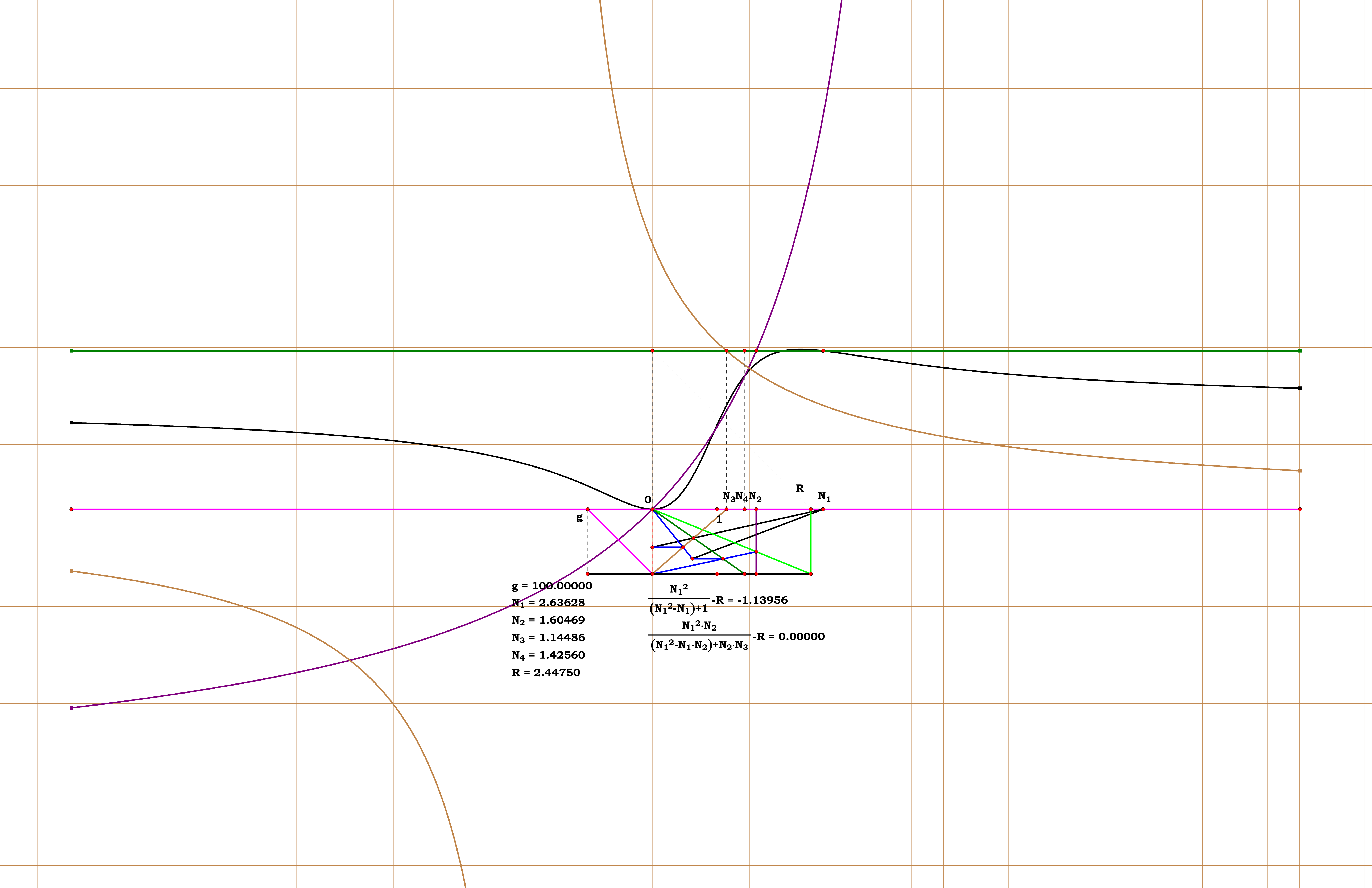
1, 2, 3, 0. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_1 \cdot N_2 + N_2 \cdot N_3}$

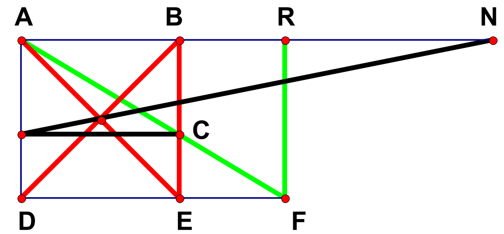
1, 0, 3, 4. $\dfrac{N_1^{\,2}}{N_1^{\,2} - N_1 + N_3}$

0, 2, 3, 4. $\dfrac{N_2}{N_2 \cdot N_3 - N_2 + 1}$

1, 2, 0, 4. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_2 \cdot N_1 + N_2}$

1, 2, 3, 4. $\dfrac{N_1^{\,2} \cdot N_2}{N_1^{\,2} - N_1 \cdot N_2 + N_2 \cdot N_3}$


0
g
1
N_3 N_4 N_2
R
N_1
g = 100.00000
N_1 = 2.63628
N_2 = 1.60469
N_3 = 1.14486
N_4 = 1.42560
R = 2.44750
$\frac{N_1^2}{(N_1^2 - N_1) + 1} - R = -1.13956$
$\frac{N_1^2 \cdot N_2}{(N_1^2 - N_1 \cdot N_2) + N_2 \cdot N_3} - R = 0.00000$



1CST7R8

Unit.

$AB := 1$

Given.

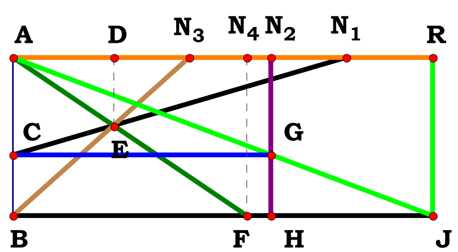
$AN := 3$

Descriptions.

$$BC := \frac{AN}{2 \cdot AN - 1} \qquad DF := \frac{AB^2}{BC} \qquad AR := DF$$

Definitions.

$$AR - \frac{2AN - 1}{AN} = 0$$

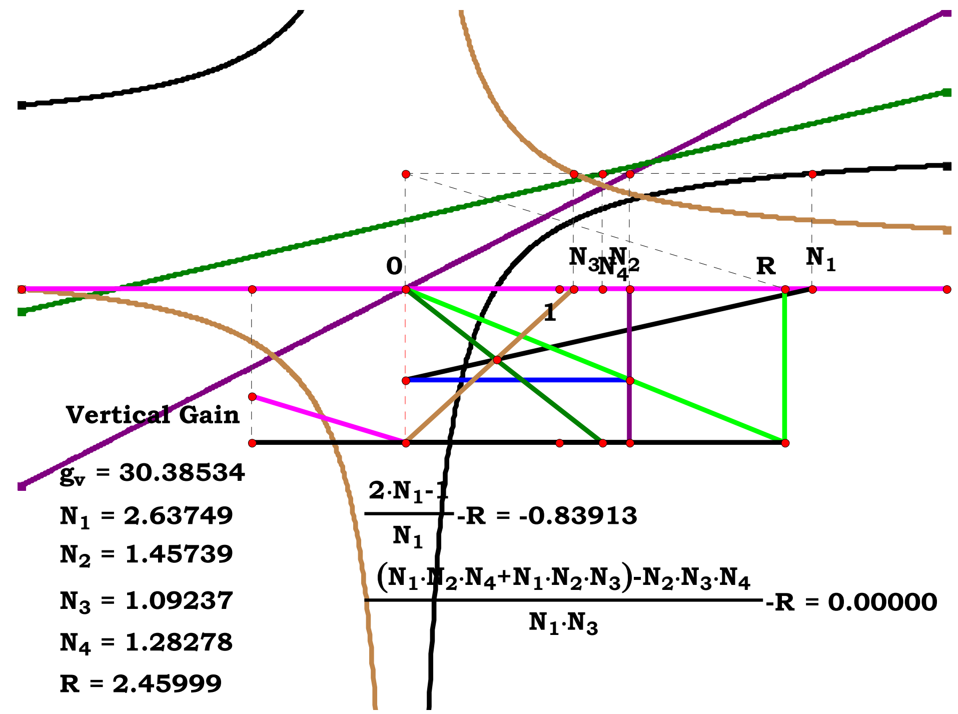


$N_1 := 5$

$N_2 := 4$

$N_3 := 3$

$N_4 := 2$



$$de := \frac{N_3}{N_3 + N_4} \qquad ad := N_4 \cdot de \qquad ac := \frac{de \cdot N_1}{N_1 - ad} \qquad ar := \frac{N_2}{ac}$$

Definitions.

$$ad - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \qquad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_3} = 0$$

Four Transforms.

0, 0, 0, 0. 1

1, 0, 0, 0. $\dfrac{2 \cdot N_1 - 1}{N_1}$

0, 2, 0, 0. N_2

0, 0, 3, 0. $\dfrac{1}{N_3}$

0, 0, 0, 4. 1

1, 2, 0, 0. $\dfrac{2 \cdot N_1 \cdot N_2 - N_2}{N_1}$

1, 0, 3, 0. $\dfrac{N_1 - N_3 + N_1 \cdot N_3}{N_1 \cdot N_3}$

1, 0, 0, 4. $\dfrac{N_1 - N_4 + N_1 \cdot N_4}{N_1}$

0, 2, 3, 0. $\dfrac{N_2}{N_3}$

0, 2, 0, 4. N_2

0, 0, 3, 4. $\dfrac{N_3 + N_4 - N_3 \cdot N_4}{N_3}$

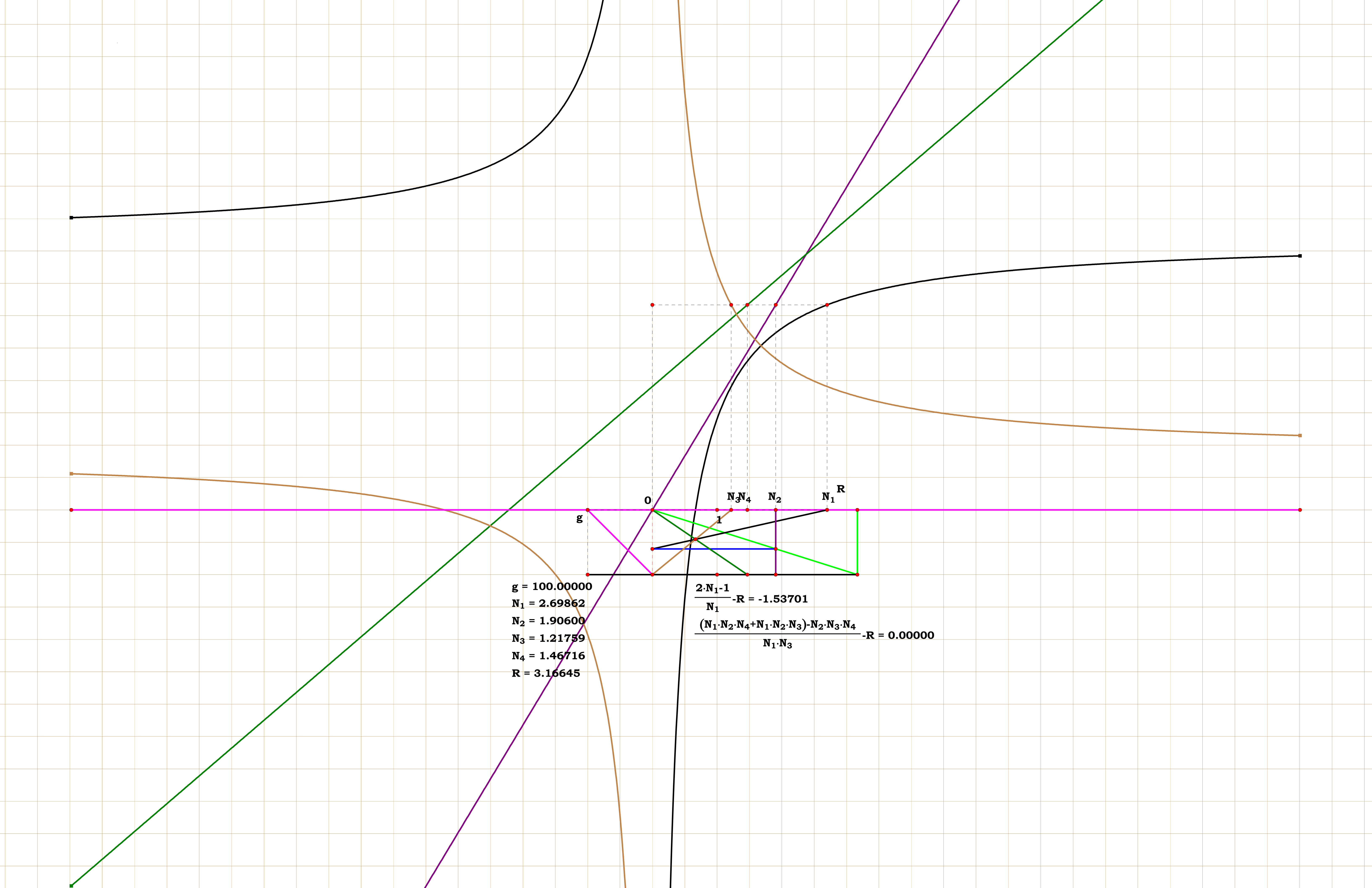
1, 2, 3, 0. $\dfrac{N_1 \cdot N_2 - N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_3}{N_1 \cdot N_3}$

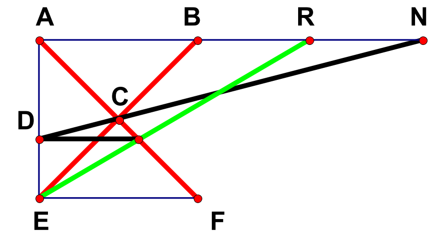
1, 0, 3, 4. $\dfrac{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4}{N_1 \cdot N_3}$

0, 2, 3, 4. $\dfrac{N_2 \cdot N_3 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_3}$

1, 2, 0, 4. $\dfrac{N_1 \cdot N_2 - N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4}{N_1}$

1, 2, 3, 4. $\dfrac{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_3}$


0
N_3 N_4
N_2
N_1
R
g
1
g = 100.00000
N_1 = 2.69862
N_2 = 1.90600
N_3 = 1.21759
N_4 = 1.46716
R = 3.16645
$\frac{2 \cdot N_1 - 1}{N_1} - R = -1.53701$
$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_3} - R = 0.00000$



1CST7R9

Unit.

$AB := 1$

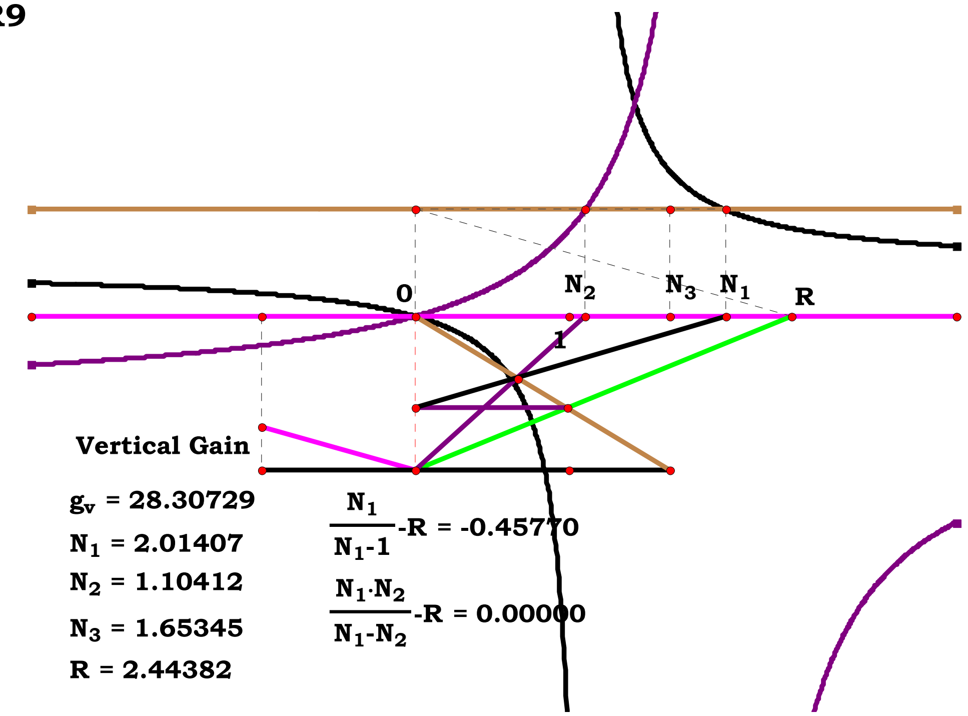
Given.

$AN := 3$

Descriptions.

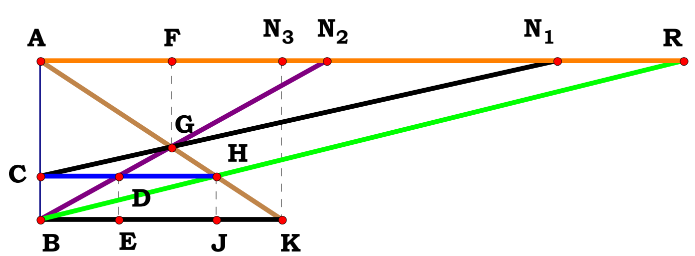
$$AD := \frac{AN}{2 \cdot AN - 1} \qquad DE := AB - AD$$

$$AR := \frac{AD \cdot AB}{DE}$$



Definitions.

$$AR - \frac{AN}{AN - 1} = 0$$



$N_1 := 4 \qquad N_2 := 3 \qquad N_3 := 2$

$$fg := \frac{N_2}{N_2 + N_3} \qquad af := N_3 \cdot fg \qquad ac := \frac{fg \cdot N_1}{N_1 - af} \qquad cd := \frac{N_2}{ac} \qquad bc := 1 - ac \qquad bj := N_3 - N_3 \cdot bc \qquad ar := \frac{bj}{bc}$$

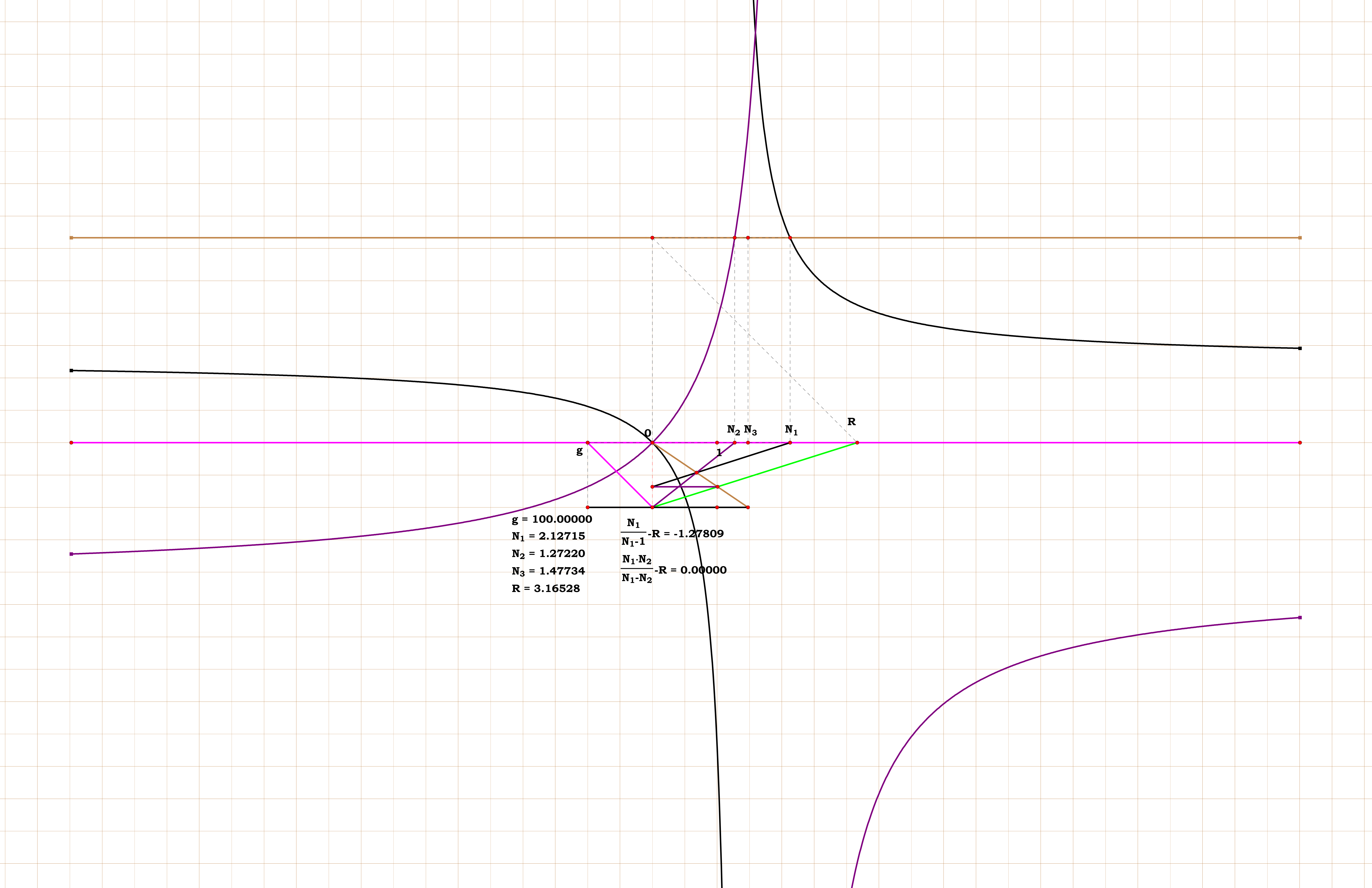
Definitions.

$$af - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0 \qquad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad cd - \frac{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} = 0 \qquad bc - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad bj - \frac{N_1 \cdot N_2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

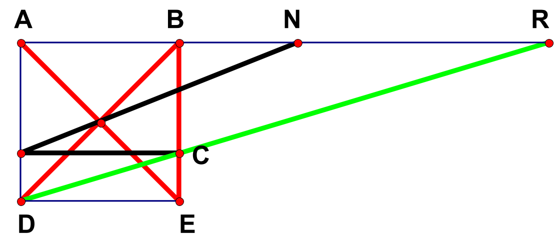
$$ar - \frac{N_1 \cdot N_2}{N_1 - N_2} = 0$$

Two Transforms.

0, 0.	**undefined**
1, 0.	$\dfrac{N_1}{N_1 - 1}$
0, 2.	$\dfrac{N_2}{1 - N_2}$
1, 0.	$\dfrac{N_1 \cdot N_2}{N_1 - N_2}$



0
g
1
N_2 N_3
N_1
R
g = 100.00000
N_1 = 2.12715
N_2 = 1.27220
N_3 = 1.47734
R = 3.16528
$\frac{N_1}{N_1-1}$-R = -1.27809
$\frac{N_1 \cdot N_2}{N_1-N_2}$-R = 0.00000



1CST7R10

Unit.

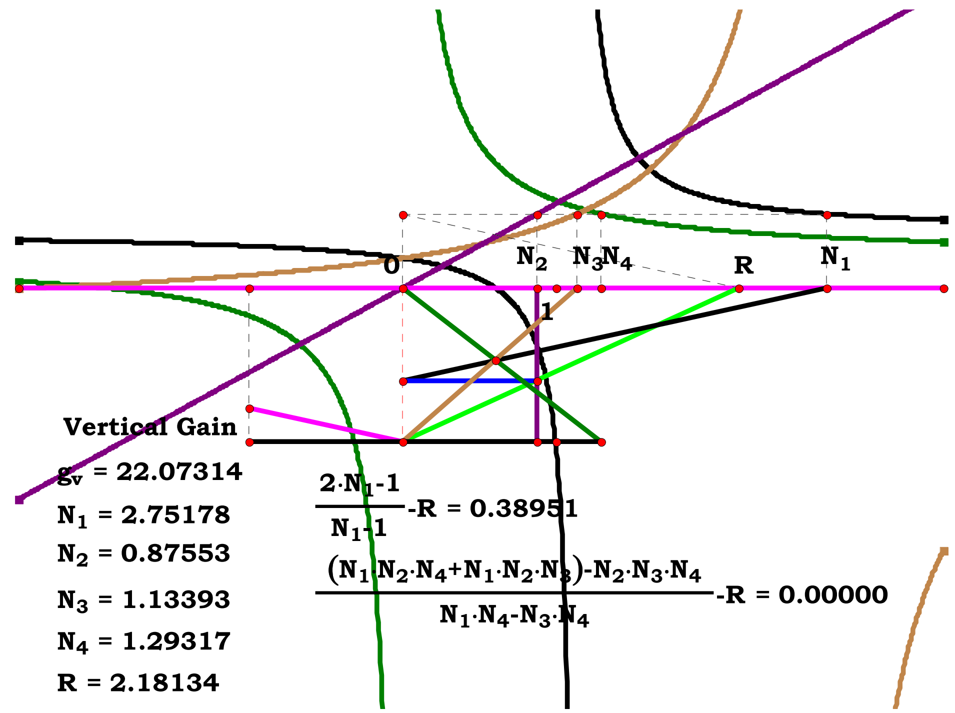
$AB := 1$

Given.

$AN := 5$

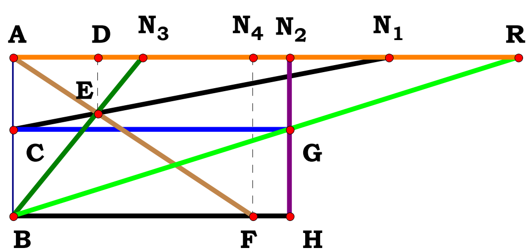
Descriptions.

$$BC := \frac{AN}{2 \cdot AN - 1} \qquad CE := AB - BC \qquad AR := \frac{AB^2}{CE}$$



Definitions.

$$AR - \frac{2AN - 1}{AN - 1} = 0$$



$N_1 := 5$

$N_2 := 4$

$N_3 := 3$

$N_4 := 2$

$$de := \frac{N_3}{N_3 + N_4} \qquad ad := N_4 \cdot de \qquad ac := \frac{de \cdot N_1}{N_1 - ad} \qquad ar := \frac{N_2}{1 - ac}$$

Definitions.

$$ad - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \qquad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. undefined

1, 0, 0, 0. $\dfrac{2 \cdot N_1 - 1}{N_1 - 1}$

0, 2, 0, 0. undefined

0, 0, 3, 0. $-\dfrac{1}{N_3 - 1}$

0, 0, 0, 4. undefined

1, 2, 0, 0. $\dfrac{2 \cdot N_1 \cdot N_2 - N_2}{N_1 - 1}$

1, 0, 3, 0. $\dfrac{N_1 - N_3 + N_1 \cdot N_3}{N_1 - N_3}$

1, 0, 0, 4. $\dfrac{N_1 - N_4 + N_1 \cdot N_4}{N_1 \cdot N_4 - N_4}$

0, 2, 3, 0. $-\dfrac{N_2}{N_3 - 1}$

0, 2, 0, 4. undefined

0, 0, 3, 4. $\dfrac{N_3 + N_4 - N_3 \cdot N_4}{N_4 - N_3 \cdot N_4}$

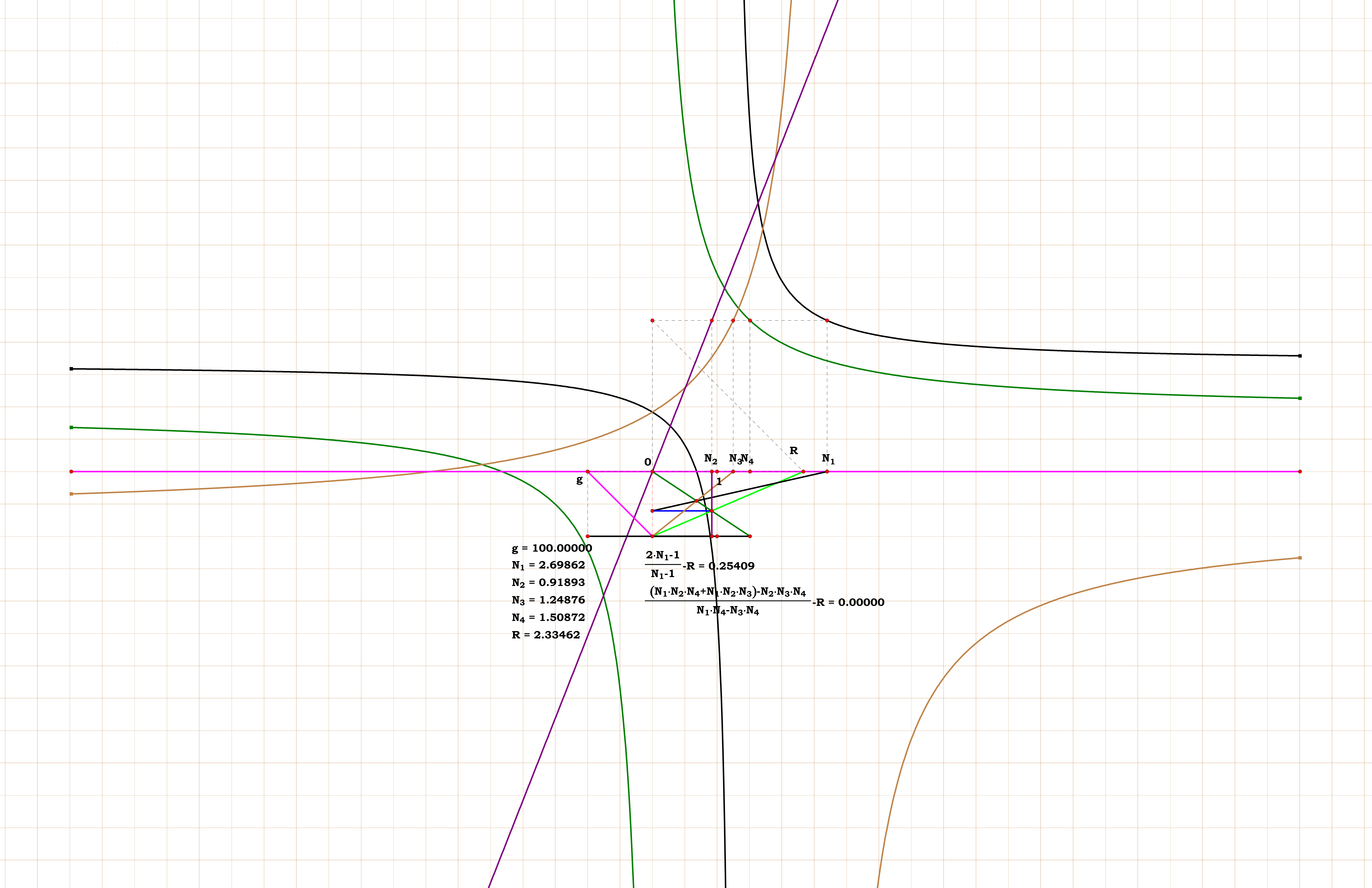
1, 2, 3, 0. $\dfrac{N_1 \cdot N_2 - N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_3}{N_1 - N_3}$

1, 0, 3, 4. $\dfrac{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4}$

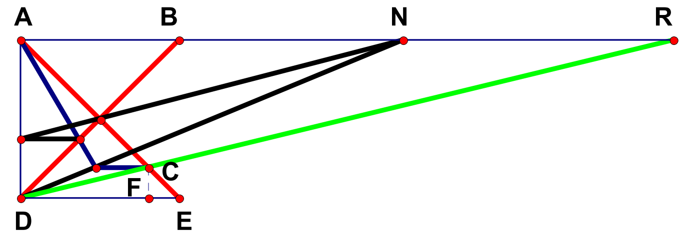
0, 2, 3, 4. $\dfrac{N_2 \cdot N_3 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_4 - N_3 \cdot N_4}$

1, 2, 0, 4. $\dfrac{N_1 \cdot N_2 - N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_4}{N_1 \cdot N_4 - N_4}$

1, 2, 3, 4. $\dfrac{N_1 \cdot N_2 \cdot N_3 + N_1 \cdot N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4}$



0
N_2
N_3 N_4
R
N_1
g
1
g = 100.00000
N_1 = 2.69862
N_2 = 0.91893
N_3 = 1.24876
N_4 = 1.50872
R = 2.33462
$\frac{2 \cdot N_1 - 1}{N_1 - 1} - R = 0.25409$
$\frac{(N_1 \cdot N_2 \cdot N_4 + N_1 \cdot N_2 \cdot N_3) - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} - R = 0.00000$



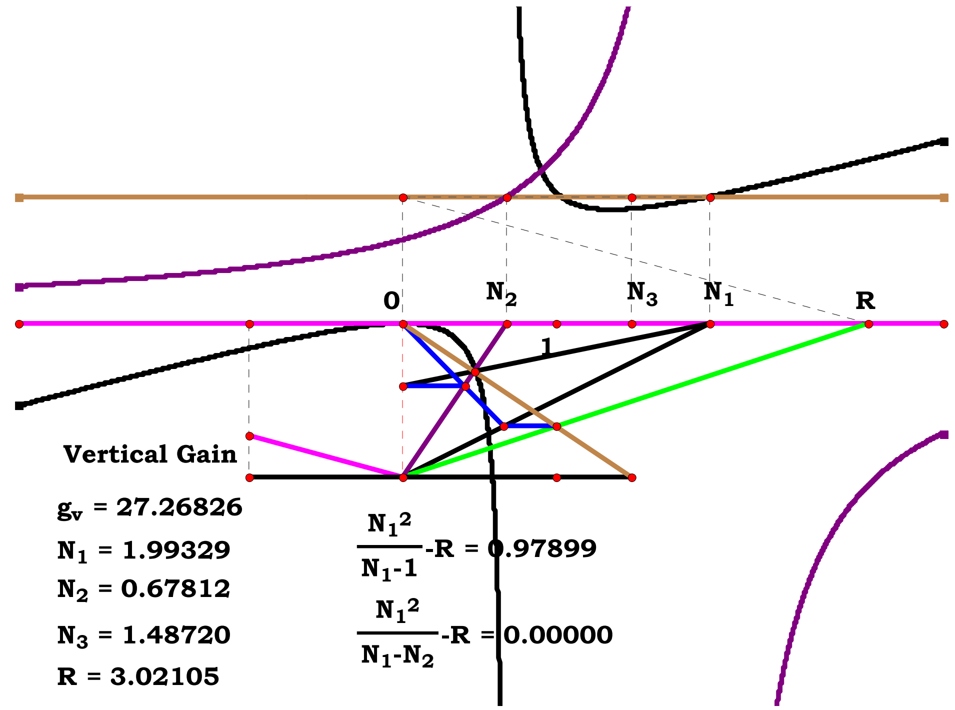
1CST7R11

Unit. $AB := 1$ **Given.** $AN := 3$

Descriptions.

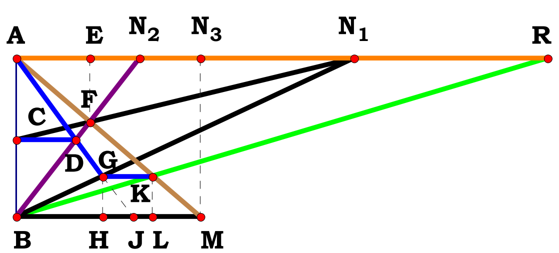
$$CF := \frac{AN - 1}{AN^2 + AN - 1} \qquad DF := AB - CF$$

$$AR := \frac{DF \cdot AB}{CF}$$



Definitions.

$$AR - \frac{AN^2}{AN - 1} = 0$$



$$N_1 := 4 \qquad N_2 := 3 \qquad N_3 := 2$$

$$ef := \frac{N_2}{N_2 + N_3} \quad ae := N_3 \cdot ef \quad ac := \frac{ef \cdot N_1}{N_1 - ae} \quad cd := N_2 - N_2 \cdot ac \quad bj := \frac{cd}{ac} \quad gh := \frac{bj}{N_1 + bj} \quad bl := N_3 - N_3 \cdot gh \quad ar := \frac{bl}{gh}$$

Definitions.

$$ae - \frac{N_2 \cdot N_3}{N_2 + N_3} = 0 \qquad ac - \frac{N_1 \cdot N_2}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad cd - \frac{N_1 \cdot N_2 \cdot N_3 - N_2^2 \cdot N_3}{N_1 \cdot N_2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad bj - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1} = 0 \qquad gh - \frac{N_1 \cdot N_3 - N_2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0$$

$$bl - \frac{N_1^2 \cdot N_3}{N_1^2 + N_1 \cdot N_3 - N_2 \cdot N_3} = 0 \qquad ar - \frac{N_1^2}{N_1 - N_2} = 0$$

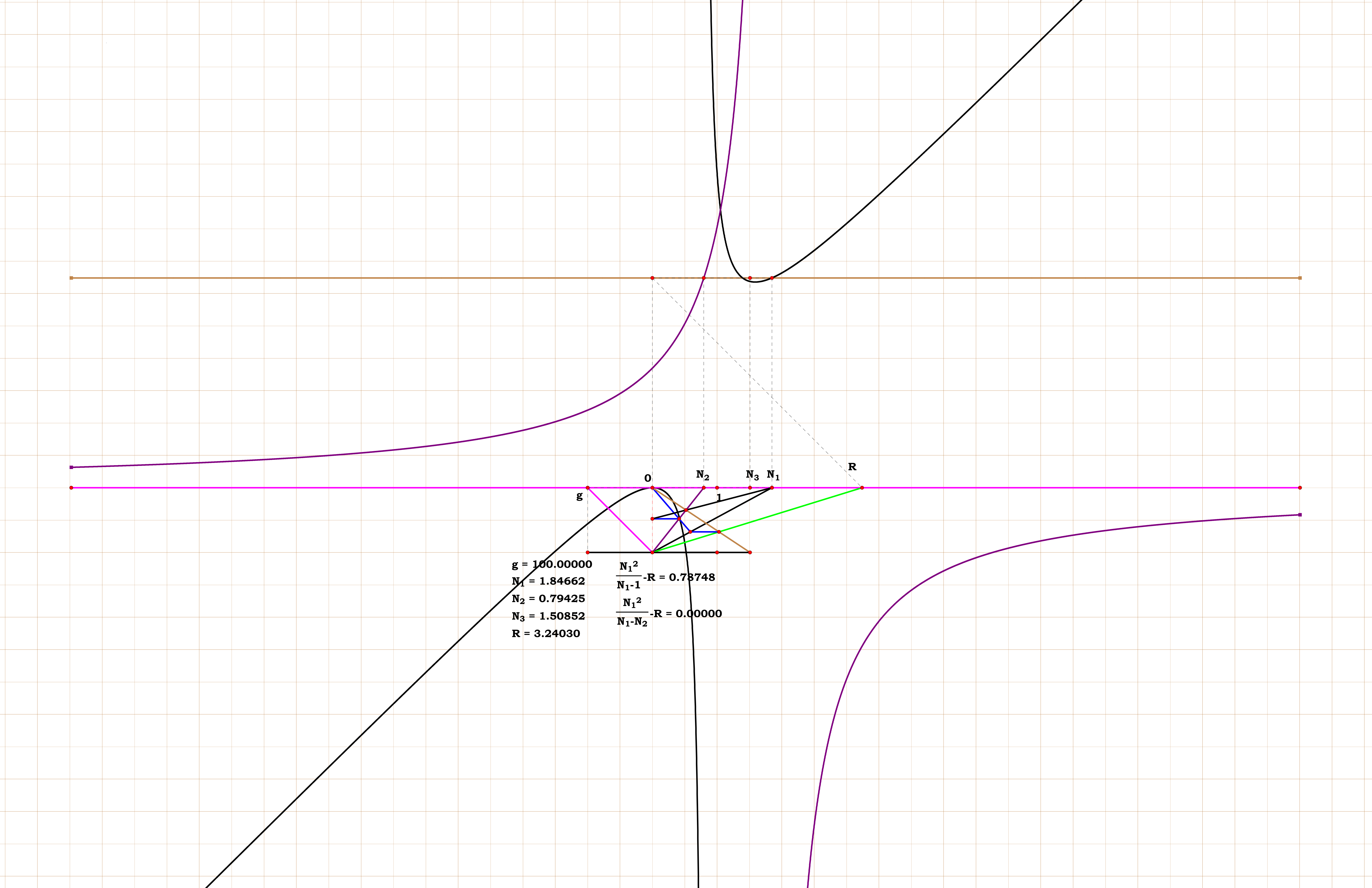
Two Transforms.

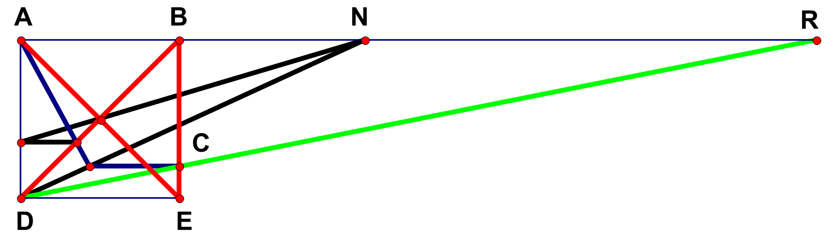
0, 0. undefined

1, 0. $\dfrac{N_1^{\,2}}{N_1 - 1}$

0, 2. $\dfrac{1}{1 - N_2}$

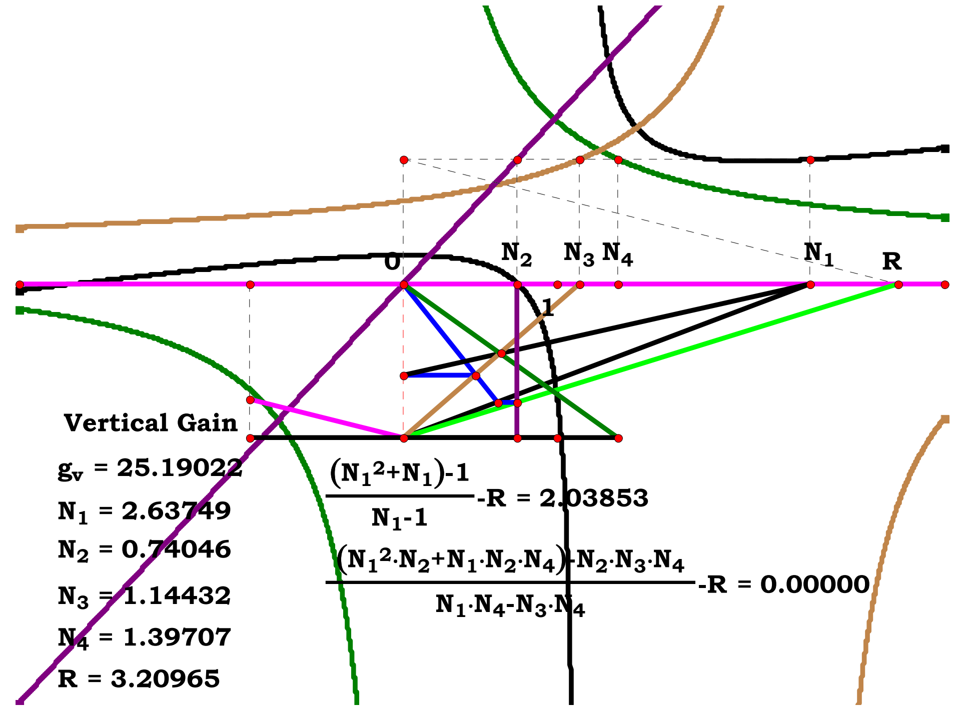
1, 2. $\dfrac{N_1^{\,2}}{N_1 - N_2}$


0
N2
N3
N1
R
g
1
g = 100.00000
N1 = 1.84662
N2 = 0.79425
N3 = 1.50852
R = 3.24030
N1²/(N1-1) -R = 0.78748
N1²/(N1-N2) -R = 0.00000



1CST7R12

Unit. $AB := 1$ **Given.** $AN := 3$

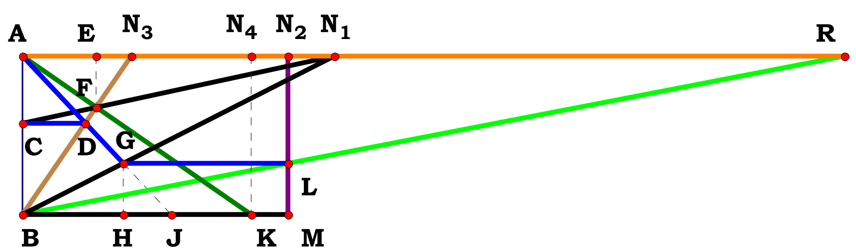


Descriptions.

$$CE := \frac{AN - 1}{AN^2 + AN - 1} \qquad AR := \frac{AB^2}{CE}$$

Definitions.

$$AR - \frac{AN^2 + AN - 1}{AN - 1} = 0$$



$N_1 := 5$

$N_2 := 4$

$N_3 := 3$

$N_4 := 2$

$$ef := \frac{N_3}{N_3 + N_4} \quad ae := N_4 \cdot ef \quad ac := \frac{ef \cdot N_1}{N_1 - ae} \quad cd := N_3 - N_3 \cdot ac \quad bj := \frac{cd}{ac} \quad gh := \frac{bj}{N_1 + bj} \quad ar := \frac{N_2}{gh}$$

Definitions.

$$ae - \frac{N_3 \cdot N_4}{N_3 + N_4} = 0 \quad ac - \frac{N_1 \cdot N_3}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad cd - \frac{N_1 \cdot N_3 \cdot N_4 - N_3^2 \cdot N_4}{N_1 \cdot N_3 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \quad bj - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1} = 0$$

$$gh - \frac{N_1 \cdot N_4 - N_3 \cdot N_4}{N_1^2 + N_1 \cdot N_4 - N_3 \cdot N_4} = 0 \qquad ar - \frac{N_1^2 \cdot N_2 + N_2 \cdot N_1 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4} = 0$$

Four Transforms.

0, 0, 0, 0. undefined

1, 0, 0, 0. $\dfrac{N_1^2 + N_1 - 1}{N_1 - 1}$

0, 2, 0, 0. undefined

0, 0, 3, 0. $\dfrac{N_3 - 2}{N_3 - 1}$

0, 0, 0, 4. undefined

1, 2, 0, 0. $\dfrac{N_2 \cdot N_1^2 + N_2 \cdot N_1 - N_2}{N_1 - 1}$

1, 0, 3, 0. $\dfrac{N_1^2 + N_1 - N_3}{N_1 - N_3}$

1, 0, 0, 4. $\dfrac{N_1^2 + N_4 \cdot N_1 - N_4}{N_1 \cdot N_4 - N_4}$

0, 2, 3, 0. $\dfrac{N_2 \cdot N_3 - 2 \cdot N_2}{N_3 - 1}$

0, 2, 0, 4. undefined

0, 0, 3, 4. $\dfrac{N_4 - N_3 \cdot N_4 + 1}{N_4 - N_3 \cdot N_4}$

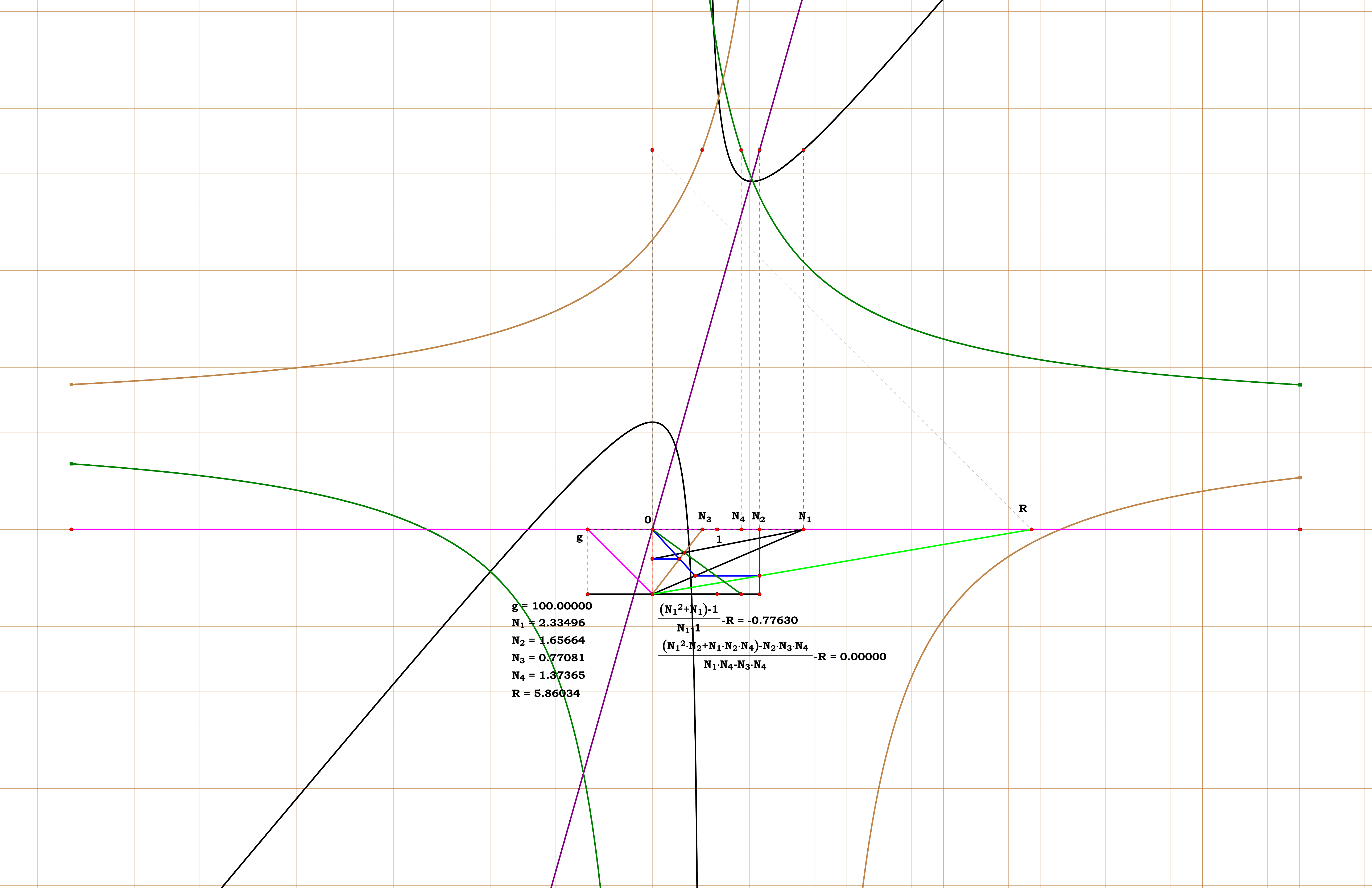
1, 2, 3, 0. $\dfrac{N_2 \cdot N_1^2 + N_2 \cdot N_1 - N_2 \cdot N_3}{N_1 - N_3}$

1, 0, 3, 4. $\dfrac{N_1^2 + N_4 \cdot N_1 - N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4}$

0, 2, 3, 4. $\dfrac{N_2 + N_2 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_4 - N_3 \cdot N_4}$

1, 2, 0, 4. $\dfrac{N_2 \cdot N_1^2 + N_2 \cdot N_4 \cdot N_1 - N_2 \cdot N_4}{N_1 \cdot N_4 - N_4}$

1, 2, 3, 4. $\dfrac{N_1^2 \cdot N_2 + N_2 \cdot N_1 \cdot N_4 - N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4 - N_3 \cdot N_4}$


0
N_3
N_4
N_2
N_1
R
g
1
$g = 100.00000$
$N_1 = 2.33496$
$N_2 = 1.65664$
$N_3 = 0.77081$
$N_4 = 1.37365$
$R = 5.86034$
$\frac{(N_1^2+N_1)-1}{N_1-1}-R = -0.77630$
$\frac{(N_1^2 \cdot N_2+N_1 \cdot N_2 \cdot N_4)-N_2 \cdot N_3 \cdot N_4}{N_1 \cdot N_4-N_3 \cdot N_4}-R = 0.00000$